Spin and the 3D Structure of the Nucleon

Huey-Wen Lin
Lattice Parton Physics Project (LP3)

https://www.pa.msu.edu/~hwlin/LP3/

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Parton Distribution Functions

PDFs are universal quark/gluon distributions of nucleon

Many ongoing/planned experiments
(BNL, JLab, J-PARC, COMPASS, GSI, EIC, LHeC, ...)

Electron Ion Collider:
The Next QCD Frontier

Imaging of the proton

How are the *sea* quarks and gluons, and their spins, distributed in space and momentum inside the nucleon?

EIC White Paper, 1212.1701
PDFs are universal quark/gluon distributions of nucleon

- Many ongoing/planned experiments
  (BNL, JLab, J-PARC, COMPASS, GSI, EIC, LHeC, ...)

Important inputs to discern new physics at LHC
- Currently dominate errors in Higgs production

### Parton Distribution Functions

<table>
<thead>
<tr>
<th>Process</th>
<th>Cross Section (pb)</th>
<th>Uncertainty (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( gg \to H )</td>
<td>19.5</td>
<td>14.7</td>
</tr>
<tr>
<td>VBF</td>
<td>1.56</td>
<td>2.9</td>
</tr>
<tr>
<td>WH</td>
<td>0.70</td>
<td>3.9</td>
</tr>
<tr>
<td>ZH</td>
<td>0.39</td>
<td>5.1</td>
</tr>
<tr>
<td>( ttH )</td>
<td>0.13</td>
<td>14.4</td>
</tr>
</tbody>
</table>

(J. Campbell, HCP2012)
Discrepancies appear when data is scarce
Many groups have tackled the analysis

CTEQ, MSTW, ABM, JR, NNPDF, etc.

A first joint workshop with global-fitting community to address key LQCD inputs

http://www.physics.ox.ac.uk/confs/PDFlattice2017

Whitepaper will study the needed precision of lattice PDFs in the large-\(x\) region
A Promising New Direction
Long existing obstacles!

§ Lattice calculations rely on operator product expansion, only provide moments \( \langle x^n \rangle \)

\[
\langle x^n \rangle_q = \int_{-1}^{1} dx \; x^n q(x)
\]

§ For higher moments, all ops mix with lower-dimension ops

 No practical proposal to overcome this

New Strategy (LaMET):

§ Calculate finite-momentum boosted quark distribution

 In \( P_z \to \infty \) limit, parton distribution is recovered

 For finite \( P_z \), corrections are needed

§ Feasible with today’s resources!

Large-Momentum Effective Theory for PDFs

\[
\int \frac{dz}{4\pi} e^{-izk_z} \left\langle P \left| \overline{\psi}(z) \Gamma \exp\left( -ig \int_0^z dz' A_z(z') \right) \psi(0) \right| P \right\rangle
\]

\[
P^2 = 0 \quad \Gamma \in \{ \gamma_z, \gamma_z \gamma_5, \gamma_z \gamma_\perp \gamma_5 \}
\]

\[
q(x, \mu) = \tilde{q}(x, \mu, P_z) + \mathcal{O}(\alpha_s) + \mathcal{O}\left( \frac{M_N^2}{P_z^2} \right) + \mathcal{O}\left( \frac{\Lambda_{QCD}^2}{P_z^2} \right)
\]

X. Xiong et al., 1310.7471; J.-W. Chen et al, 1603.06664
First time in LQCD history to study antiquark distribution!

$\mathcal{M}_\pi \approx 310$ MeV

$\bar{q}(x) = -q(-x)$

Lost resolution in small-$x$ region

Future improvement: larger lattice volume

$$\int dx \left( \bar{u}(x) - \bar{d}(x) \right) \approx -0.16(7)$$

<table>
<thead>
<tr>
<th>Experiment</th>
<th>$x$ range</th>
<th>$\int_0^1 [\bar{d}(x) - \bar{u}(x)] dx$</th>
</tr>
</thead>
<tbody>
<tr>
<td>E866</td>
<td>$0.015 &lt; x &lt; 0.35$</td>
<td>$0.118 \pm 0.012$</td>
</tr>
<tr>
<td>NMC</td>
<td>$0.004 &lt; x &lt; 0.80$</td>
<td>$0.148 \pm 0.039$</td>
</tr>
<tr>
<td>HERMES</td>
<td>$0.020 &lt; x &lt; 0.30$</td>
<td>$0.16 \pm 0.03$</td>
</tr>
</tbody>
</table>

R. Towell et al. (E866/NuSea), Phys.Rev. D64, 052002 (2001)
§ First time in LQCD history to study antiquark distribution!

\[ M_\pi \approx 310 \text{ MeV} \]

\[ \bar{q}(x) = -q(-x) \]

Lost resolution in small-\(x\) region:

\[ \bar{d}(x) \approx -0.16(7) \]

A milestone for lattice QCD!

