QCD thermodynamics

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OUTLINE:

- Equation of state and transition temperature
- QCD phase diagram close to the chiral limit
- Charge fluctuations and the RHIC search for the critical point
- Outlook: Thermal QCD on next generation hardware
QCD thermodynamics on appropriate hardware

- the QCD equation of state and calculation of the QCD transition temperature
  requires large zero (and finite) temperature lattices, $48^3 \times 64$ lattice generated on leadership class computers (BlueGene/P) at ALCF using USQCD INCITE resources

- studies of the QCD phase diagram at vanishing chemical potential
  moderate size, finite temperature lattices ($\lesssim 48^3 \times 12$)
  use clusters operated for USQCD at JLab and FNAL

- calculating Taylor expansion coefficients for studies of finite density QCD
  large number of matrix inversions on single gauge field configurations are done on GPUs at JLab (and soon at FNAL)
Thermodynamics projects in 2010/11

Equation of state and the transition temperature:

M. Cheng et al., The finite temperature QCD using 2 + 1 flavors of domain wall fermions at Nt = 8, Phys. Rev. D81, 054510 (2010)


H.-T. Ding et al., Quark number susceptibilities at high temperature, ongoing hotQCD project (INCITE), eg:


W. Soldner, Chiral Aspects of Improved Staggered Fermions with 2+1-Flavors from the HotQCD Collaboration, PoS LATTICE2010, 215 (2010)

- type-A cluster and INCITE projects in 2011/12 (R. Soltz)
- type-B cluster project in 2011/12 (H.-T. Ding)
Thermodynamics projects in 2010/11

QCD at non-zero baryon chemical potential or baryon number:

C. DeTar et al., QCD thermodynamics with nonzero chemical potential at Nt=6 and effects from heavy quarks, Phys. Rev. D81, 114504 (2010)

O. Kaczmarek et al., Phase boundary for the chiral transition in (2+1) -flavor QCD at small values of the chemical potential, Phys. Rev. D83, 014504 (2011)


A. Li, A. Alexandru, K.-F. Liu, Critical point of Nf = 3 QCD from lattice simulations in the canonical ensemble, arXiv:1103.3045 [hep-ph]

- type-A cluster and GPU projects in 2011/12 (S. Mukherjee | P. Hegde)
- type-B cluster project in 2011/12 (A. Li)
...some statistics from SPIRES

- 50 top cited papers in hep-lat during each year in 2006-2010

<table>
<thead>
<tr>
<th>year</th>
<th># Thermo papers (WW)</th>
<th># Thermo papers (US)</th>
<th>rank of the top-cited thermo paper (WW)</th>
<th>rank of the top-cited thermo paper (US)</th>
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<tr>
<td>2006</td>
<td>14</td>
<td>1</td>
<td>2 (MEM)</td>
<td>6 (Quarkonium)</td>
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<td>2007</td>
<td>24</td>
<td>8</td>
<td>2 ((2+1)-f, $T_c$)</td>
<td>4 ((2+1)-f, $T_c$)</td>
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<td>2008</td>
<td>20</td>
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<td>2010</td>
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<td>1 ((2+1)-f, EoS,II)</td>
<td>1 ((2+1)-f, EoS,II)</td>
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</tbody>
</table>

- about 1/3 of the 50 top cited papers in hep-lat deal with topics in QCD thermodynamics
- about 1/3 of these papers involve authors from the US (=USQCD)
most cited US-Thermo papers 2010

1) A. Bazavov et al (hotQCD Collaboration), Equation of state and QCD transition at finite temperature, Phys.Rev.D80, 014504 (2009) [75 citations]


28) O. Kaczmarek and F. Zantow (Bielefeld-BNL), Static quark anti-quark interactions in zero and finite temperature QCD. I. Heavy quark free energies, running coupling and quarkonium binding, Phys. Rev. D71, 114510 (2005) [31 citations]

29) M. Cheng et al (RBC-Bielefeld collaboration), The QCD equation of state for physical quark masses, Phys. Rev. D81, 054504 (2010) [29 citations]


QCD equation of state and the QCD (phase) transition temperature

EoS controls hydrodynamic expansion of matter created in HIC

hotQCD collaboration: systematic study of cut-off and quark mass dependence of thermodynamic observables; use asqtad and HISQ actions with different light/strange quark mass ratios \( \frac{m_l}{m_s} \) and different values of the cut-off \( aT \equiv 1/N_\tau \)

\[ \frac{m_l}{m_s} = 0.1, 0.05, 0.025 \quad \Rightarrow \quad 100\text{MeV} \lesssim m_\pi \lesssim 200\text{MeV} \]

most T>0 lattices on cluster

\[ N_\tau = 6, 8, 12 \]
\[ N_\sigma = 32, 48 \]

large T=0 lattices on BlueGene P (INCITE)

\[ 48^4, 48^3 \times 64 \]
QCD equation of state and the QCD (phase) transition temperature

