High-Statistics Calculation of Nucleon Structure and Matrix Elements on Isotropic Clover Lattices

Combined LHP & NME Proposal

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(|LHP❭ + |NME❭): K.Orginos

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Nucleon Structure with Isotropic Wilson Lattices

Goal : Compute Nucleon Structure and Quark Matrix Elements with high statistical precision and robust control of systematic errors

Wilson fermions are economical and permit

- higher statistics for better precision and noisy observables (TMDs, GPDs)
- experiments with newer techniques
  - controlling excited states
  - computing disconnected diagrams
  - exploring hadron states with high momentum

JLab Isotropic clover-improved Wilson lattices:

ID	a[fm]	Volume	m_π	$m_\pi L$	Traj. available	Conn.cost per conf.[NMEp]	%%	
D4	0.085	32^3x64	400	5.5	5100	500	~20%	Systematics study [NMEp]
D5	0.081	32^3x64	300	4.0	2600	825		
D6	0.080	48^3x96	190	3.7	700	7,125		
D7	0.080	64^3x128	190	4.9	900 (++ by 07/01)	32,055	~80%	proposed in [LHPp]
D8	0.080	64^3x128	140	4.1	Started	Next Year (hopefully)		

Nucleon Structure Scientific Objectives

In the Joint proposal, we will study (topics as expressed by in the initial proposals)

<table>
<tr><th>LHP (before'15 : DWF with RBC)</th><th>NME (before'15 : Wilson on HISQ)</th></tr>
<tr><td colspan="2">Vector (EM) Form Factors $G_{E,M}$ (including high momenta Q^2) and Radii $(r^2_{E,M})^{p,n}$</td></tr>
<tr><td colspan="2">Axial Vector Form Factors $G_{A,P}$ and Axial Coupling g_A</td></tr>
<tr><td colspan="2">Scalar and Tensor Charges $(g_{S,T})^{u-d}$</td></tr>
<tr><td>Generalized Form Factors,
Moments of PDFs, Nucleon Spin</td><td></td></tr>
<tr><td>Ordinary and Transverse Momentum-Dependent Parton Distributions</td><td></td></tr>
<tr><td></td><td>Quark (chromo)EDM-induced nEDM</td></tr>
</table>

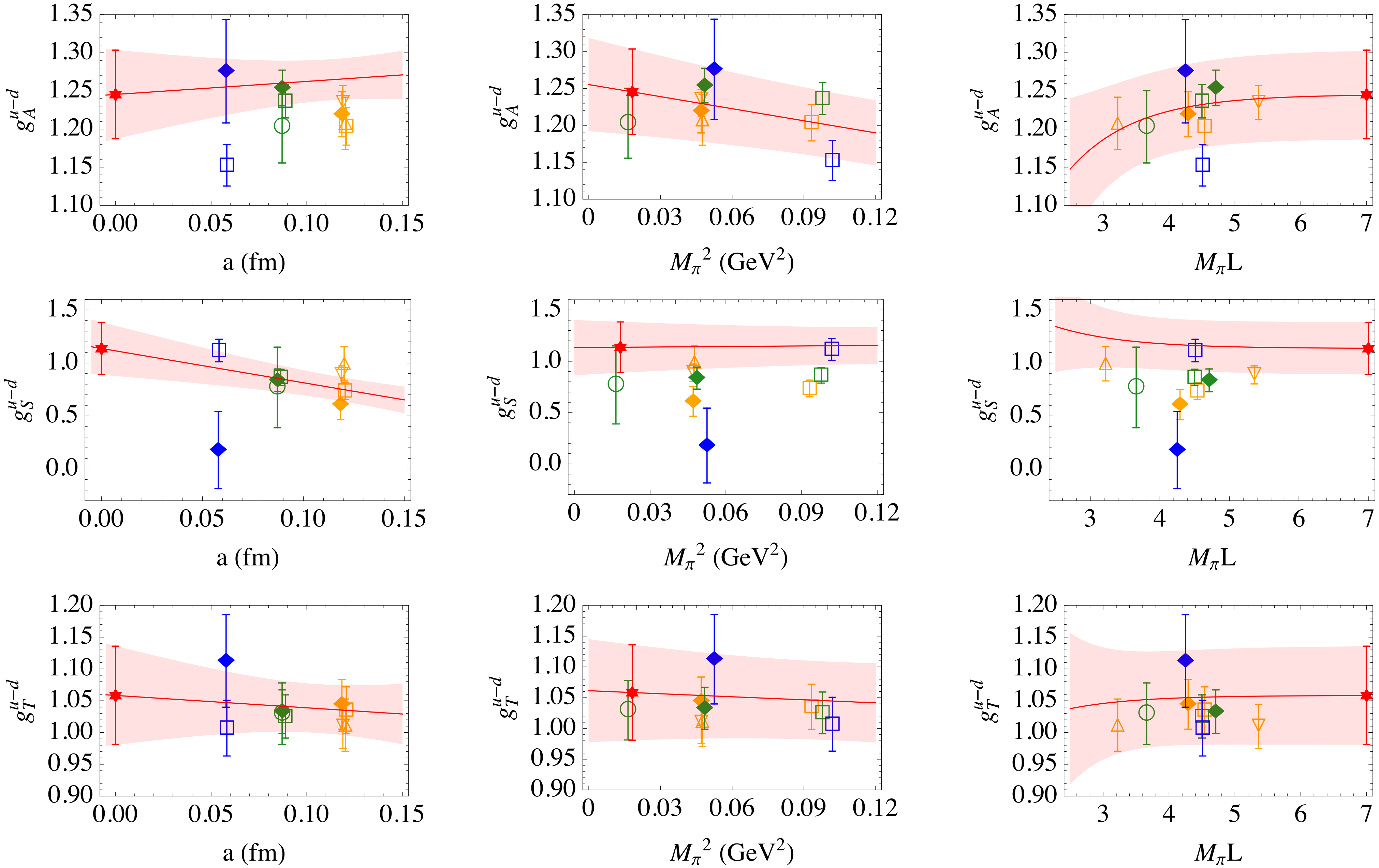
Wilson Fermions will make affordable

<table>
<tr><td>Variational analysis of Exc.States</td><td></td></tr>
<tr><td></td><td>Study dep. on a, L, m_π (≥190 MeV)</td></tr>
<tr><td colspan="2">Including Disconnected (light & strange) Quark Contractions</td></tr>
</table>

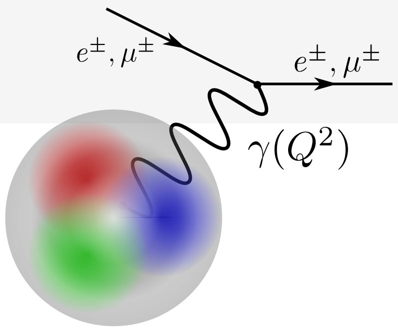
Nucleon "Charges" $g_{A,S,T}$

[(P)N(D)ME, Lattice'14]

- $g_{S,T}$ "charges" = couplings to BSM physics in precision meas. of β-decay [LANL]
- Clover-improved Wilson valence quarks on HISQ lattices
- Extrapolation in a, L, m_π : $g(a, m_\pi, L) = g^0 + \alpha a + \beta m_\pi^2 + \gamma e^{-m_\pi L}$

