Quark Spin from Anomalous Ward Identify and Chiral Axial-vector Current

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USQCD all hands meeting, April 29 2017
Definitions

The vector conserved current for the wilson/clover action,

\[ \bar{\psi}_y \frac{1 - \gamma_\mu}{2} U_\mu(y) \psi_{y+\bar{\mu}} - \bar{\psi}_{y+\bar{\mu}} U_\mu^+(y) \frac{1 + \gamma_\mu}{2} \psi_y \]

with

\[ \partial_\mu^* \left[ \bar{\psi}_y \frac{1 - \gamma_\mu}{2} U_\mu(y) \psi_{y+\bar{\mu}} - \bar{\psi}_{y+\bar{\mu}} U_\mu^+(y) \frac{1 + \gamma_\mu}{2} \psi_y \right] = 0. \]

\( \partial_\mu^* \) is the backward lattice derivative.

That for the overlap fermion is,

\[ V^a_\mu(x) = J^a_R(x) + J^a_L(x) = \bar{\psi} \left( P_L K_\mu(x) \hat{P}_R + P_R K_\mu(x) \hat{P}_L \right) T^a \psi \]

\[ = \frac{1}{2} \bar{\psi} \left( K_\mu(x) - \gamma_5 K_\mu(x) \hat{\gamma}_5 \right) T^a \psi, \]

with

\[ K_\mu(x) = -i \left. \frac{\delta D(U^{(\alpha)}_\mu)}{\delta \alpha_\mu(x)} \right|_{\alpha=0} \]

and

\[ \hat{\gamma}_5 = \gamma_5 (1 - D). \]

\[ U_\mu(x) \rightarrow U^{(\alpha)}_\mu(x) = e^{i \alpha_\mu(x)} U_\mu(x) \]
Definitions

**Exact chiral axial-vector current** can be defined with **chiral fermions**,

\[ A^a_\mu(x) = J^a_{R_\mu}(x) - J^a_{L_\mu}(x) = \overline{\psi} \left( P_L K_\mu(x) \hat{P}_R - P_R K_\mu(x) \hat{P}_L \right) T^a \psi \]
\[ = \frac{1}{2} \overline{\psi} (-\gamma_5 K_\mu(x) + K_\mu(x) \gamma_5) T^a \psi , \]

which satisfies the **Anomalous Ward Identity (AWI)**,

\[ \left\{ - \left[ \frac{\delta^a_\mu \mathcal{O}}{\delta \epsilon(z)} \right]_F - \left[ \mathcal{O} \partial^*_\mu A^a_\mu(x) \right]_F + 2m \left[ \mathcal{O} P^a(x) \right]_F - \delta^a_0 2N_f \left[ \mathcal{O} q(x) \right]_F \right\}_F = 0 \]

where

\[ P^a(x) = \overline{\psi} \left[ \frac{1}{2} E(x) \gamma_5 \left( 1 - \frac{1}{2} D \right) + \frac{1}{4} \left( 1 - \frac{1}{2} D \right) \left( \gamma_5 E(x) + E(x) \gamma_5 \right) \right] T^a \psi \]

and

\[ q(x) = \frac{1}{2} \text{tr}(\gamma_5 D(x, x)) \]

\(\mathcal{O}\) can be an arbitrary operator, likes the meson interpolation field or nucleon correlators, etc..
Why we need the chiral axial vector current?

On DWF $2+1$ $24^3 \times 64$ lattice with $a=0.111(3)$ fm:

- $Z_V$ from the vector charge is $1.096(6)$, 
  
  Yi-Bo Yang, et. al, $\chi$QCD collaboration, Phys.Rev. D93 (2016) no.3, 034503

- $Z_A$ from Ward identity in pion two point function is $1.105(4)$,
  

- But our improved current work shows that additional operator is needed to make the axial charge from $A_i$ and $A_4$ to be the same.
  
  Jian Liang et. al, $\chi$QCD collaboration, arXiv: 1612.04388

\[ J_A^\mu = Z_A \left( \bar{\psi} \gamma_5 \gamma_\mu \psi + f' \partial_\mu (\bar{\psi} \gamma_5 \psi) + g \bar{\psi} \gamma_5 \sigma_{\mu\nu} \overleftrightarrow{D}_\nu \psi \right) \]

- No effect in the forward matrix element
- The $O(a)$ correction terms
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- The improved current can enlarge the iso-vector $g_A$ by 2%.
- We will check the same quantity with the chiral axial vector current

\[
A^a_\mu(x) = J^a_{R\mu}(x) - J^a_{L\mu}(x) = \bar{\psi} \left( P_L K_\mu(x) \hat{P}_R - P_R K_\mu(x) \hat{P}_L \right) T^a \psi = \frac{1}{2} \bar{\psi} (-\gamma_5 K_\mu(x) + K_\mu(x) \gamma_5) T^a \psi,
\]

To confirm:
1. **whether** the result is also **larger** than that with the local current.
2. **whether** the value from $A_i$ and $A_4$ can be **the same**.
Why we need the chiral axial vector current?

From the Anomalous Ward Identity (AWI),

\[ \kappa_A \partial^\mu A^0_\mu = 2 \sum_{f=1}^{N_f} m_f \bar{q}_f \gamma_5 q_f + N_f 2q \]

we can have the following relation,

\[ \langle ps | A_\mu | ps \rangle s_\mu = \lim_{\vec{q} \to 0} \frac{i|\vec{s}|}{\vec{q} \cdot \vec{s}} \langle p', s | 2m_f \mathcal{P} + 2i q | p, s \rangle \]

\[ = 2m_f \langle p, s | \int d^3x \bar{x} \cdot s \mathcal{P}(x) | p, s \rangle + 2i \langle p, s | \int d^3x \bar{x} \cdot s q(x) | p, s \rangle \]

\[ \kappa_A \sim 1.36 \]
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$$= 2m_f \langle p, s | \int d^3 x \; \bar{x} \cdot \bar{s} \; \mathcal{P}(x) | p, s \rangle + 2i \langle p, s | \int d^3 x \; \bar{x} \cdot \bar{s} \; q(x) | p, s \rangle$$

- The improved current can enlarge the strange spin by 36%.
- We will check the AWI with the chiral axial vector current to confirm:
  1. whether $\kappa_A = 1$ in such a case.
  2. whether the AWI also holds perfectly in the connected insertion case.
The test

with the conserved vector current

The vector conserved current for the overlap action,

\[ V_\mu^a(x) = J^a_{R\mu}(x) + J^a_{L\mu}(x) = \overline{\psi} \left( P_L K_\mu(x) \hat{P}_R + P_R K_\mu(x) \hat{P}_L \right) T^a \psi \]

\[ = \frac{1}{2} \overline{\psi} (K_\mu(x) - \gamma_5 K_\mu(x) \gamma_5) T^a \psi , \]

With the conversed vector current:

- The matrix elements with \( V_4 \) are perfectly agree with the vector charges of u/d in proton,
  - except the possible counteract term effect in the boundary \( t=10 \).
  - We are still working on this!

Andrei Alexandru, et. al, \( \chi \)QCD collaboration, in preparation.
The **Exact chiral axial-vector current** can be constructed with the **chiral fermion**.

We need the chiral axial vector current to control the **systematic uncertainties** from kinds of the **discrepancies** we confirmed with the **local currents**.

The **kernel** needed by the vector conserved current has been **constructed** and we are making **progress on the tests**.