- **Trace anomaly closer to the continuum limit**

- **Chiral susceptibility**

- **Quark mass and cut-off dependence of the transition temperature**

- **HotQCD vs. Budapest-Wuppertal**

A. Bazavov et al. (HotQCD), PRD80, 014504 (2009)
Thermal Dileptons
probing the structure of the QGP

\[ V = \pi R^2 \Delta y \]

thermal dilepton rate:

\[ \frac{dW}{d\omega d^3p d^4x} = \frac{5}{9} \frac{\alpha_{em}^2}{6\pi^3} \rho_V(\omega, \vec{p}, T) \frac{\omega^2(e^\omega/T - 1)}{\omega^2(e^\omega/T - 1)} \]

thermal emission during expansion:

\[ \frac{dW}{d\omega d^3p} = \pi R^2 \Delta y \frac{5}{9} \frac{\alpha_{em}^2}{6\pi^3} \int_{\tau_0}^{\tau_c} d\tau \frac{\rho_V(\omega, \vec{p}, T(\tau))}{\omega^2(e^\omega/T(\tau) - 1)} \]

future:

need information on temperature and momentum dependence of the vector spectral function.

from EoS using Bjorken model or BETTER 3d-hydro
Transition temperature and O(N) scaling

quark mass dependence of the chiral condensate: fit to O(N) scaling functions

$$z = z_0 \left| \frac{T - T_c}{T_c} \right| \left( \frac{m_l}{m_s} \right)^{-1/\beta \delta}$$

$$\langle \bar{\psi} \psi \rangle \sim m_l^{1/\delta} f_G(z)$$

parameter free comparison with chiral susceptibility asqtad, Nt=8, 12

$$\chi_h \sim m_l^{1/\delta - 1} f_\chi(z)$$

\[
\begin{align*}
\text{asqtad} & \\
N_t & = 12, 0.05 \\
& = 8, 0.05 \\
& = 0.1
\end{align*}
\]
"thermal" fluctuations of the order parameter

\[ t \equiv \frac{1}{t_0} \left( \frac{T}{T_c} - 1 \right) - \kappa_q \left( \frac{\mu_q}{T} \right)^2, \quad z = t/h^{1/\beta\delta} \]

\[ M_b = h^{1/\delta} f_G(z), \quad \chi_{m,q} = \frac{\partial^2 M_b}{\partial (\mu_q/T)^2} = \frac{2\kappa_q}{t_0 T_c} h^{(\beta-1)/\delta\beta} f'_G(z) \]

compare to freeze-out curve:

\[ \kappa_q = 0.059 \pm 0.006 \]
Finite density QCD calculations in the canonical ensemble

Maxwell construction

3-f QCD: simulations at fixed baryon number
determinant ratio

\[
\frac{1}{T} \langle \mu_B \rangle_B = - \ln \langle \gamma(U) \rangle_0 / \langle \alpha(U) \rangle_0
\]

phase!

16^3 \times 4 : 72 parameter sets

A. Li, A. Alexandru, K-F Liu, arXiv:1103.3045

A. Li, A. Alexandru, K-F Liu, X. Meng, PRD82, 054502 (2010)
QCD phase diagram (close to the chiral limit)

RHIC low energy runs:
\[ \sqrt{s} = (9 - 200) \text{GeV}/A \]

- charge fluctuations along the freeze-out line
- higher moments of charge fluctuations, e.g.

Skewness
\[ S_q \equiv \frac{\langle (\delta N_q)^3 \rangle}{\sigma_q^3} \]

Kurtosis
\[ \kappa_q \equiv \frac{\langle (\delta N_q)^4 \rangle}{\sigma_q^4} - 3 \]
Mean, variance, skewness & kurtosis

STAR results from RHIC low energy run

Generalized Quark number susceptibilities

- Taylor expansion of the pressure

\[ \frac{p}{T^4} = \sum_{n=0}^{\infty} \frac{1}{n!} \chi_{B,0}^{(n)}(T) \left( \frac{\mu_B}{T} \right)^n \]

- Generalized quark number susceptibilities

\[ \chi_{B,0}^{(n)} = \frac{1}{VT^3} \left. \frac{\partial^n \ln Z}{\partial (\mu_B/T)^n} \right|_{\mu_B=0} \]

- Taylor expansion of quark number susceptibilities

\[ \chi_{B,\mu}^{(n)} = \sum_{k=0}^{\infty} \frac{1}{k!} \chi_{B,0}^{(k+n)}(T) \left( \frac{\mu_B}{T} \right)^k \]

Skewness

\[ S_B \sigma_B \equiv \frac{\chi_{B,\mu}^{(3)}}{\chi_{B,\mu}^{(2)}} \]

Kurtosis

\[ \kappa_B \sigma_B^2 \equiv \frac{\chi_{B,\mu}^{(4)}}{\chi_{B,\mu}^{(2)}} \]
The RHIC low energy runs

Moments of charge fluctuations

charge fluctuations at freeze-out agree with HRG model predictions and lattice calculations
Higher moments of charge fluctuations at RHIC and LHC

