Kaon Physics in the Standard Model

RBC and UKQCD Collaborations

USQCD All Hands Meeting

April 28, 2017

Norman H. Christ

Columbia University

Five Topics

- 2^{nd} order weak with charm
  - Long distance contribution rare kaon decays, $K \to \pi \nu \bar{\nu}$
  - $K_L - K_S$ mixing:
    - $K_L - K_S$ mass difference
    - Long distance contribution to ε_K
- $K \to \pi \pi$ decay and ε'
  - G-parity boundary conditions
  - periodic boundary conditions

RBC Collaboration

- BNL
  - Chulwoo Jung
  - Taku Izubuchi
  - Luchang Jin
  - Christoph Lehner
  - Meifeng Lin
  - Amarjit Soni
- RBRC
  - Mattia Bruno
  - Shigemi Ohta (KEK)
  - Sergey Syritsyn
- Beijing
  - Xu Feng
- Columbia
  - Ziyuan Bai
  - Norman Christ
  - Duo Guo
  - Chris Kelly
  - Robert Mawhinney
  - David Murphy
  - Masaaki Tomii
  - Jiqun Tu
  - Bigeng Wang
  - Tianle Wang
- Connecticut
  - Tom Blum
  - Dan Hoying

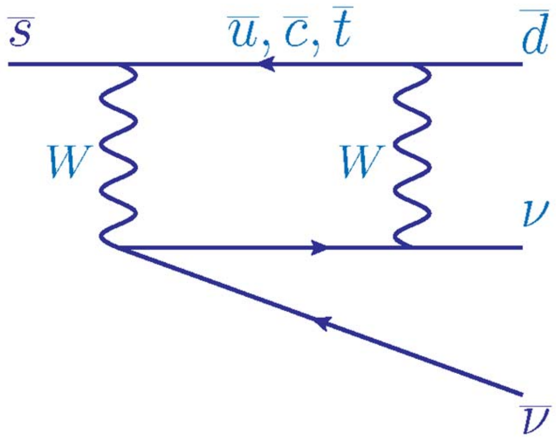
UKQCD Collaboration

- Southampton
  - Jonathan Flynn
  - Vera Guelpers
  - James Harrison
  - Andreas Juttner
  - Andrew Lawson
  - Edwin Lizarazo
  - Chris Sachrajda
- Edinburgh
  - Peter Boyle
  - Guido Cossu
  - Julien Frison (KEK)
  - Nicolas Garron (Plymouth)
  - Julia Kettle
  - Ava Khamseh
  - Antonin Portelli
  - Tobias Tsang
  - Oliver Witzel

Rare Kaon Decays

$K^+ \rightarrow \pi^+ \nu \bar{\nu}$

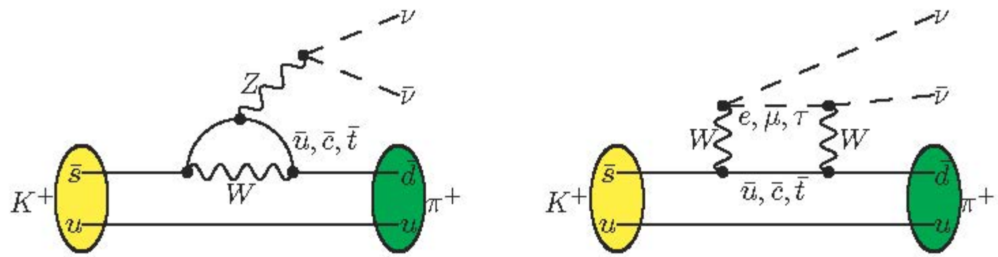
- Flavor changing neutral current
  - Allowed in the Standard Model only in second order
  - Short distance dominated



- Target of NA62 at CERN
  - 100 events in 2-3 years
  - Test Standard Model prediction at 10% level
  - Use lattice for long distance part: 5% effect ?

$K^+ \rightarrow \pi^+ \nu \bar{\nu}$

(Xu Feng)

