High-Statistics Calculation of Nucleon Structure and Matrix Elements on Isotropic Clover Lattices

Combined LHP & NME Proposal

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(|LHP⟩ + |NME⟩): K.Orginos

USQCD All-Hands Meeting, Fermilab
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Nucleon Structure with Isotropic Wilson Lattices

**Goal:** Compute Nucleon Structure and Quark Matrix Elements with high statistical precision and robust control of systematic errors

Wilson fermions are economical and permit

- higher statistics for better precision and noisy observables (TMDs, GPDs)
- experiments with newer techniques
  - controlling excited states
  - computing disconnected diagrams
  - exploring hadron states with high momentum

**JLab Isotropic clover-improved Wilson lattices:**

<table>
<thead>
<tr>
<th>ID</th>
<th>a[fm]</th>
<th>Volume</th>
<th>$m_\pi$</th>
<th>$m_\pi L$</th>
<th>Traj. available</th>
<th>Conn. cost per conf.[NMEp]</th>
<th>%%</th>
</tr>
</thead>
<tbody>
<tr>
<td>D4</td>
<td>0.085</td>
<td>32$^3$x64</td>
<td>400</td>
<td>5.5</td>
<td>5100</td>
<td>500</td>
<td>~20%</td>
</tr>
<tr>
<td>D5</td>
<td>0.081</td>
<td>32$^3$x64</td>
<td>300</td>
<td>4.0</td>
<td>2600</td>
<td>825</td>
<td>~20%</td>
</tr>
<tr>
<td>D6</td>
<td>0.080</td>
<td>48$^3$x96</td>
<td>190</td>
<td>3.7</td>
<td>700</td>
<td>7,125</td>
<td>~20% Systematics study [NMEp]</td>
</tr>
<tr>
<td>D7</td>
<td>0.080</td>
<td>64$^3$x128</td>
<td>190</td>
<td>4.9</td>
<td>900 (++ by 07/01)</td>
<td>32,055</td>
<td>~80% proposed in [LHPp]</td>
</tr>
<tr>
<td>D8</td>
<td>0.080</td>
<td>64$^3$x128</td>
<td>140</td>
<td>4.1</td>
<td>Started</td>
<td>Next Year (hopefully)</td>
<td>~80% Systematics study [NMEp]</td>
</tr>
</tbody>
</table>
## Nucleon Structure Scientific Objectives

In the Joint proposal, we will study (topics as expressed by in the initial proposals)

<table>
<thead>
<tr>
<th>LHP (before’15 : DWF with RBC)</th>
<th>NME (before’15 : Wilson on HISQ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vector (EM) Form Factors $G_{E,M}$ (including <strong>high</strong> momenta $Q^2$) and Radii $(r_{E,M}^p,n)$</td>
<td></td>
</tr>
<tr>
<td>Axial Vector Form Factors $G_{A,P}$ and Axial Coupling $g_A$</td>
<td></td>
</tr>
<tr>
<td>Scalar and Tensor Charges $(g_{S,T})^{u-d}$</td>
<td></td>
</tr>
<tr>
<td>Generalized Form Factors, Moments of PDFs, Nucleon Spin</td>
<td>Quark (chromo)EDM-induced nEDM</td>
</tr>
<tr>
<td>Ordinary and Transverse Momentum-Dependent Parton Distributions</td>
<td></td>
</tr>
</tbody>
</table>

**Wilson Fermions will make affordable**

<table>
<thead>
<tr>
<th>Variational analysis of Exc. States</th>
<th>Study dep. on $a, L, m_\pi$ ($\geq 190$ MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td><strong>Including Disconnected (light &amp; strange) Quark Contractions</strong></td>
</tr>
</tbody>
</table>

S. Syritsyn (LHP), R. Gupta (NME)
Nucleon “Charges” $g_{A,S,T}$

[*(P)N(D)ME, Lattice’14]*

- $g_{S,T}$ “charges” = couplings to BSM physics in precision meas. of $\beta$-decay [LANL]
- Clover-improved Wilson valence quarks on HISQ lattices
- Extrapolation in $a, L, m_\pi$: $g(a, m_\pi, L) = g^0 + \alpha a + \beta m_\pi^2 + \gamma e^{-m_\pi L}$
Scientific Objectives

Nucleon Vector (EM) Form Factors

\[ \langle P + q | \bar{q} \gamma^\mu q | P \rangle = \bar{U}_{P+q} \left[ F_1(Q^2) \gamma^\mu + \frac{F_2(Q^2)}{2M_N} i \sigma^{\mu\nu} q_\nu \right] U_P \]