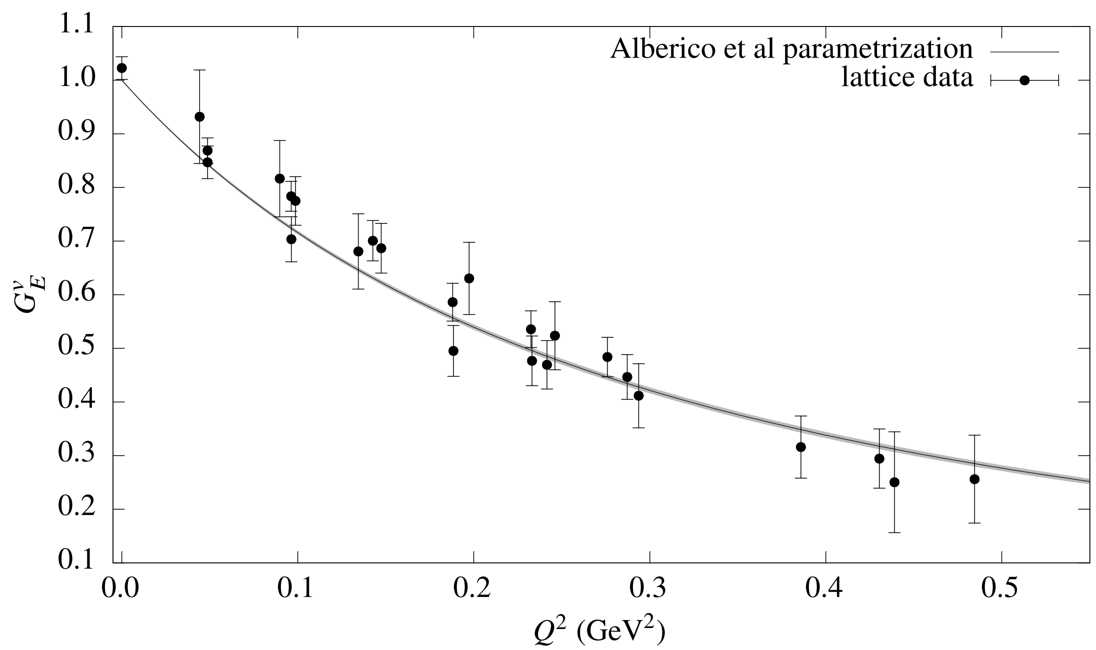


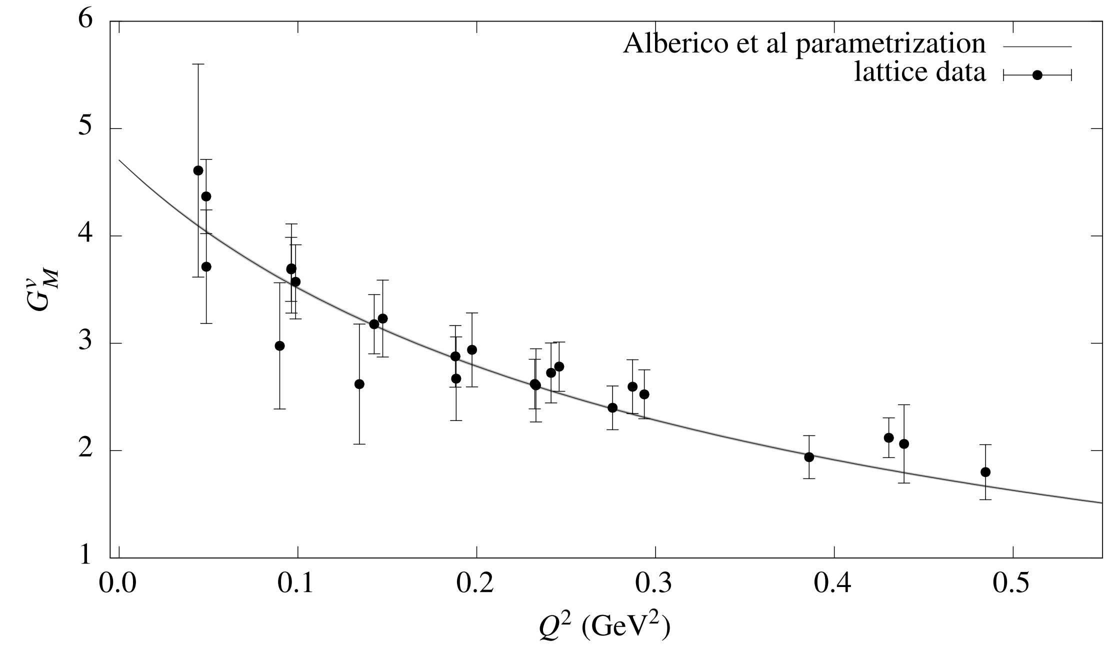
Nucleon Vector (EM) Form Factors



$$\langle P+q|\,\bar{q}\gamma^\mu q\,|P\rangle = \bar{U}_{P+q}\Big[F_1(Q^2)\,\gamma^\mu + F_2\,(Q^2)\frac{i\sigma^{\mu\nu}q_\nu}{2M_N}\Big]U_P$$

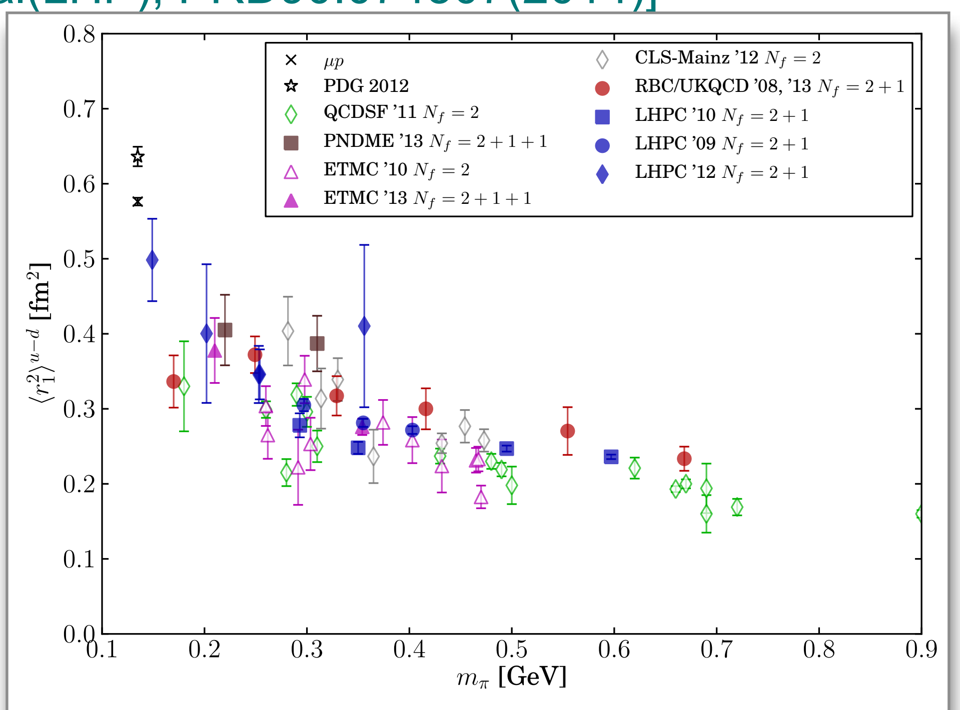
- Form Factors: (F_1 / F_2) scaling, (G_E/G_M), *u-*,*d*-contributions