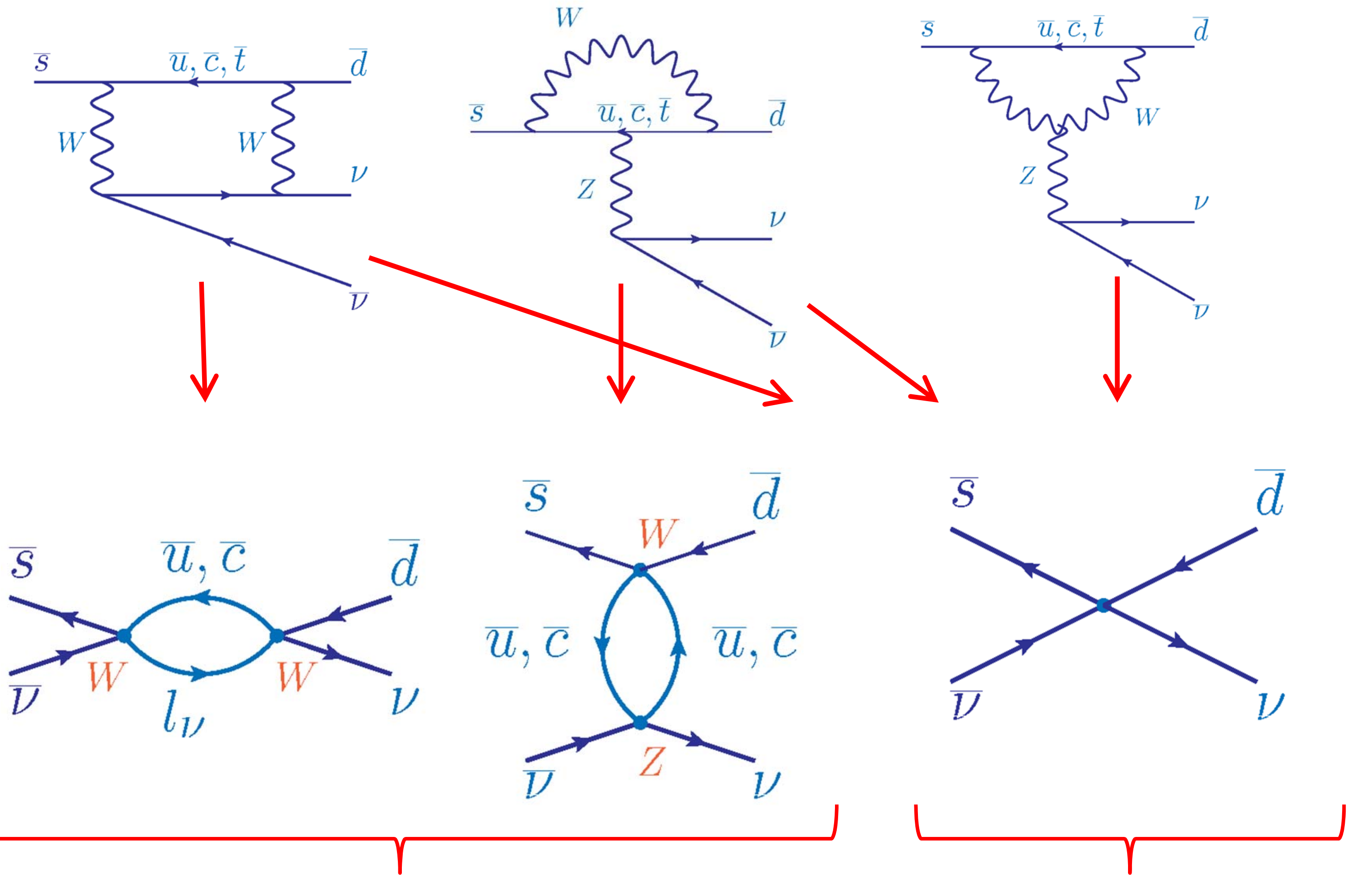


- Estimate 3 contributions: top : charm-*sd* : charm-*ld*
  [Cirigliano et.al. Rev. Mod. Phys.]

$$\lambda_t \frac{m_t^2}{M_W^2} : \lambda_c \frac{m_c^2}{M_W^2} \ln \frac{M_W}{m_c} : \lambda_u \frac{\Lambda_{\rm QCD}^2}{M_W^2} = 68\% : 29\% : 3\%$$

- Expect ≤ 10% long-distance uncertainty
- Important target for lattice calculation

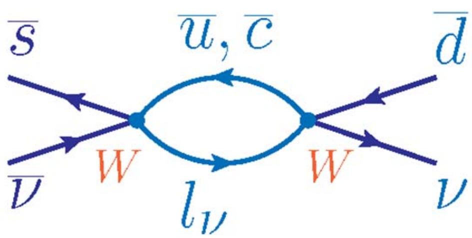
$K^+ \rightarrow \pi^+ \nu \bar{\nu}$ at long distance

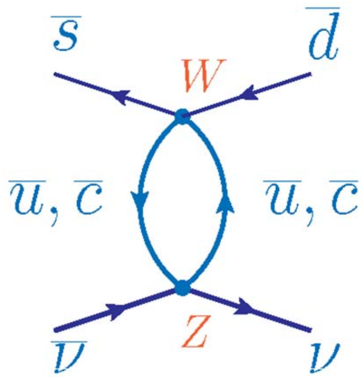


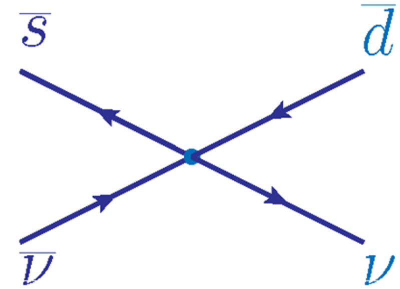
Use lattice QCD

Known $\langle \pi^+ | V_\mu | K^+ \rangle$

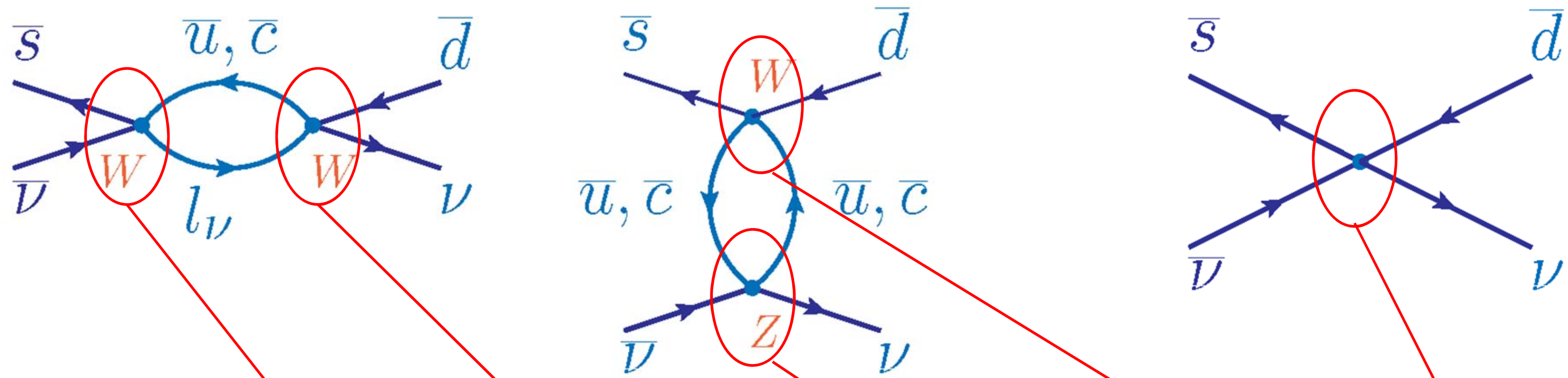
H_{eff} for $K^+ \rightarrow \pi^+ \nu \bar{\nu}$





