LATTICE SUSY

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DESPERATELY SEEKING SUSY (ELISE TOO…)

WILSON FERMION N=4

Can impose SO(4), gauge invariance

$$
\begin{aligned}
S \;=\; & \int d^4x \; \mathrm{Tr}\Big\{ \frac{1}{2g_r^2} F_{\mu\nu}F_{\mu\nu} + \frac{i}{g_r^2}\overline{\lambda}_i \overline{\sigma}^\mu D_\mu \lambda_i + \frac{1}{g_r^2} D_\mu\phi_m D_\mu\phi_m + m_\phi^2 \phi_m\phi_m \\
& + m_\lambda(\lambda_i\lambda_i + \overline{\lambda}_i\overline{\lambda}_i) + \kappa_1 \phi_m\phi_m\phi_n\phi_n + \kappa_2 \phi_m\phi_n\phi_m\phi_n + y_1(\lambda_i[\phi_{ij},\lambda_j] + \overline{\lambda}_i[\phi_{ij},\overline{\lambda}_j]) \\
& + y_2 \epsilon_{ijkl}(\lambda_i[\phi_{jk},\lambda_l] + \overline{\lambda}_i[\phi_{jk},\overline{\lambda}_l]) \Big\} \\
& + \int d^4x \; \Big\{ \kappa_3 (\mathrm{Tr}\phi_m\phi_m)^2 + \kappa_4 \mathrm{Tr}\phi_m\phi_n \mathrm{Tr}\phi_m\phi_n \Big\}
\end{aligned}
$$

8-dimensional parameter space to fine-tune in

Notice rescaling of fields exploited for first three terms---we do that in our twisted theory too.

The twisted, Q invariant lattice action takes the form

$$
\begin{aligned}
S &= \frac{1}{2g^2}(\mathcal{Q}\lambda + S_{\text{closed}}) \\
\lambda &= \sum_x a^4 \text{Tr}(\chi_{ab}\mathcal{F}_{ab} + \eta\overline{\mathcal{D}}_a^{(-)}\mathcal{U}_a - \frac{1}{2}\eta d) \\
S_{\text{closed}} &= -\frac{1}{4}\sum_x a^4 \epsilon_{abcde}\text{Tr}\chi_{de}\overline{\mathcal{D}}_c^{(-)}\chi_{ab}(x)
\end{aligned}
$$

$$\mathcal{Q}S = 0$$

Observes the notion that anything correct should be simple.
Four terms --- will result in four coefficients to fine tune.

WHAT'S THE DIFFERENCE?

Of course we have to say how the derivatives are implemented:

$$\begin{aligned}
\mathcal{F}_{ab}(x) &= \mathcal{D}_a^{(+)}\mathcal{U}_b(x) = \mathcal{U}_a(x)\mathcal{U}_b(x+e_a) - \mathcal{U}_b(x)\mathcal{U}_a(x+e_b) \\
\overline{\mathcal{D}}_a^{(-)}\mathcal{U}_a(x) &= \mathcal{U}_a(x)\overline{\mathcal{U}}_a(x) - \overline{\mathcal{U}}_a(x-e_a)\mathcal{U}_a(x-e_a) \\
\epsilon_{abcde}\chi_{de}\overline{\mathcal{D}}_c^{(-)}\chi_{ab}(x) &= \epsilon_{abcde}\chi_{de}(x+e_a+e_b)[\chi_{ab}(x)\overline{\mathcal{U}}_c(x-e_c) \\
&\quad -\overline{\mathcal{U}}_c(x-e_c+e_a+e_b)\chi_{ab}(x-e_c)]
\end{aligned}$$

$$\mathcal{U}_a = \frac{1}{a} + \mathcal{A}_a \qquad\qquad e_a \leftrightarrow A_4^*$$

$$gl(N,\mathbb{C}) \qquad \mathcal{A}_a = A_a + iB_a$$

SCALAR SUPERSYMMETRY

The supersymmetry transformation is:

$$\mathcal{Q}\mathcal{U}_a = \psi_a, \quad \mathcal{Q}\psi_a = 0, \quad \mathcal{Q}\overline{\mathcal{U}}_a = 0$$
$$\mathcal{Q}\chi_{ab}(x) = -\overline{\mathcal{F}}_{ab}(x) \equiv \overline{\mathcal{U}}_b(x+e_a)\overline{\mathcal{U}}_a(x) - \overline{\mathcal{U}}_a(x+e_b)\overline{\mathcal{U}}_b(x)$$
$$\mathcal{Q}\eta = d, \quad \mathcal{Q}d = 0$$