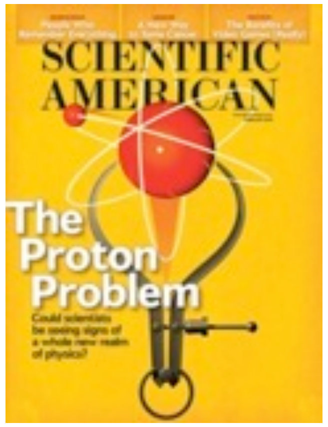




m_π=149 MeV data vs Phenomenology [J.Green et al(LHP), PRD90:074507(2014)]

- Proton radius: 7σ difference; JLab pRAD, MUSE ($e^\pm$,$\mu^\pm$ –p)

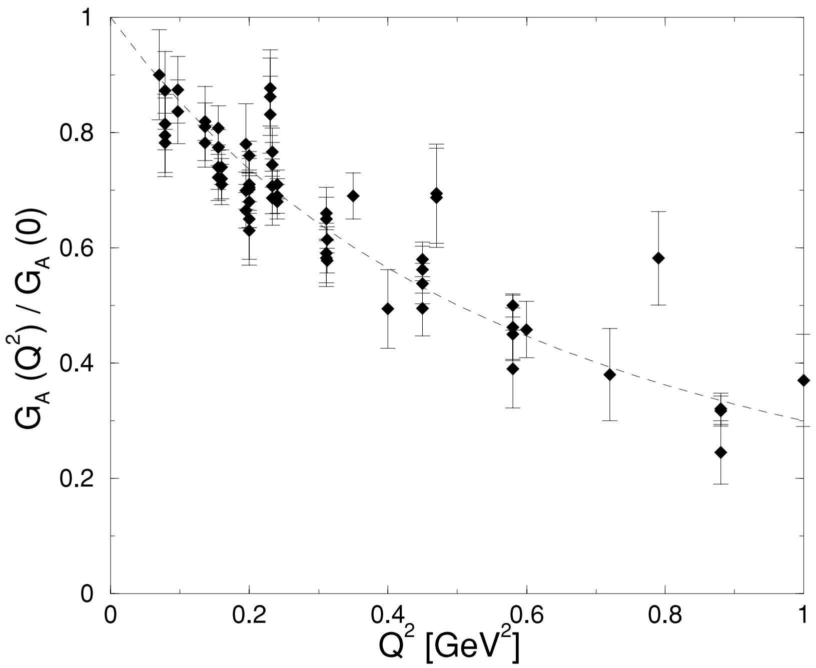




Nucleon Axial Form Factors

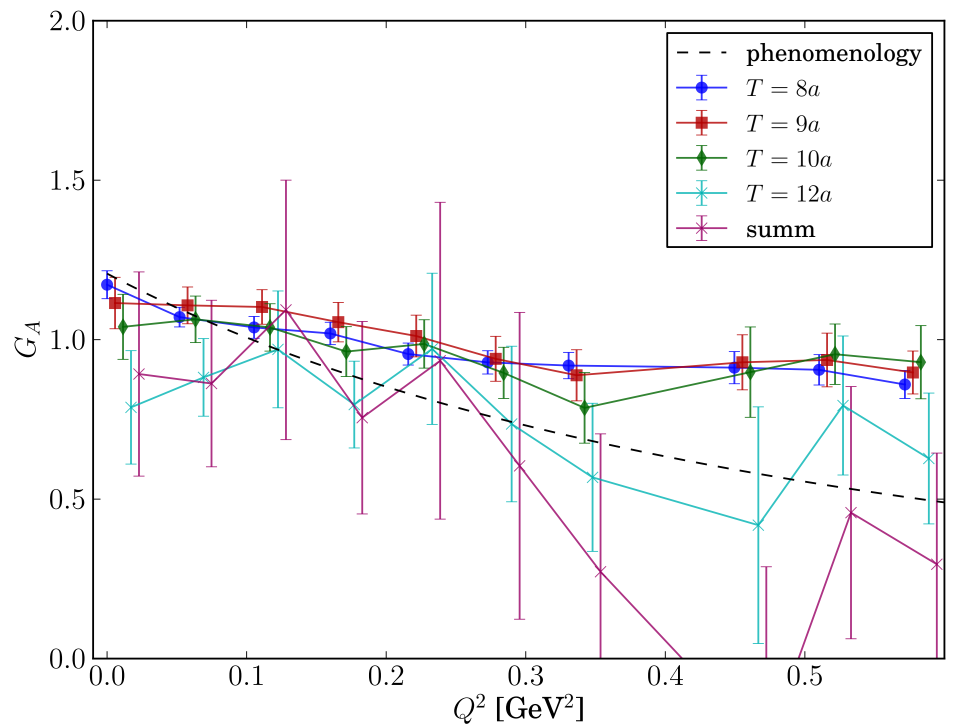
$$\langle P+q|\,\bar{q}\gamma^\mu\gamma^5 q\,|P\rangle = \bar{U}_{P+q}\Big[G_A(Q^2)\gamma^\mu\gamma^5 + G_P(Q^2)\frac{\gamma^5 q^\mu}{2M_N}\Big]U_P$$

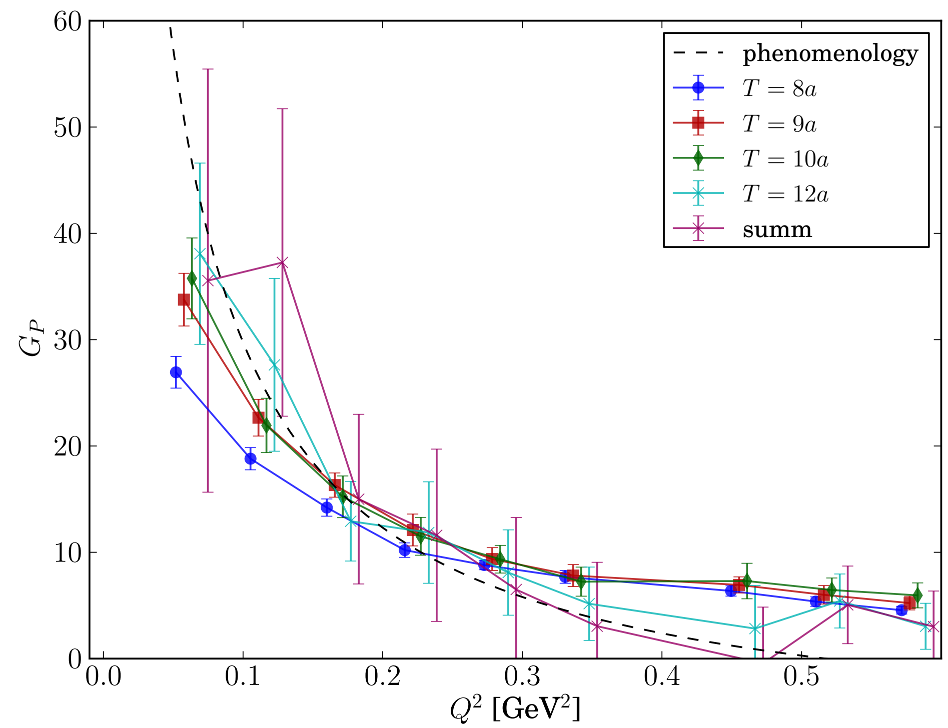
- *$G_A(Q^2)$* are measured in ν-scattering, π-production;
  implications for neutrino flux norm. in IceCube, etc
- Axial radius $\langle r_A^2\rangle=12\,/\,m_A^2$: model dependence
  varying nuclear / G_A shape models: m_A=0.9 ... 1.4 GeV
- Strange quark $G^s_{A,P}(Q^2)$: MiniBooNE
- *$G_P(Q^2)$* induced pseudoscalar : μ capture (MuCAP)