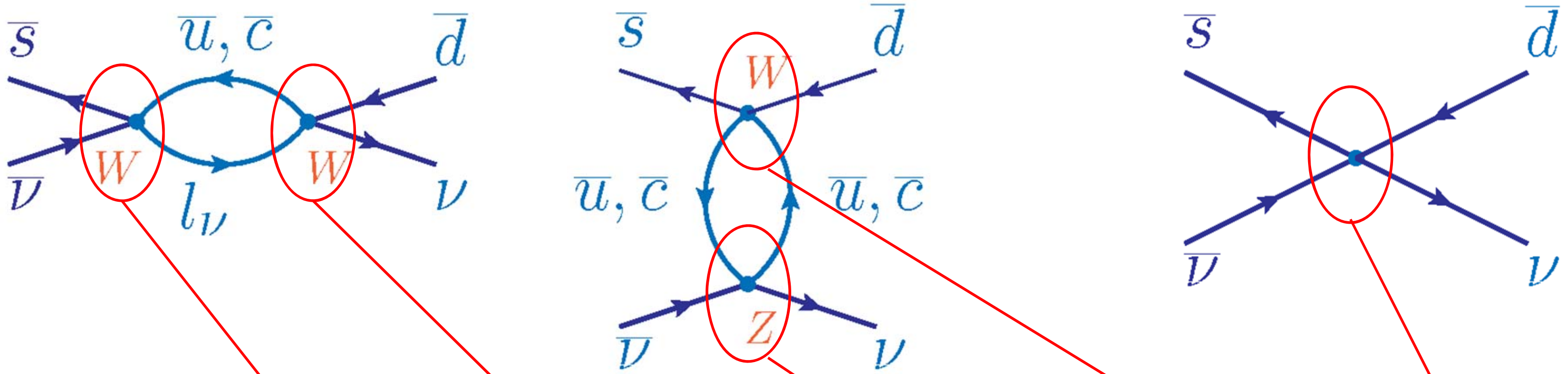


H_{eff} for $K^+ \rightarrow \pi^+ \nu \bar{\nu}$



$$\mathcal{H}_{\text{eff}} = +\frac{G_F}{\sqrt{2}} \left\{ \sum_{\substack{q=u,c \\ \ell=e,\mu,\tau}} \left(V_{qs}^* O_{q\ell}^{\Delta S=1} + V_{qd} O_{q\ell}^{\Delta S=0} \right) + \sum_{\ell=e,\mu,\tau} O_\ell^Z + \sum_{q=u,c} \lambda_q O_q^W \right\} + O_0$$

$H_{\rm eff}$ for $K^+ \to \pi^+ \nu \bar{\nu}$



$$\mathcal{H}_{\rm eff} = +\frac{G_F}{\sqrt{2}}\left\{\sum_{\substack{q=u,c\\ \ell=e,\mu,\tau}}\left(V_{qs}^* O_{q\ell}^{\Delta S=1} + V_{qd}\, O_{q\ell}^{\Delta S=0}\right) + \sum_{\ell=e,\mu,\tau} O_\ell^Z + \sum_{q=u,c}\lambda_q O_q^W\right\} + O_0$$

$$O^{\Delta S=1} = C_{\Delta S=1}(\bar{s}q)_{V-A}(\bar{\ell}\nu_\ell)_{V-A}$$

$$O_q^W = C_1(\bar{s}_a q_b)_{V-A}(\bar{q}_b d_a)_{V-A} + C_2(\bar{s}_a q_a)_{V-A}(\bar{q}_b d_b)_{V-A}$$

$$O^{\Delta S=0} = C_{\Delta S=0}(\bar{q}d)_{V-A}(\bar{\ell}\nu_\ell)_{V-A}$$

$$O_0 = C_0 \sum_{\ell=e,\mu,\tau} (\bar{s}d)_{V-A}(\bar{\nu}_\ell \nu_\ell)_{V-A}$$

$$O_\ell^Z = C_Z \sum_{q=u,c,d,s}\left(T_3^q \bar{q}\gamma_\mu(1-\gamma_5)q - Q_{{\rm em},q}\sin^2\theta_W \bar{q}\gamma_\mu q\right)\overline{\nu_\ell}\gamma_\mu(1-\gamma_5)\ell$$

Euclidean-space Lattice Calculation

- Unphysical terms growing exponentially with time

$$\int_{-T}^{T} dt \langle \pi \nu \overline{\nu} | T\Big(O_A(t)\, O_B(0)\Big) | K\rangle$$

$$= \sum_n \left\{ \frac{\langle \pi \nu \overline{\nu} | O_A | n\rangle \langle n | O_B | K\rangle}{M_K - E_n} + \frac{\langle \pi \nu \overline{\nu} | O_B | n\rangle \langle n | O_A | K\rangle}{M_K - E_n} \right\} \left(1 - e^{(M_K - E_n)T}\right)$$

- Terms with $M_K > E_n$ must be removed.
- GIM cancellation leaves a logarithmic divergence which must be replaced by a perturbative, electro-weak LEC.

Exploratory Lattice Calculations

- 16^3 x 32, RBC-UKQCD ensemble
  - 2+1 flavor DWF, $1/a$ = 1.73 GeV
  - M_π = 420 MeV, M_K = 540 MeV,
  - $m_c(2\ \text{GeV})^{\overline{MS}}$ = 863 GeV
  - 800 configurations, time translate 32 times
  - P_c = 0.253(6) + 0.004(6) (large cancellation)
- USQCD Project:
  - 32^3 x 64 Iwasaki-DSDR, m_π = 170 MeV
  - Find weak dependence on Dalitz variables
- 2018 Incite target: M_π = 135 MeV, 64^3 x 128

Kaon Mixing

CP violation

$K \to \pi\pi$ decay

$K \rightarrow \pi\,\pi$ and CP violation

- Final $\pi\pi$ states can have I = 0 or 2.

$$\langle \pi\,\pi(I=2)|H_w|K^0\rangle = A_2 e^{i\delta_2} \qquad \Delta I = 3/2$$
$$\langle \pi\,\pi(I=0)|H_w|K^0\rangle = A_0 e^{i\delta_0} \qquad \Delta I = 1/2$$