Higher moments (e.g. 6th order) are drastically different in QCD close to criticality and in a hadron resonance gas, e.g.

\[
\frac{\chi_B^{(6)}}{\chi_B^{(2)}} = \begin{cases} 
1 & \text{hadron resonance gas} \\
< 0 & \text{QCD at the crossover transition}
\end{cases}
\]

\[
\chi_B^{(6)} \sim A_\pm \left| \frac{T - T_c}{T_c} \right|^{-(1-\alpha)}
\]

green = regions of negative sixth order moments

B. Friman et al., arXiv:1103.3511;

M. Cheng et al., PRD 79 (2009) 074505
Outlook: Thermodynamics on next generation hardware

- Getting ready for petascale-thermodynamics
  - Thermodynamics with chiral fermions
  - Spectral properties of light and heavy quark correlation functions

State-of-the-art: Quenched QCD on large thermal lattices

128^3 \times 48

Thermodynamics with Domain Wall Fermions

- Using a chiral fermion formulation for thermodynamic calculations allows to perform a more rigorous analysis of the role of the axial $U_A(1)$ symmetry close to $T_c$.

- Chiral fermions have an exact $SU_L(2) \times SU_R(2)$ flavor symmetry.

ongoing project of hotQCD on BG/P;

preparatory work of RIKEN/BNL for thermodynamics on BlueGene/Q

**Chiral susceptibility**

- DWF, DSDR, $16^2, L_\pi=32, 48$, $m_\pi=200$ MeV
- DWF, DSDR, $32^3, L_\pi=32$, $m_\pi=200$ MeV
- $N_t=12$, Asqtad, $m_s/m_{\pi}=0.05$, $m_\pi=179$ MeV
- $N_t=8$, HISQ/tree, $m_s/m_{\pi}=0.05$, $m_\pi=160$ MeV

**Topology and axial anomaly**

- $0.5(PS+S)$
- $\Omega_{top}(+4)$
- $180$ MeV
Thermal Dilepton and Photon rates and transport coefficients in the QGP

Thermal dilepton rate:

\[
\frac{dW}{d\omega d^3p} = \frac{5}{9} \frac{\alpha^2_{em}}{6\pi^3} \rho_v(\omega, \vec{p}, T) \frac{\omega^2(e^{\omega/T} - 1)}
\]

Thermal Dilepton and Photon rates

and transport coefficients in the QGP

Vector meson correlation function:

\[ G_V(\tau, \vec{p}, T) = \int_0^\infty \frac{d\omega}{2\pi} \rho_V(\omega, \vec{p}, T) \frac{\cosh(\omega(\tau - 1/2T))}{\sinh(\omega/2T)} \]

\[ \vec{p} = 0 , \; T = 1.5T_c \]

quenched QCD, clover action:

- volume dependence \( N_{\sigma}^3 \times 16 , \; N_{\sigma} = 48 - 128 \)
- cut-off dependence \( 128^3 \times N_\tau , \; N_\tau = 16 - 48 \)
- quark mass dependence \( m_{q_{\overline{MS}}} / T = 0.02, 0.11 \)

BNL-Bielefeld-GSI, Vector current correlator in quenched QCD, Phys.Rev. D83 (2011) 034504
Thermal Dileptons & electrical conductivity

Thermal dilepton rate:

\[
T = 1.5 T_c \\
\frac{dW}{d\omega d^3p} = \frac{5}{9} \frac{\alpha_{em}^2}{6\pi^3} \frac{\rho_V(\omega, \vec{p}, T)}{\omega^2(e^{\omega/T} - 1)}
\]

\[
\frac{\sigma}{T} = \lim_{\omega \to 0} \frac{\rho_{ii}(\omega)}{(6\omega T)}
\]

- vector spectral function
- HTL resummation, perturbation theory
- Breit-Wigner ansatz
- low energy enhancement

arXiv:0912.0244v1
Conclusions

LGT calculations start to produce quantitative predictions on QCD thermodynamics that provide input to the interpretation of heavy ion experiments

- EoS, Tc, transport coefficients, spectral functions, phase boundary, charge fluctuations,.....

How sensitive is the QCD transition with physical quark masses to universal properties at the chiral phase transition?
How does the phase transition vary with chemical potentials?

Use staggered fermion action with reduced taste violation (HISQ) with physical quark masses close to the cont. limit
Use chiral fermion formulations (DWF) in thermodynamic calculations
QCD equation of state and the QCD (phase) transition temperature

trace anomaly and its parametrization from L-QCD

\[(\varepsilon-3p)/T^4\]

- p4, \(N_t=6\)
- p4, \(N_t=8\)
- asqtad, \(N_t=8\)

A. Bazavov et al. (HotQCD), PRD80, 014504 (2009)

hydrodynamic modelling of elliptic flow spectra

\[v_2\]

- s95p-v1
- EoS Q
- STAR (open)
- PHENIX (filled)

P. Huovinen, P. Petreczky, NP A837, 26 (2010)