[V.Bernard et at, J.Phys.G28:R1(2002)]

Physical m_π, chiral quarks [LHP & RBC collabs, Lattice'14]





Nucleon Gen. F.F.s and Nucleon Spin

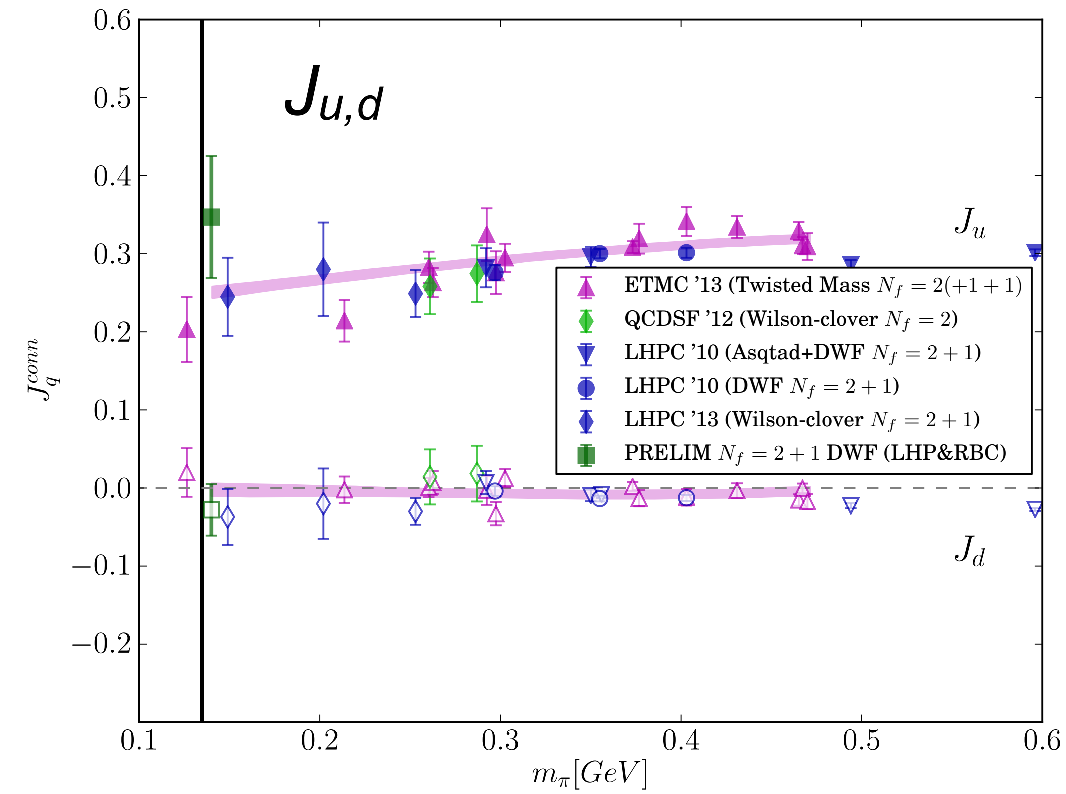
$$\langle N(p+q)|\,T^{q,glue}_{\mu\nu}\,|N(p)\rangle \to \Big\{A_{20},\,B_{20},\,C_{20}\Big\}(Q^2)$$