- CP symmetry requires A_0 and A_2 be real.
- Direct CP violation in this decay is characterized by:

$$\epsilon' = \frac{i\,e^{\delta_2-\delta_0}}{\sqrt{2}}\left|\frac{A_2}{A_0}\right|\left(\frac{\mathrm{Im}A_2}{\mathrm{Re}A_2}-\frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}\right)$$

Direct CP violation

$K^0 - \overline{K^0}$ mixing

- ΔS=1 weak decays allow K^0 and $\overline{K^0}$ to decay to the same $\pi-\pi$ state.
- Resulting mixing described by Wigner-Weisskopf:

$$i\frac{d}{dt}\begin{pmatrix} K^0 \\ \overline{K}^0 \end{pmatrix} = \left\{ \begin{pmatrix} M_{00} & M_{0\bar{0}} \\ M_{\bar{0}0} & M_{\bar{0}\bar{0}} \end{pmatrix} - \frac{i}{2} \begin{pmatrix} \Gamma_{00} & \Gamma_{0\bar{0}} \\ \Gamma_{\bar{0}0} & \Gamma_{\bar{0}\bar{0}} \end{pmatrix} \right\} \begin{pmatrix} K^0 \\ \overline{K}^0 \end{pmatrix}$$

- Decaying states are mixtures of K^0 and K^0

$$|K_S\rangle = \frac{K_+ + \overline{\epsilon} K_-}{\sqrt{1+|\overline{\epsilon}|^2}}$$

$$|K_L\rangle = \frac{K_- + \overline{\epsilon} K_+}{\sqrt{1+|\overline{\epsilon}|^2}}$$

$$\overline{\epsilon} = \frac{i}{2}\left\{ \frac{\mathrm{Im}\, M_{0\bar{0}} - \frac{i}{2}\mathrm{Im}\, \Gamma_{0\bar{0}}}{\mathrm{Re}\, M_{0\bar{0}} - \frac{i}{2}\mathrm{Re}\, \Gamma_{0\bar{0}}} \right\}$$

Indirect CP violation

CP violation

- CP violating, experimental amplitudes:

$$\eta_{+-} \equiv \frac{\langle \pi^+\pi^- | H_w | K_L \rangle}{\langle \pi^+\pi^- | H_w | K_S \rangle} = \epsilon + \epsilon'$$

$$\eta_{00} \equiv \frac{\langle \pi^0\pi^0 | H_w | K_L \rangle}{\langle \pi^0\pi^0 | H_w | K_S \rangle} = \epsilon - 2\epsilon'$$

- Where:

$$\epsilon = \overline{\epsilon} + i\,\frac{\mathrm{Im} A_0}{\mathrm{Re} A_0}$$

Indirect: $|\varepsilon| = (2.228 \pm 0.011) \times 10^{-3}$

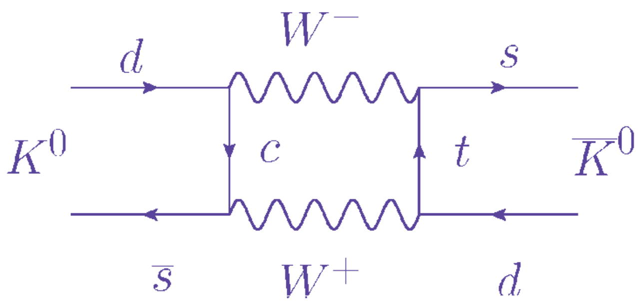
Direct: $\mathrm{Re}(\varepsilon'/\varepsilon) = (1.66 \pm 0.23) \times 10^{-3}$

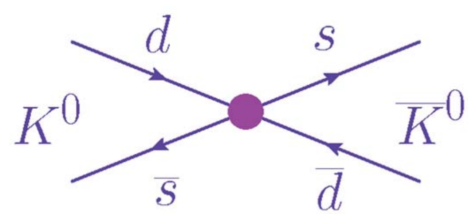
$K^0 - \overline{K^0}$ mixing

$K^0 - \overline{K}^0$ Mixing

- CP violating: $p \sim m_t$

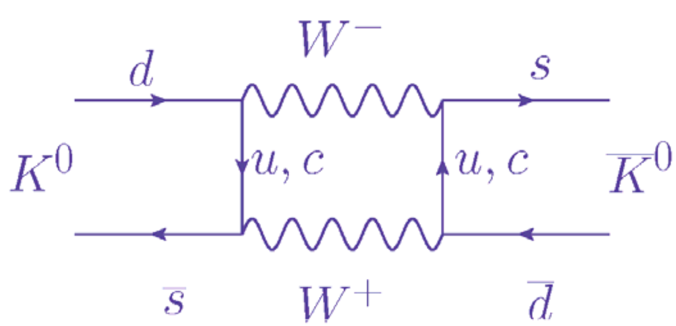
$$\epsilon_K = \frac{i}{2}\left\{\frac{\mathrm{Im}\,M_{0\bar{0}} - \frac{i}{2}\mathrm{Im}\,\Gamma_{0\bar{0}}}{\mathrm{Re}\,M_{0\bar{0}} - \frac{i}{2}\mathrm{Re}\,\Gamma_{0\bar{0}}}\right\} + i\,\frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}$$

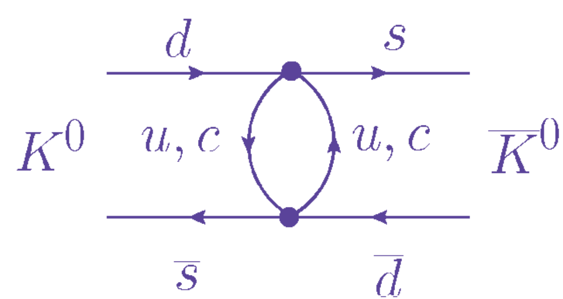




- CP conserving: $p \leq m_c$

$$m_{K_S} - m_{K_L} = 2\mathrm{Re}\{M_{0\bar{0}}\}$$





$K_L - K_S$ mass difference

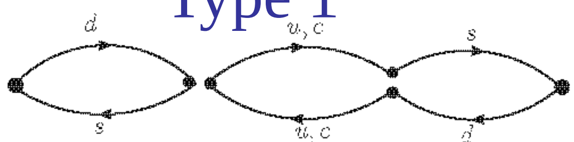
Lattice calculations

- Must include charm quark (GIM *u*–*c* cancellation)
- Three calculations performed:

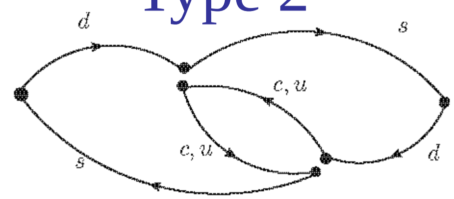
Jianglei Yu
- – 16^3 x 32, m_p = 420 MeV, types 1 & 2 (arXiv:1212.5931)
- – 24^3 x 64, m_p = 330 MeV, all graphs (arXiv:1406.0916)

Ziyuan Bai
- – 32^3 x 64, m_p = 170 MeV, all graphs
- – 64^3 x 128 m_p = 135 MeV, all graphs, physical m_c
  2016-2017 Incite project

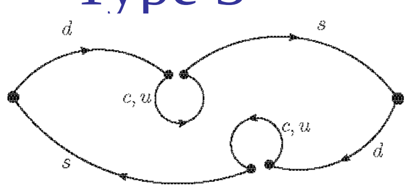
Type 1



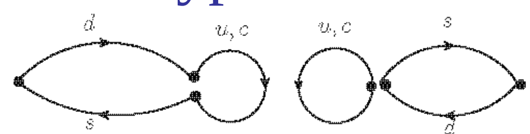
Type 2



Type 3



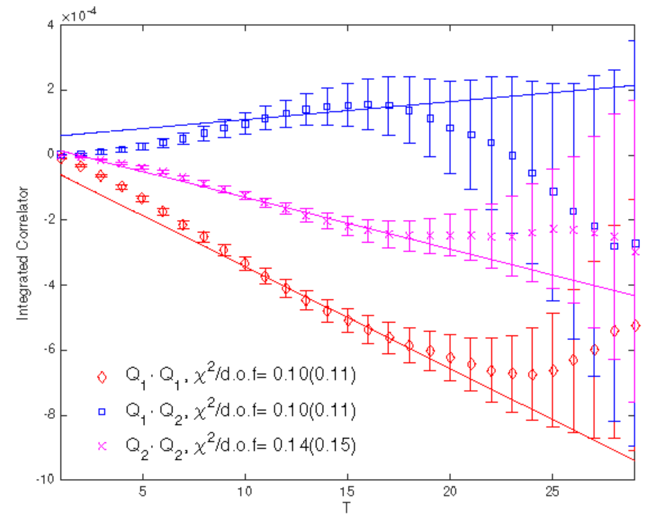
Type 4



m_π = 135 MeV – 64^3 x 128 result

(Ziyuan Bai)

- 2+1 flavor, DWF ensemble:
  - 64^3 x 128, 1/a=2.4 GeV
  - Valence, physical mass charm
  - 160 configurations
- 2016 and 2017 Incite project
  - Type 3 & 4, AMA:
    52 sloppy, 7 exact
  - Type 1 & 2: 11 (exact)
  - All-2-all without time dilution, 2,000 eigenvectors
  - Single integration method



- Current result:
  - ΔM_K = 4.88(170) x 10^{-12} MeV
  - Expt: 3.483(6) x 10^{-12} MeV

Long distance part of ε_K

ε_K – overview

- ε_K : Indirect CP violation in $K^0 \rightarrow \pi\pi$ decay.
  - Experiment: 2.228(11) x 10^{-3}
  - Theory: 2.13(23) x 10^{-3} [SWME arXiv:1503.06613]
- Theory error dominated by $(V_{cb})^4$ factor
- ε_K decomposition:

$$\lambda_q = V_{qs}^* V_{qd}$$

  - 2/3 top x top ($\lambda_t \lambda_t$)
  - 1/3 top x charm ($\lambda_t \lambda_c$)

  } short distance

  - 2 - 4% long distance contribution

ΔS = 1, Four flavor operators

(Ziyuan Bai)

- Choose appropriate $N_f = 4$ effective Hamiltonian:

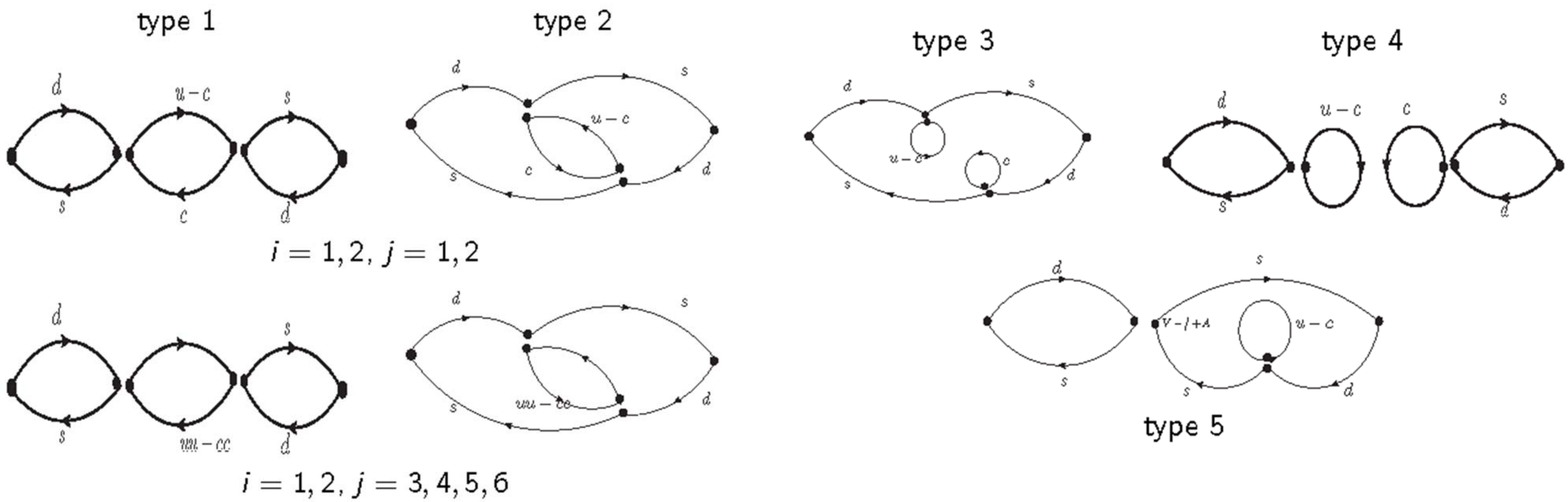
$$H_W^{\Delta S=1;\Delta C=\pm 1,0} = \frac{G_F}{\sqrt{2}} \left\{ \sum_{q,q'=u,c} V_{q's}^* V_{qd} \sum_{i=1}^{2} C_i Q_i^{q'q} + V_{ts}^* V_{td} \sum_{i=3}^{6} C_i Q_i \right\}$$