BIANCHI IDENTITY FOR Q CLOSED TERM

$$\epsilon_{mncde}\overline{\mathcal{D}}_c^{(-)}\overline{\mathcal{F}}_{mn}(x+e_c) = 0$$

$$\epsilon_{mncde}\overline{\mathcal{D}}_c^{(-)}\overline{\mathcal{F}}_{mn}(x+e_c) = \epsilon_{mncde}[\overline{\mathcal{F}}_{mn}(x+e_c)\overline{\mathcal{U}}_c(x) - \overline{\mathcal{U}}_c(x+e_m+e_n)\overline{\mathcal{F}}_{mn}(x)]$$

$$\overline{\mathcal{F}}_{mn}(x) = \overline{\mathcal{U}}_n(x+e_m)\overline{\mathcal{U}}_m(x) - \overline{\mathcal{U}}_m(x+e_n)\overline{\mathcal{U}}_n(x)$$

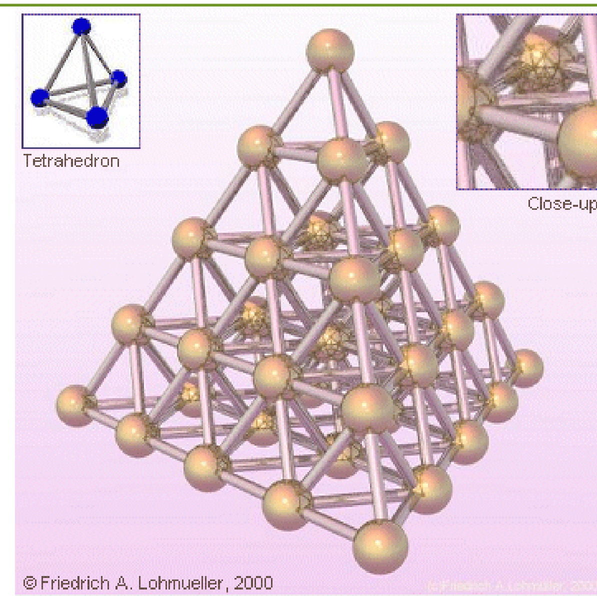
then algebra

PRIMITIVE VECTORS

$$
\begin{aligned}
e_1 &= \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{12}}, \frac{1}{\sqrt{20}}\right) \\
e_2 &= \left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{12}}, \frac{1}{\sqrt{20}}\right) \\
e_3 &= \left(0, -\frac{2}{\sqrt{6}}, \frac{1}{\sqrt{12}}, \frac{1}{\sqrt{20}}\right) \\
e_4 &= \left(0, 0, -\frac{3}{\sqrt{12}}, \frac{1}{\sqrt{20}}\right) \\
e_5 &= \left(0, 0, 0, -\frac{4}{\sqrt{20}}\right)
\end{aligned}
$$