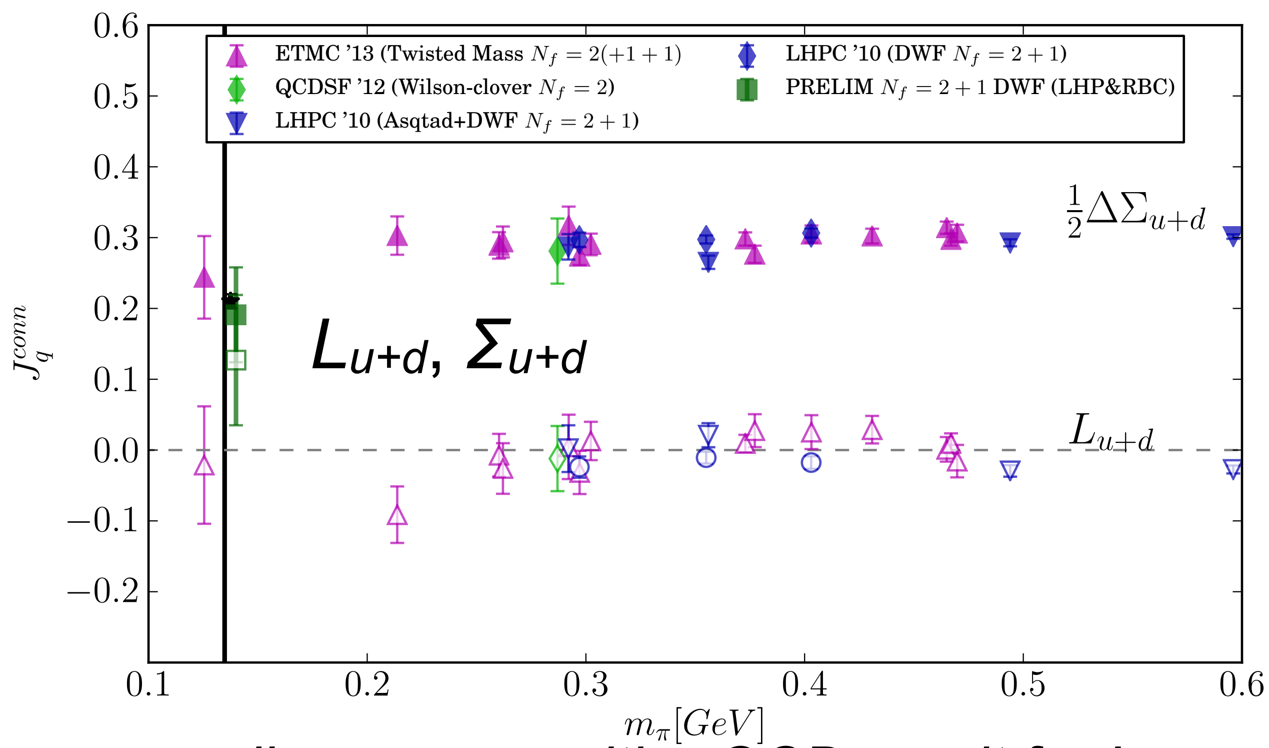
$$\langle x\rangle_q = A^q_{20}(0)$$

$$J_{q,glue} = \frac{1}{2}\left[A^{q,glue}_{20}(0) + B^{q,glue}_{20}(0)\right]$$

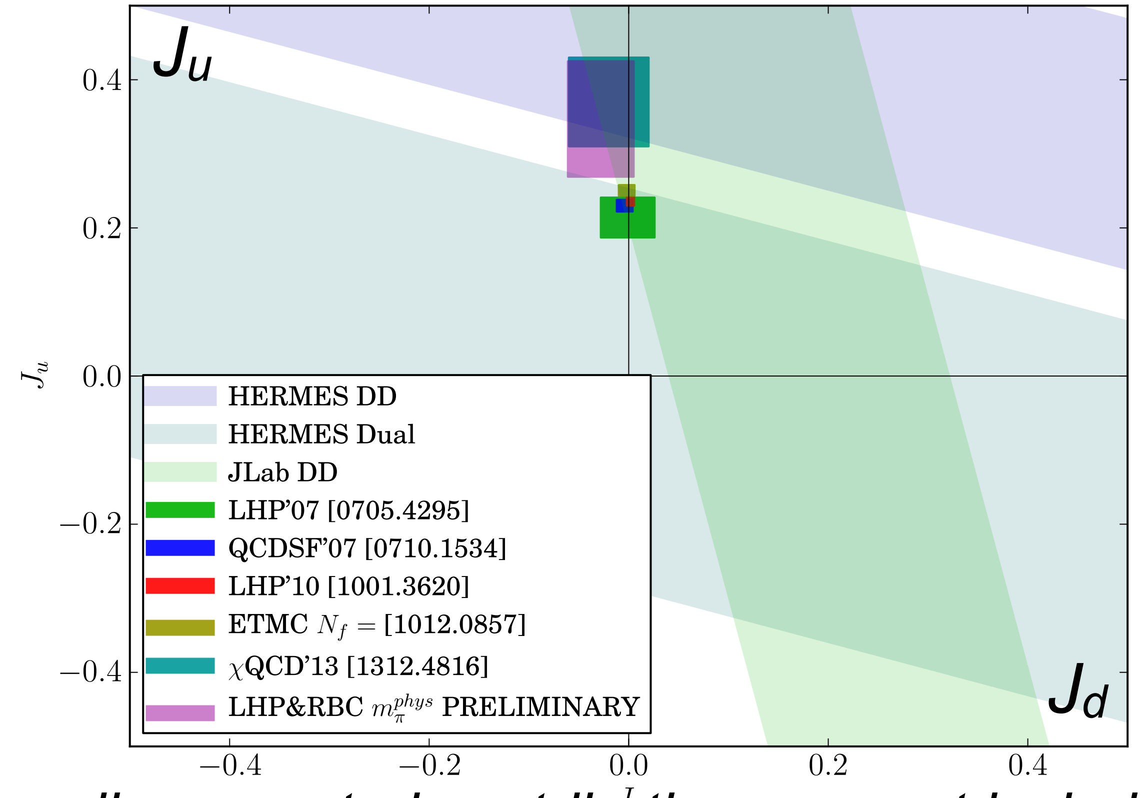
$$J_{\rm glue} + \sum_q J_q = \frac{1}{2}\,,$$

$$J_q = \frac{1}{2}\Delta\Sigma_q + L_q$$





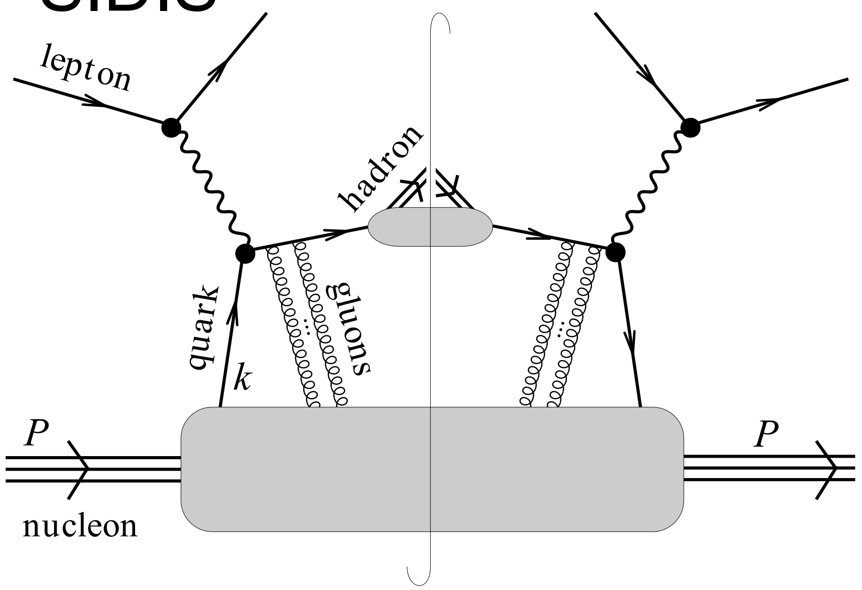
discrepancy with χQCD result for L_{u+d}



disconnected contributions are not included

Transverse Momentum-Dependent Distributions

SIDIS

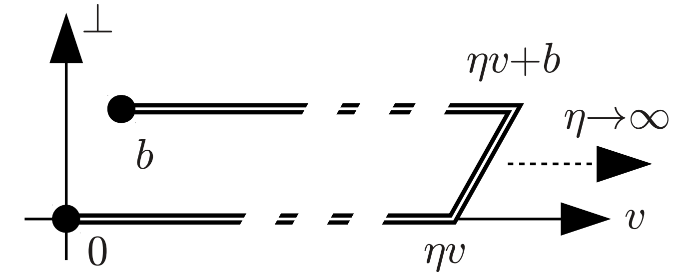


$$l + N(P) \longrightarrow l' + N(P_h) + X$$

Non-local lattice operator

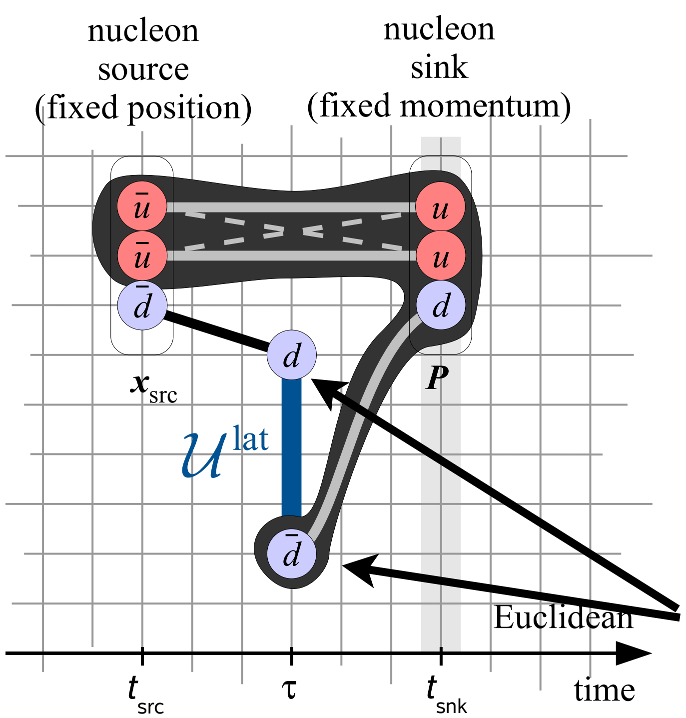
$$\Phi(b, P, S, \hat{\zeta}, \mu) = \frac{1}{2} \langle P, S |\, \bar{q}(0)\, \Gamma\, \mathcal{U}(\eta v, b)\, q(b)\, | P, S \rangle$$