$$\left.\begin{aligned} Q_1^{q'q} &= (\bar{s}_i q'_j)_{V-A} (\bar{q}_j d_i)_{V-A} \\ Q_2^{q'q} &= (\bar{s}_i q'_i)_{V-A} (\bar{q}_j d_j)_{V-A} \end{aligned}\right\} \text{current x current}$$

$$\left.\begin{aligned} Q_3 &= (\bar{s}_i d_i)_{V-A} \sum_{q=u,d,s,c} (\bar{q}_j q_j)_{V-A} \\ Q_4 &= (\bar{s}_i d_j)_{V-A} \sum_{q=u,d,s,c} (\bar{q}_j q_i)_{V-A} \\ Q_5 &= (\bar{s}_i d_i)_{V-A} \sum_{q=u,d,s,c} (\bar{q}_j q_j)_{V+A} \\ Q_6 &= (\bar{s}_i d_j)_{V-A} \sum_{q=u,d,s,c} (\bar{q}_j q_i)_{V+A} \end{aligned}\right\} \text{QCD penguin}$$

Diagrams for $\lambda_t \lambda_u$ contribution to ε_K
(Ziyuan Bai)

- Identify five types of diagrams



Exploratory calculation

(Ziyuan Bai)

- 243 x 64, 1/a=1.73 GeV, M_π = 340 MeV, m_c=968 MeV
- Perturbative LEC added to correct log divergence
- Preliminary results from 200 configurations.

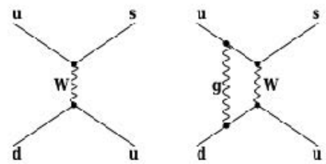
μ_{RI}	$\mathrm{Im}\, M_{\bar{0}0}^{ut,RI}$	$\mathrm{Im}\, M_{\bar{0}0}^{ut,RI\to\overline{MS}}$	$\mathrm{Im}\, M_{\bar{0}0}^{ut,ld\,corr}$	$\epsilon_K^{ut,ld\,corr}$
1.54 GeV	-0.746(0.389)	0.282	-0.464 (0.389)	0.0911(0.076)
1.92 GeV	-0.912(0.389)	0.384	-0.527 (0.389)	0.104(0.076)
2.11 GeV	-0.986(0.389)	0.434	-0.551 (0.389)	0.108(0.076)
2.31 GeV	-1.050(0.390)	0.486	-0.565 (0.390)	0.111(0.077)
2.56 GeV	-1.115(0.390)	0.548	-0.568 (0.390)	0.111(0.077)

- $|\varepsilon_K|$ = 2.228(11) x 10^{-3} expt.
- 2018 Incite target: 64^3 x 128, physical mass result

$K \rightarrow \pi\pi$ decay

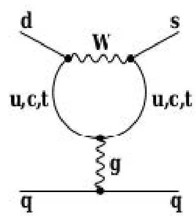
Four quark operators (N_f=3)

- **Current-current operators**



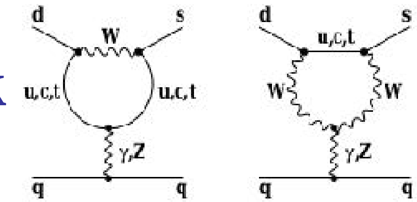
$$Q_1 \equiv (\bar{s}_\alpha d_\alpha)_{V-A}(\bar{u}_\beta u_\beta)_{V-A}$$
$$Q_2 \equiv (\bar{s}_\alpha d_\beta)_{V-A}(\bar{u}_\beta u_\alpha)_{V-A}$$

- **QCD Penguins**



$$Q_3 \equiv (\bar{s}_\alpha d_\alpha)_{V-A} \sum_{q=u,d,s} (\bar{q}_\beta q_\beta)_{V-A}$$
$$Q_4 \equiv (\bar{s}_\alpha d_\beta)_{V-A} \sum_{q=u,d,s} (\bar{q}_\beta q_\alpha)_{V-A}$$
$$Q_5 \equiv (\bar{s}_\alpha d_\alpha)_{V-A} \sum_{q=u,d,s} (\bar{q}_\beta q_\beta)_{V+A}$$
$$Q_6 \equiv (\bar{s}_\alpha d_\beta)_{V-A} \sum_{q=u,d,s} (\bar{q}_\beta q_\alpha)_{V+A}$$

