Neutron electric dipole moment from lattice QCD with the theta term

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Motivations

• Baryon asymmetry in the universe suggests other sources of CP-violation besides the weak CP phase, BSM high dimensional terms or the “strong CP-violation” coming from the $\theta$ term?

• Direct consequence of “strong CP-violation”, nEDM, has not yet been discovered by experiments.

• The recent upper limit is $2.9 \times 10^{-13}$ e·fm and hopefully the next generation of experiments will detect it at some stage.

• We will focus on the 4-dimensional $\theta$ term in this project.

• An *ab-initio lattice QCD* study provides quantificational relation between the nEDM and the parameter $\theta$, help us to understand more about the “strong CP-violation”.

*C.A. Baker et al., PRL 97, 131801 (2006)*
Motivations

• The CP-odd form factor $F_3$ is noisy at large volume, so far most lattice nEDM calculations are done on relatively small volumes with $L_s \leq 2.6$ fm.

• The only large-volume study ($L_s = 4.6$ fm) gives two-sigma result for the nEDM and one-sigma result for the proton one.

• Additionally, it is recently claimed that all the previous lattice results of nEDM need to be corrected by

$$F'_3 = F_3 + 2 \alpha_1 F_2$$

• All the previous results are reduced to one-sigma signal or less after this correction according to their estimation.

• It becomes more significant to reduce the error of lattice calculation.

E. Shintani et al., PRD 93, 094503 (2016)

M. Abramczyk et al., arXiv:1701.07792
Disconnected correlations

DI of nucleon states with a scalar loop

\[ C_3(R_s, \tau) = \langle \sum_{\vec{x}} \sum_{|\vec{r}|<R_s} \mathcal{O}_N(\vec{x}, t) S(\vec{x} + \vec{r}, \tau) \bar{\mathcal{O}}_N(\mathcal{G}, 0) \rangle \]

The correlation decays exponentially.

The error of correlation keeps constant.
Cluster decomposition principle

The signal decays exponentially.

\[ |\langle 0 | \mathcal{B}_1(x_1) \mathcal{B}_2(x_2) | 0 \rangle_s| \leq Ar^{-\frac{3}{2}} e^{-Mr} \]


The noise keeps constant.

vacuum contribution

\[ V_{ar} = \langle \sum \sum \mathcal{B}_1 \dagger \mathcal{B}_1 (\vec{x} + \vec{r}, t) \mathcal{B}_2 \dagger \mathcal{B}_2 (\vec{x}, 0) \rangle \]

\[ = \sum \sum \langle \mathcal{B}_1 \dagger \mathcal{B}_1 (\vec{x} + \vec{r}, t) \rangle \langle \mathcal{B}_2 \dagger \mathcal{B}_2 (\vec{x}, 0) \rangle + \ldots \propto VV_{R,S} \]

K.F. Liu, J. Liang, Y.B. Yang., submitting to arXiv

After certain distance, the signal falls below the noise.

S/N can be improved by the cutoff

\[ \frac{S/N(R_s)}{S/N(L)} \sim \sqrt{\frac{V}{V_{R,s}}} \]
Cluster decomposition in the EDM calculation

To calculate \( \alpha \):

\[
C^Q(t) = \sum_{\bar{x}} \langle S_N(\bar{x}, t) \rangle
\]

\[
C^Q(t) = \sum_{\bar{x}} \langle S_N(\bar{x}, t) \sum_y q(y) \rangle = \sum_{\bar{x}} \langle S_N(\bar{x}, t) \sum_r q(x + r) \rangle
\]

\[
C^Q(t, R) = \sum_{\bar{x}} \langle S_N(\bar{x}, t) \sum_{|r| \leq R} q(x + r) \rangle
\]

The double-summation is handled by convolution theorem plus FFT effectively.
Preliminary result on 24I lattice

Preliminary result on the small 24I lattice after the correction.

\[ R = 28a \]

\[ d_N = \theta F_3(0)/2m_N = -0.054(25)\theta \text{ e}\cdot\text{fm} \]

- There are indeed some improvements even on this small lattice.
- More statistics are on the way.
- The sea quark mass dependence will be studied with new lattices.
Proposed project

- In the next year, we would like to concentrate on the project of calculating the neutron electric dipole moment on the RBC/UKQCD 32ID lattice.
- We will use the local topological charge defined from the overlap operator and compare it with other definitions.
- We will calculate the CP-violation phase and the CP-odd form factor using our new method on this larger lattice.
- Our goal is to have more than 4-sigma result on the 32ID lattice around the physical pion mass, then carry out the continuum extrapolation and study the sea quark mass dependence using the results from both the 24I and 32ID lattices.

<table>
<thead>
<tr>
<th>name</th>
<th>size</th>
<th>M_pi (sea)</th>
<th>a</th>
<th>Ls</th>
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</thead>
<tbody>
<tr>
<td>32ID</td>
<td>32^3*96</td>
<td>~170 MeV</td>
<td>0.14 fm</td>
<td>4.6 fm</td>
</tr>
</tbody>
</table>

R. Arthur et al., PRD87, 094514 (2013)
Resource requirement

• For the local topological charge calculation we need 0.32 million Jpsi-equivalent core hours on 12s.

• For the 3-point functions part we need 11.04 million Jpsi-equivalent core hours on 12s.

• For the 4-point functions part we apply for 32.40 million Jpsi-equivalent core hours on the KNL cluster.

• As far as storage is concerned we will need 110 TB disk space for the production.

• All together, we request ~ 44 million Jpsi-equivalent core hours on 12s and the KNL cluster at JLAB. We also request equivalently $110 \times 40K = 4.4$ million Jpsi core hours for disk space.
Thank you!
Use of overlap fermion

Theoretical side:

Both the valence and sea are chiral fermions satisfying the Ginsparg-Wilson relation, they do not have $O(a)$ errors and the $O(a^2)$ errors in many physical quantities studied so far turn out to be small.

The overlap operator can be used to define the topological charge which is related to the fermion zero modes through the Atiya-Singer theorem, such that the topological susceptibility approaches zero when the quark mass approaches zero at finite lattice spacing.

Numerical side:

Deflation and Multi-mass algorithm to accelerate the inversions.

LMS for the nucleon propagator and connected 3-point functions.

LMA for the quark loops.

With the above improvements, the overlap fermions can be as efficient as non-chiral fermions in calculating the three-point functions.

Keh-Fei Liu et al., arXiv: 1702.04384