with spacelike link path $\mathcal{U}$ =



probes $k_\perp$-moments ("shifts") of TMDs

$$\sim \int dx \int d^2\vec{k}_\perp\, k_i\, f(x, \vec{k}_\perp)$$



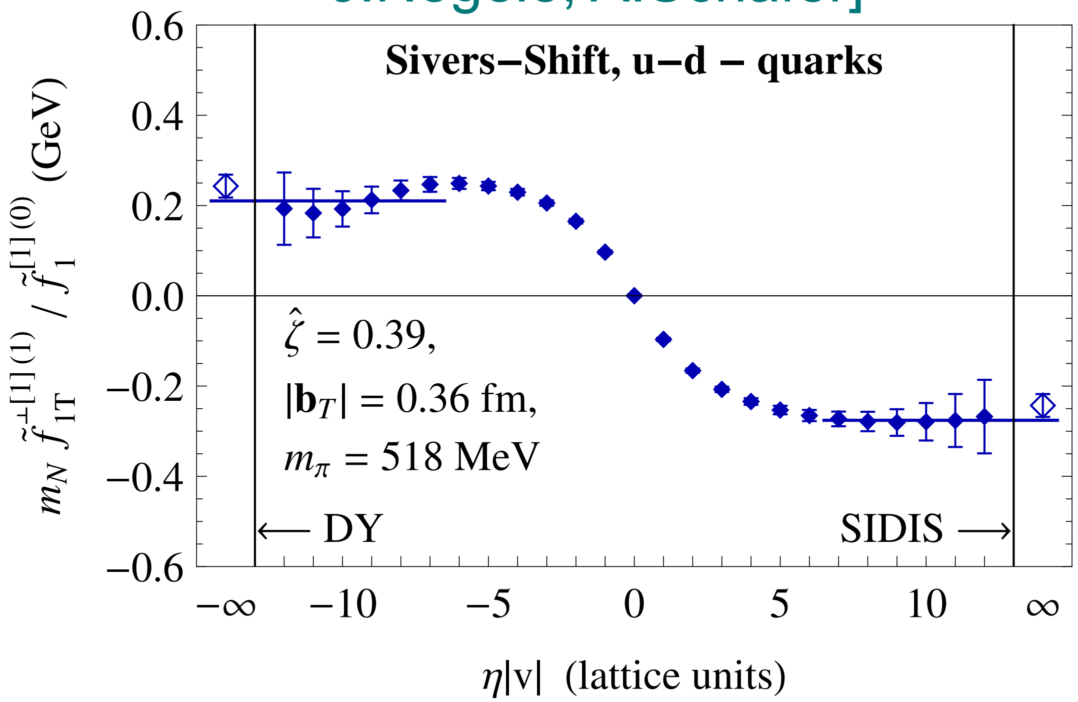
"light-cone" limit

$$\hat{\zeta} = \frac{P \cdot v}{m_N\, |v|} \to \infty$$

operator localized at Euclidean time τ

valence DWF on Asqtad

[B.Musch, P.Hägler, M.Engelhardt, J.Negele, A.Schäfer]



Calculation Details and Improvements

- Kinematics to access high-momentum form factors :
  - include $|p_{sink}|$~1 GeV^2 (up to Q^2~4 GeV^2 in Breit frame)
  - TMDs also require high momentum in-,out-states $|p_{sink}|=|p_{source}|$
- Variational method to reduce excited states :
  - 2x2 nucleon correlators with varied source smearing
  - optimize nucleon operators both zero/low and high momentum states
- Improved sampling with *All-Mode-Averaging* :
  - exact low-mode deflation OR truncated multigrid solver
  - ~4,000 (exact+sloppy) samples for the lightest m_π=190 MeV
- Disconnected quark loops (light and strange) with variance reduction :
  - hierarchical probing
  - low-eigenmode deflation

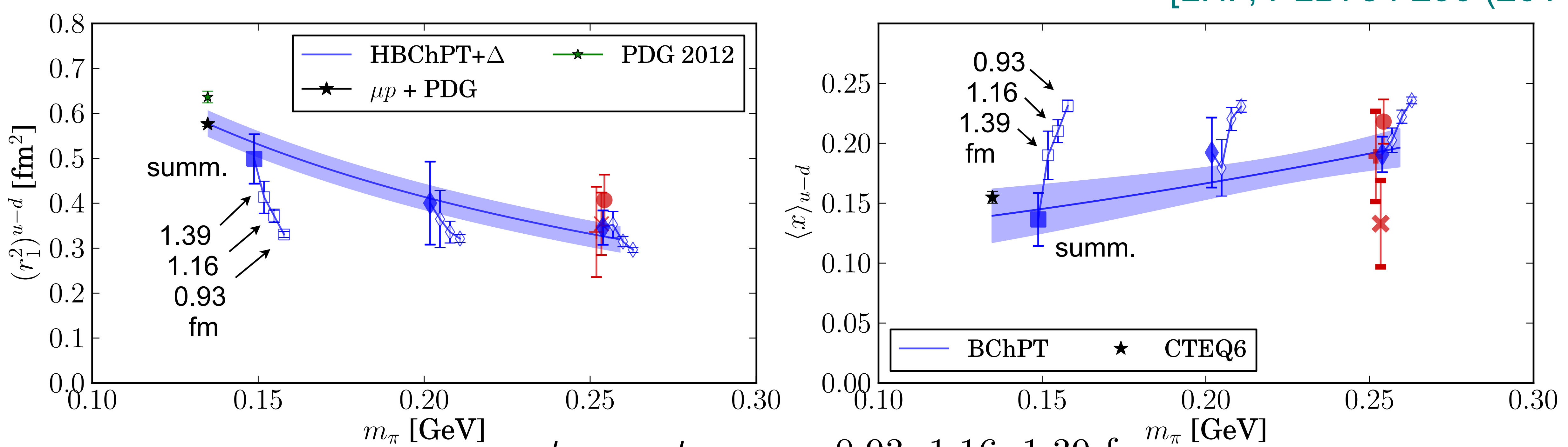
Nucleon Excited States and SNR

- Stochastic noise grows rapidly with *T*, especially with light pions [Lepage'89]:

$$\begin{array}{lll} \text{Signal} & \langle N(T)\bar{N}(0)\rangle & \sim e^{-M_N T} \\ \text{Noise} & \langle |N(T)\bar{N}(0)|^2\rangle - |\langle N(T)\bar{N}(0)\rangle|^2 & \sim e^{-3m_\pi T} \\ \text{Signal/Noise} & & \sim e^{-(M_N - \frac{3}{2}m_\pi)T} \end{array}$$

Physical point: SNR ~ x(1/2) every (2a)

[LHP, PLB734 290 (2014)]



$$t_{sink} - t_{source} = 0.93,\ 1.16,\ 1.39\ \text{fm}$$

- Multi-exp. fits of *T*-dependence : determined by the largest *T*
- Variational method: **(–)**expensive ~$(N_{op})^2$, **(+)***greatly extend plateaus* [CSSM]
- Proposal : explore and compare cost / benefit ***variational*** vs ***traditional***

Disconnected Quark Contractions

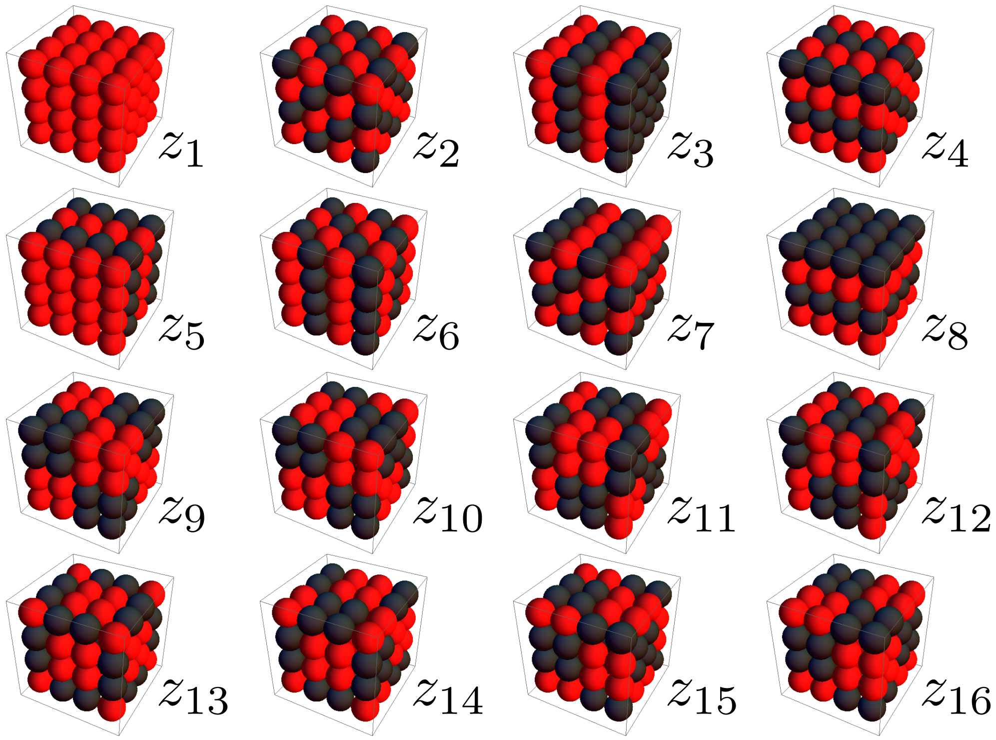
Hierarchical probing [K.Orginos, A.Stathopoulos, '13] :