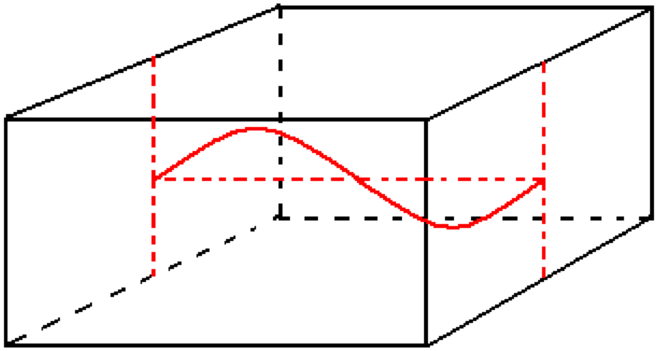
- **Electro-Weak Penguins**



$$Q_7 \equiv \frac{3}{2}(\bar{s}_\alpha d_\alpha)_{V-A} \sum_{q=u,d,s} e_q(\bar{q}_\beta q_\beta)_{V+A}$$
$$Q_8 \equiv \frac{3}{2}(\bar{s}_\alpha d_\beta)_{V-A} \sum_{q=u,d,s} e_q(\bar{q}_\beta q_\alpha)_{V+A}$$
$$Q_9 \equiv \frac{3}{2}(\bar{s}_\alpha d_\alpha)_{V-A} \sum_{q=u,d,s} e_q(\bar{q}_\beta q_\beta)_{V-A}$$
$$Q_{10} \equiv \frac{3}{2}(\bar{s}_\alpha d_\beta)_{V-A} \sum_{q=u,d,s} e_q(\bar{q}_\beta q_\alpha)_{V-A}$$

Physical $\pi\pi$ states Lellouch-Luscher

- Euclidean $e^{-H_{QCD}t}$ projects onto $|\pi\pi(\vec{p}=0)\rangle$
- Exploit finite-volume quantization.
  - Adjust volume so 1st or 2nd excited state has correct $\vec{p}$.



  - Introduce boundary conditions so that the ground state has physical $\vec{p}$
- Discuss two complimentary projects:
  - Status of calculation with G-parity boundary conditions.
  - New project with periodic boundary conditions

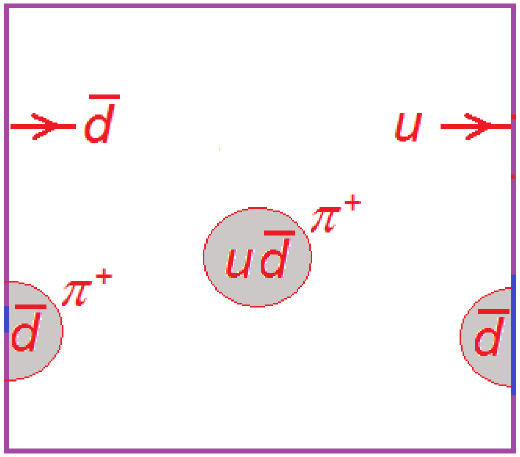
Overview of G-parity calculation
(Chris Kelly, Daiqian Zhang)

- Use 32^3 x 64 ensemble
  - *1/a* = 1.3784(68) GeV, *L* = 4.53 fm.
  - 216 configurations separated by 4 time units
  - 900 low modes for all-to-all propagators
  - Solve for $\pi\pi$ and kaon sources on each of 64 time slices

$\Delta I = ½ \;\; K \rightarrow \pi\pi$ – above threshold

(Chris Kelly & Daiqian Zhang)

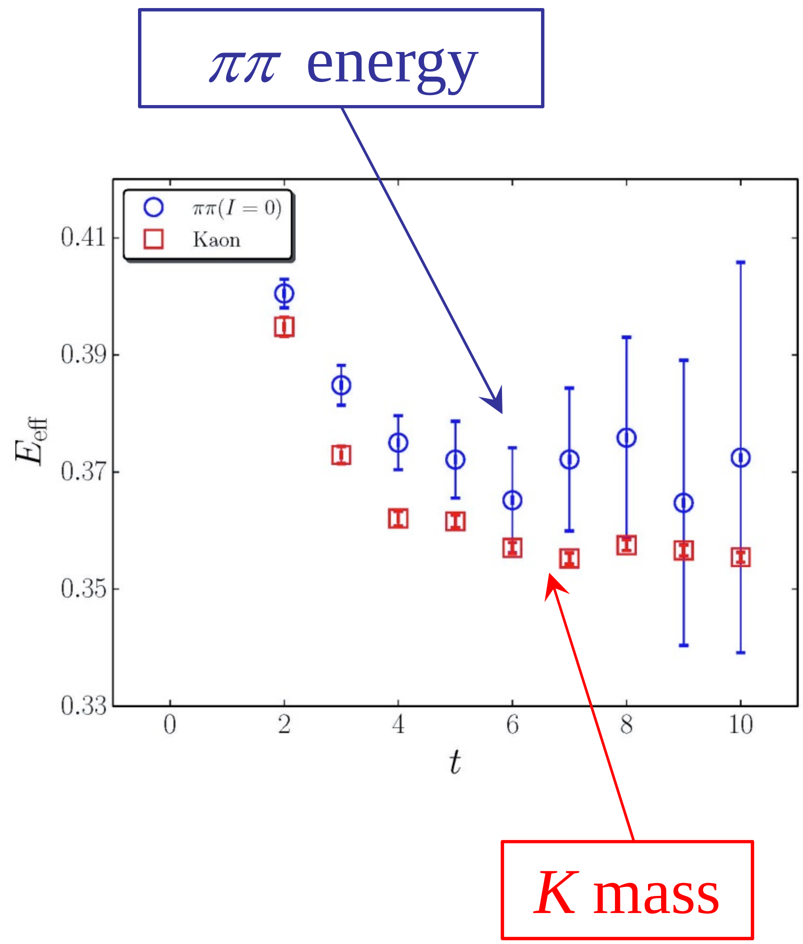
- **G-parity** BC give p_π = 205 MeV (Changhoan Kim, hep-lat/0210003)
  - $G = C\, e^{i\pi I_y}$
  - Non-trivial: $\begin{pmatrix} u \\ d \end{pmatrix} \rightarrow \begin{pmatrix} \bar{d} \\ -\bar{u} \end{pmatrix}$
  - Extra I = 1/2, s' quark adds $e^{-m_K L}$ error.
  - Tests: f_K and B_K correct within errors.