First sea flavor asymmetry ever calculated!

Huey-Wen Lin — USQCD All Hands' Meeting
§ Lattice exploratory study

\[ M_\pi \approx 310 \text{ MeV} \]

Compared with E866
Too good to be true?
Lost resolution in small-\(x\) region

 Similar results repeated by ETMC, at \( M_\pi \approx 373 \text{ MeV} \)

\[ \frac{1}{d} \approx -0.16 \pm 0.03 \]

\( x \)

R. Towell et al. (E866/NuSea), Phys.Rev. D64, 052002 (2001)
\[ \int dx \left( \delta \bar{u}(x) - \delta \bar{d}(x) \right) \approx -0.10(8) \]

The calculation was also repeated by ETMC, at \( M_\pi \approx 373 \text{ MeV} \)

ETMC, 1610.03689

\[ \delta \bar{q}(x) = -\delta q(-x^\dagger) \]

1505.05589; 1503.03495

\( M_\pi \approx 310 \text{ MeV} \)

P. Schweitzer et al., PRD 64, 034013 (2001)

\[ \int dx \left( \delta \bar{u}(x) - \delta \bar{d}(x) \right) \approx -0.082 \]

Huey-Wen Lin — USQCD All Hands’ Meeting
Progress Last Year: Systematics
§ Improved quasi-quark distribution

\[ \tilde{q}_{\text{imp}}(x, \Lambda, p_z) = \int_{-\infty}^{\infty} \frac{dz}{4\pi} e^{izk_z - \delta m|z|} \langle p | \bar{\psi}(0, 0, z) \gamma_z L(z, 0) \psi(0) | p \rangle \]

§ Wilson-line renormalization to remove power divergence

\[ a \approx 0.09 \text{ fm}, \; L \approx 6 \text{ fm}, \; M_\pi \approx 130 \text{ MeV}, \; \text{clover/HISQ} \]
§ Ongoing investigation of renormalization

Investigating RI’MOM scheme renormalization (Yang)
RI’MOM to \( \overline{\text{MS}} \) matching (Zhao)

§ Preliminary result

Test case: \( a \approx 0.12 \) fm, \( M_\pi \approx 310 \) MeV, clover/HISQ

Plot by Yi-Bo Yang
2017/18 Allocation
Proposed Calculation

§ Proposed ensemble

\[ N_f = 2 + 1 + 1 \] clover/HISQ lattices (MILC)
\[ M_\pi \approx 130 \text{ MeV}, \ a \approx 0.09 \text{ fm (} L \approx 6 \text{ fm)} \]

§ Proposed physics case

\[ \text{At least } 3 \ t_{\text{sep}}, \text{ multiple mom. source smearing} \]
\[ \text{Extended unpol. PDFs to larger } p_z \ (w/\text{mom. source}) \]
\[ \text{and polarized structure (helicity + transversity)} \]
\[ \text{First LQCD study of } x\text{-dep’}t \text{ GPDs (3D Spatial Maps of the Nucleon)} \]

\[
F_q(x, \xi, t) = \int \frac{dz^-}{4\pi} e^{i\mathbf{p}^+ \mathbf{z}^-} \langle \bar{q}(\mathbf{z}) \gamma^+ L(-\frac{z^-}{2}, \frac{z^+}{2}, \frac{z}{2}) q(\mathbf{z}) | p' \rangle_{z^+ = 0, z_\perp = 0} \\
= \frac{1}{2p^+} \left[ H(x, \xi, t) \bar{u}(p'') \gamma^+ u(p') + E(x, \xi, t) \bar{u}(p'') \frac{i\sigma^{\nu\mu} \Delta^\nu}{2m} u(p') \right] \\
p^\mu = \frac{p''^\mu + p'^\mu}{2}, \quad \Delta^\mu = p''^\mu - p'^\mu, \quad t = \Delta^2, \quad \xi = \frac{p''^+ - p'^+}{p''^+ + p'^+} 
\]

“The Tomography of the Nucleon — Spatial Imaging of Gluons and Sea Quarks”
§ Proposed ensemble

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\[ M_\pi \approx 130 \text{ MeV, } a \approx 0.09 \text{ fm (} L \approx 6 \text{ fm)} \]

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