In sum over 2^{dk+1} vectors (d=3),

dist(x,y) ≤ 2^k terms cancel exactly:

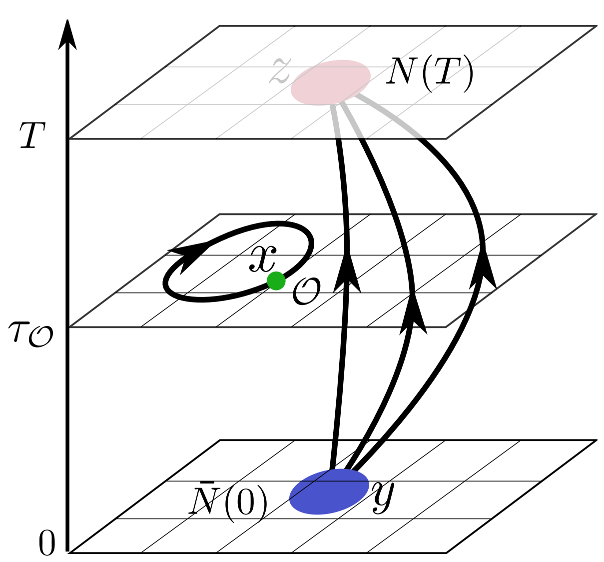
$$1 \le \sum_a |x_a - y_a| \le 2^k \; : \qquad \frac{1}{N}\sum_i^N z_i(x) z_i(y)^\dagger \equiv 0$$

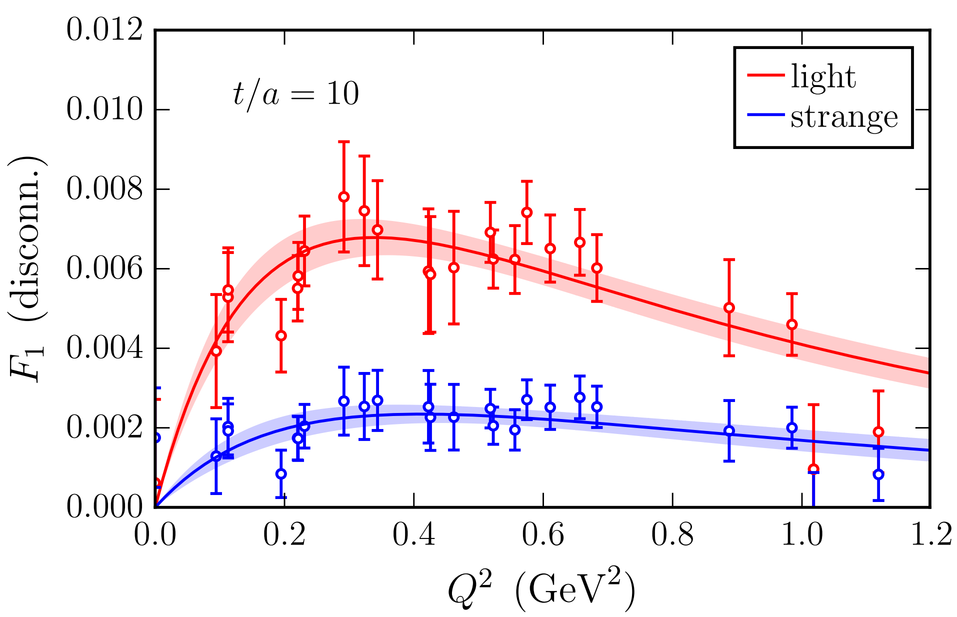
$$z_i \longrightarrow z_i \odot \xi\,, \quad \xi(x) = \text{ random } Z_2\text{-vector}$$

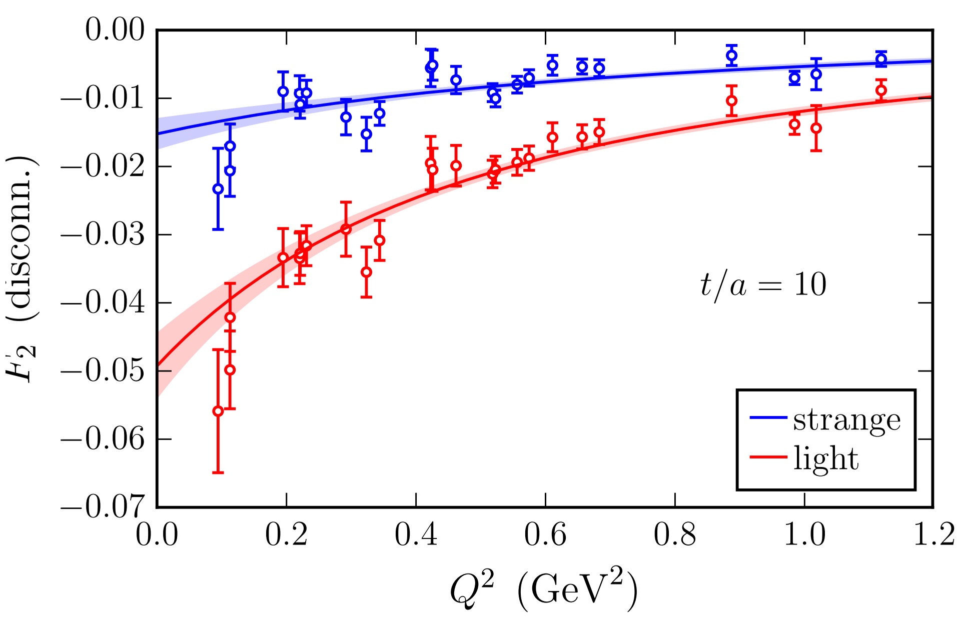


- NEW: reduce variance by treating low modes of $(\not{D}^\dagger \not{D})$ exactly [K.Orginos et al]

Disconnected diagrams with JLab isotropic Clover [S.Meinel's USQCD project '13; *in prep.*]







Total Request for the Joint Proposal

Computing resources request was updated to reflect non-overlapping goals in the proposals :

- LHP requested 43M
- NME requested 47M

Computing resources request was updated to reflect non-overlapping goals in the proposals :

- [common] connected and disconnected 3pt correlators on the lightest pion ensemble m_{π}=190 MeV : **32.8M**
- [NMEp] calculations with the heavier pion masses: **+8.2M**
- [LHPp] additional contractions (GFFs, TMDs) the lightest pion ensemble: **+9.5M**
- [common] exploration of variational method and source tuning: **+6M**