- Physical kinematics:
  - M_π = 143.1(2.0)
  - M_K = 490.6(2.2) MeV
  - $E_{\pi\pi}$ = 498(11) MeV

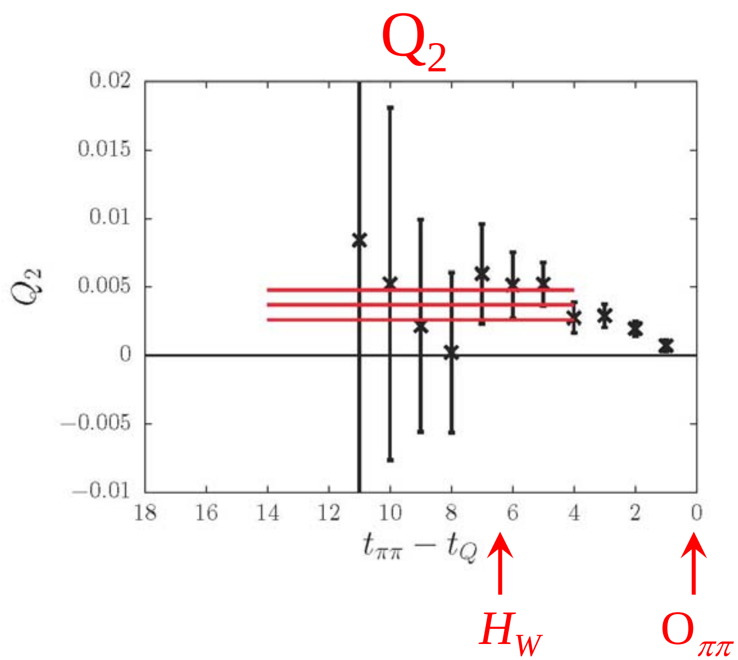
I = 0, $\pi\pi - \pi\pi$ correlator

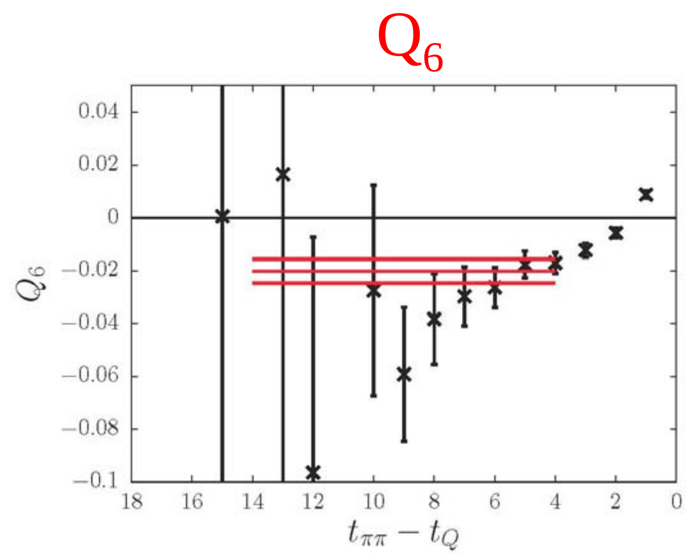
- Determine normalization of $\pi\pi$ interpolating operator
- Determine energy of finite volume, I = 0, $\pi\pi$ state: $E_{\pi\pi}$ = 498(11) MeV
- Determine I = 0 $\pi\pi$ phase shift: δ_0 = 23.8(4.9)(2.2)^o
- Phenomenological result: δ_0 = 38.0(1.3)^o [G. Colangelo]



ΔI = ½ $K \rightarrow \pi\pi$ matrix elements

- Vary time separation between H_W and $\pi\pi$ operator.
- Show data for all $K - H_W$ separations t_Q - $t_K \geq 6$ and $t_{\pi\pi}$ - t_K = 10, 12, 14, 16 and 18.
- Fit correlators with $t_{\pi\pi}$ - $t_Q \geq 4$
- Obtain consistent results for $t_{\pi\pi}$ - $t_Q \geq 3$ or 5





G-Parity 2015 Results and Outlook

- $\mathrm{Re}(\varepsilon'/\varepsilon) = (1.38 \pm 5.15_{stat} \pm 4.59_{sys}) \times 10^{-4}$
  - Expt.: $(16.6 \pm 2.3) \times 10^{-4}$ (2.1 σ difference)
- Substantial improvements underway
  - Use step-scaling to reduce operator normalization error:
    - 15% → 8% (C. Kelly, Lattice 2016)
    - Add suppressed G_1 operator (G. McGlynn)
  - Increase statistics: 216 → 1000+
    - Implement EOFA 4x speed-up (David Murphy)
    - Now with 635 new configurations (Blue Waters, BNL, Dirac, KEK, Mira, RBRC, USQCD)
    - 195 new measurements (Cori I & II → BNL KNL)
  - ALCC proposal: 1.2K, 24^3x64, $1/a$=1.0 GeV, meas'nts.
  - Requesting 17M BG/Q hrs and 70% of zero-priority time.

Periodic Boundary Conditions

- New USQCD proposal (T. Blum PI)
- Exploit methods of Jlab & Trinity groups
  - Multiple sources
  - Non-zero cm momenta
  - GEVP analysis
- Explore many $\pi\pi$ energies, interpolate to $E_{\pi\pi} = M_K$.
- Initial, exploratory proposal:
  - $24^3 \times 64$, $1/a$=1.0 GeV, L= 4.7 fm, physical masses
  - $\pi\pi$ scattering and $K \rightarrow \pi\pi$ and on 100 configurations
  - Use highly optimized Grid KNL code now running on Cori II
  - Request 63 M J/psi hours on Jlab KNL cluster.
- Important opportunity to develop & test new methods

The Future

- Early science time on Aurora:
  - New 2+1+1 ensembles with $1/a$ = 3 & 4 GeV
  - Allow 5% calculations with charm:
    - ΔM_K – a potential source of new physics
    - Long distance parts of $K \rightarrow \pi \nu \bar{\nu}$ and ε_K
- Increase accuracy of $K \rightarrow \pi\pi$ and ε'
  - Use Aurora for "small" 48^3 and 64^3 $K \rightarrow \pi\pi$ jobs
  - Use position-space NPR to cross charm threshold non-perturbatively (Masaaki Tomii)
  - Move $K \rightarrow \pi\pi$ calculations to N_f=4, include charm and GIM
  - Include E&M and m_u-m_d corrections.