A_4^* lattice

S_5 point group symmetry



A_3^* lattice

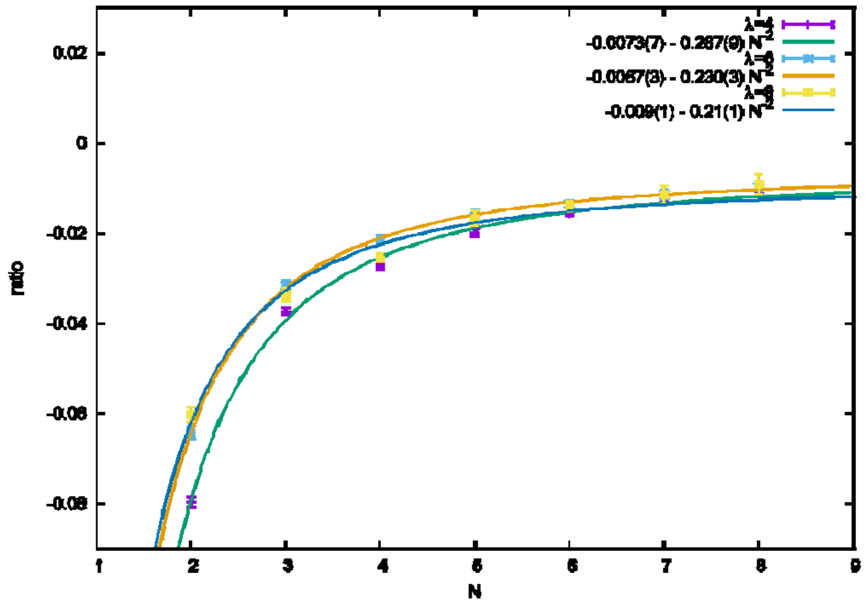
TRUNCATION

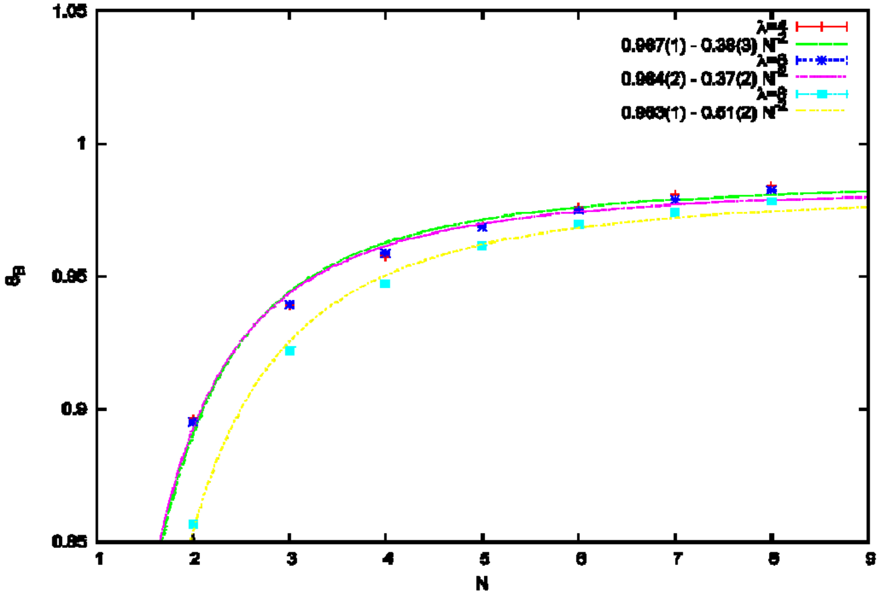
$$\det \mathcal{U}_m(x) = \det(G(x)\mathcal{U}_m(x)G^\dagger(x+e_m))$$

$$\text{provided } \det G(x) = 1 \text{ for all } x.$$

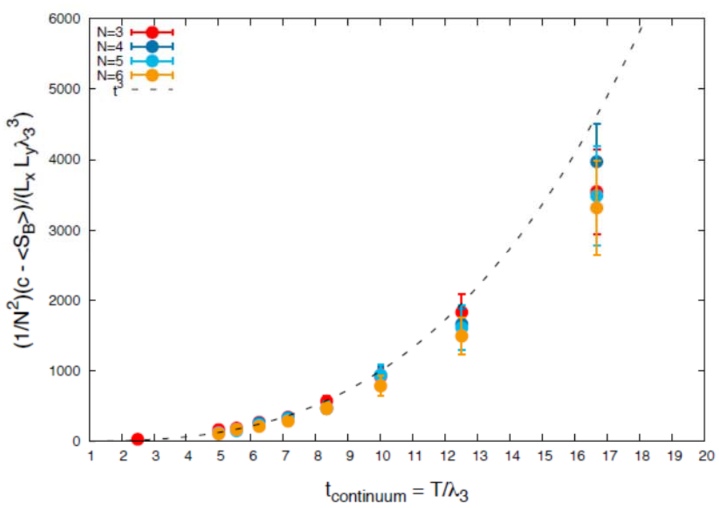
Hence we can consistently truncate to SL(N,C). All we lose is the U(1) gauge invariance, if we do it right. The surviving gauge group is SU(N). This is very helpful because the U(1) modes (scalar and gauge) are a royal pain.

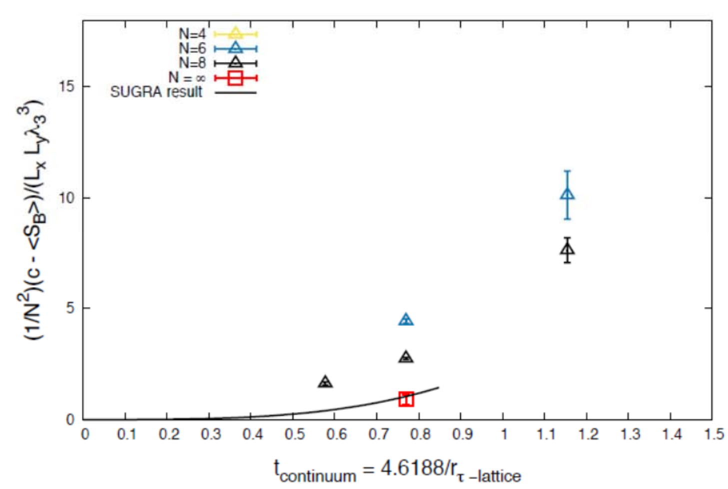
Q RESTORATION





3D HOLOGRAPHY





CONCLUSIONS

Lattice formulation with very little fine-tuning, due to symmetries

Understanding of renormalization

BPS solitons under study

Highly optimized code with good scaling

SL(N,C) truncation

Adequate computing resources

Lower-dimensional finite T/gauge-gravity duality

Operator dimensions