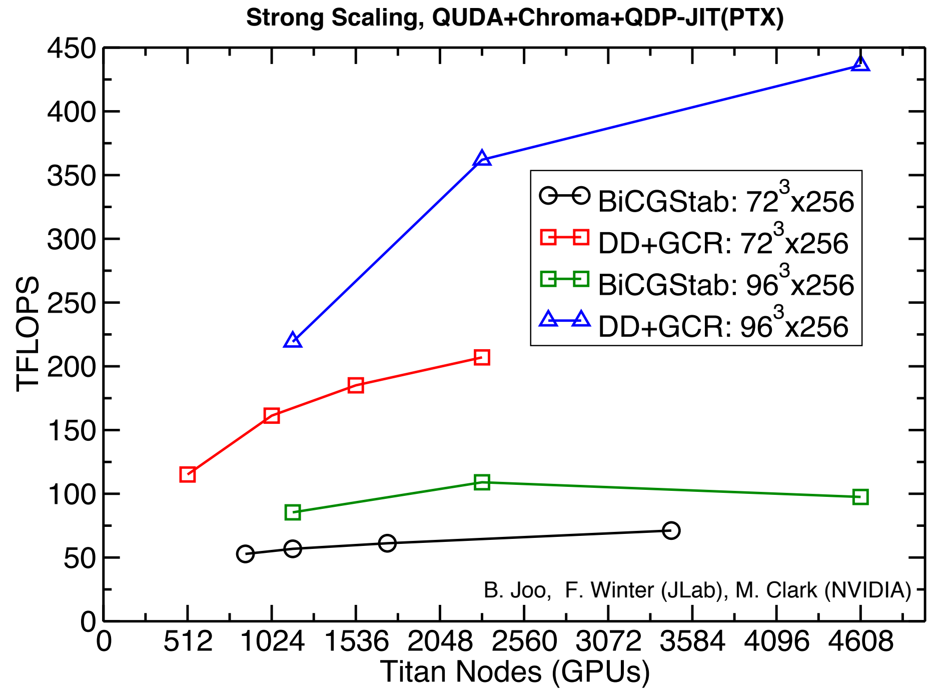
Total combined request : **56.5M**

Summary

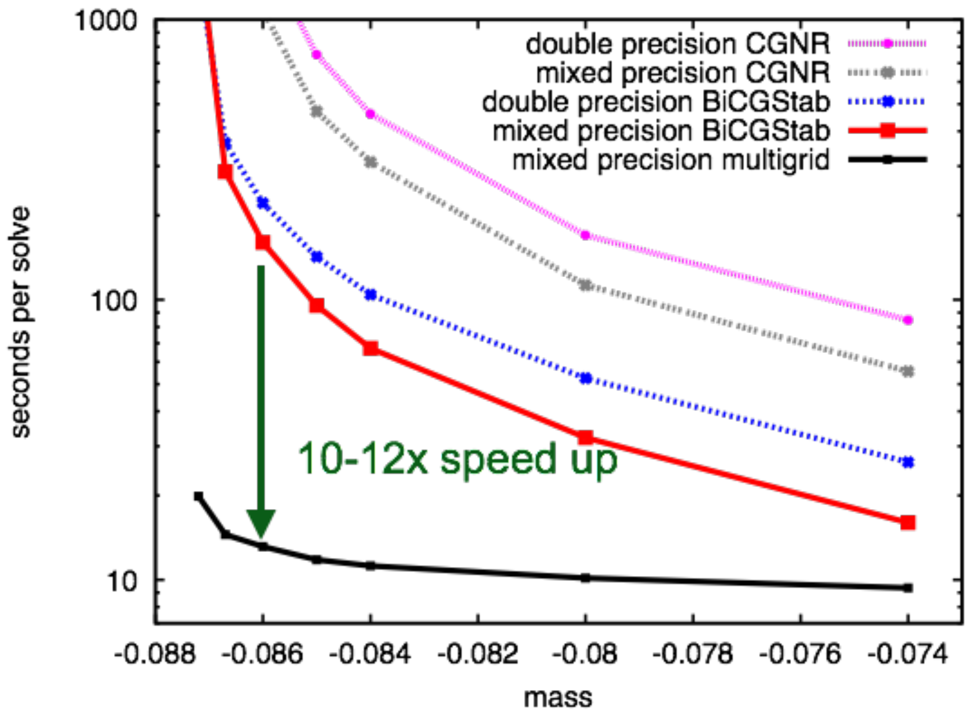
- High-statistics, high precision nucleon structure calculations with very wide scope
  - proton form factors and charge radius
  - proton spin puzzle
  - applications to BSM and CPV searches
  - parton distributions
- Exploration of new techniques crucial for calculations at the physical point
- Equal emphasis on Connected and Disconnected (Light and Strange) contributions to the nucleon structure

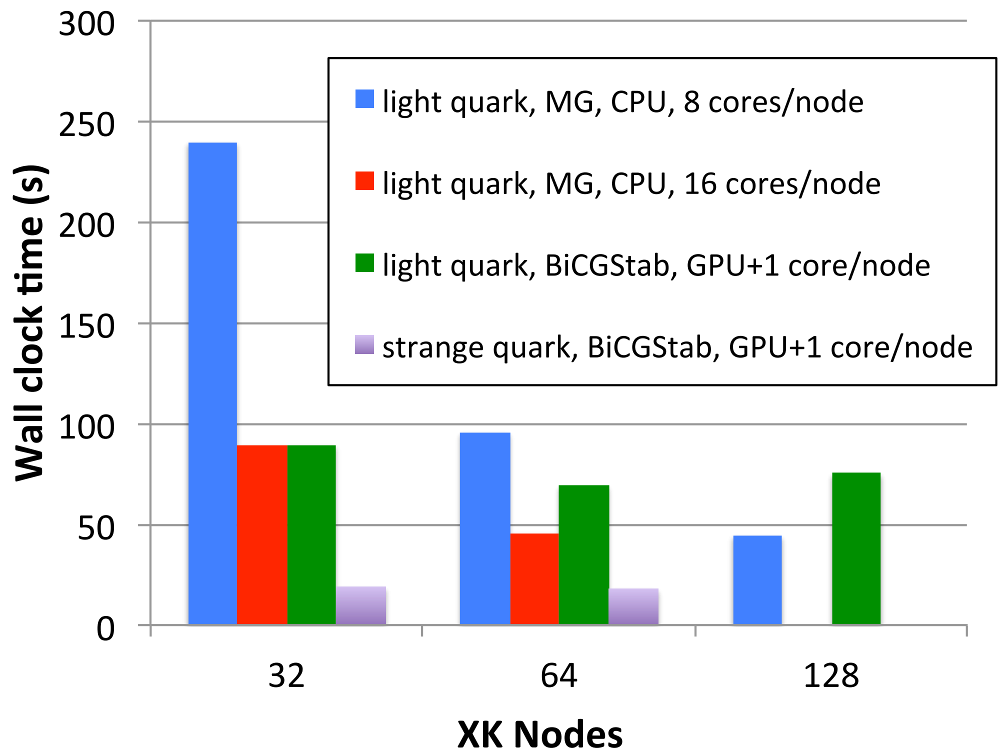
We are hopeful that the USQCD will support not only this proposal, but also generation of physical point Wilson-clover lattices

Code Performance [NMEp]



Multi-GPU DD scaling (Titan)





Comparison of AMG (QDP-MG) to usual Krylov iteration methods