- Form Factors: \((F_1 / F_2)\) scaling, \((G_E / G_M)\), \(u-, d\)-contributions

\[ m_\pi = 149 \text{ MeV data vs Phenomenology} \ [J.\text{Green et al}(LHP), \text{PRD90:074507}(2014)] \]

- Proton radius: 7σ difference; JLab pRAD, MUSE \((e^\pm, \mu^\pm-p)\)

\[ \text{(Nature) } \]

\[ \text{(Scientific American) } \]

\[ \text{(The Proton Problem) } \]
Nucleon Axial Form Factors

\[ \langle P + q | \bar{q} \gamma^\mu \gamma^5 q | P \rangle = \bar{U}_{P+q} \left[ G_A(Q^2) \gamma^\mu \gamma^5 + \frac{G_P(Q^2) \gamma^5 q^\mu}{2M_N} \right] U_P \]

- \( G_A(Q^2) \) are measured in \( \nu \)-scattering, \( \pi \)-production;
  - implications for neutrino flux norm. in IceCube, etc
- Axial radius \( (r_A^2) = \frac{12}{m_A^2} \): model dependence
  - varying nuclear / \( G_A \) shape models: \( m_A = 0.9 \ldots 1.4 \text{ GeV} \)
- Strange quark \( G_{sA,P}(Q^2) \): MiniBooNE
- \( G_P(Q^2) \) induced pseudoscalar: \( \mu \) capture (MuCAP)

Physical \( m_\pi \), chiral quarks [LHP & RBC collabs, Lattice’14]
Nucleon Gen. F.F.s and Nucleon Spin

\[ \langle N(p + q) | T_{\mu\nu}^{q,\text{glue}} | N(p) \rangle \rightarrow \left\{ A_{20}, B_{20}, C_{20} \right\} (Q^2) \]

\[ \langle x \rangle_q = A_{20}^q(0) \]

\[ J_{q,\text{glue}} = \frac{1}{2} \left[ A_{20}^{q,\text{glue}}(0) + B_{20}^{q,\text{glue}}(0) \right] \]

\[ J_{\text{glue}} + \sum_q J_q = \frac{1}{2}, \]

\[ J_q = \frac{1}{2} \Delta \Sigma_q + L_q \]

\[ \frac{1}{2} \Delta \Sigma_{u+d}, \Sigma_{u+d} \]

\[ J_{u,d} \]

\[ J_{u,d, \Sigma_{u+d}} \]

\[ \text{discrepancy with } \chi\text{QCD result for } L_{u+d} \]

\[ \text{disconnected contributions are not included} \]
**Scientific Objectives**

**Nucleon Structure with Wilson Clover Lattices**

**USQCD All-Hands Meeting, Fermilab, May 1 & 2**

**Transverse Momentum-Dependent Distributions**

**SIDIS**

\[
l + N(P) \rightarrow l' + N(P_h) + X
\]

Non-local lattice operator

\[
\Phi(b, P, S, \hat{\zeta}, \mu) = \frac{1}{2} \langle P, S | \bar{q}(0) \Gamma U(\eta v, b) q(b) | P, S \rangle
\]

with spacelike link path \(U = \)

probes \(k_\perp\)-moments (“shifts”) of TMDs

\[
\sim \int dx \int d^2 k_\perp k_i f(x, k_\perp)
\]

valence DWF on Asqtad

[B. Musch, P. Hägler, M. Engelhardt, J. Negele, A. Schäfer]

“light-cone” limit

\[
\hat{\zeta} = \frac{P \cdot v}{m_N |v|} \rightarrow \infty
\]

operator localized at Euclidean time \(\tau\)

Sivers–Shift, \(u–d\) – quarks

\[
m_{\Pi} f_1^{u,d}(0) / f_1
\]

\[
\hat{\zeta} = 0.39,
|b_T| = 0.36 \text{ fm},
m_{\pi} = 518 \text{ MeV}
\]

DY

SIDIS

\(\eta |v| \) (lattice units)
Calculation Details and Improvements

- Kinematics to access high-momentum form factors:
  - include $|p_{\text{sink}}| \sim 1 \text{ GeV}^2$ (up to $Q^2 \sim 4 \text{ GeV}^2$ in Breit frame)
  - TMDs also require high momentum in-,out-states $|p_{\text{sink}}| = |p_{\text{source}}|$

- Variational method to reduce excited states:
  - 2x2 nucleon correlators with varied source smearing
  - optimize nucleon operators both zero/low and high momentum states

- Improved sampling with *All-Mode-Averaging*:
  - exact low-mode deflation OR truncated multigrid solver
  - ~4,000 (exact+sloppy) samples for the lightest $m_{\pi}=190 \text{ MeV}$

- Disconnected quark loops (light and strange) with variance reduction:
  - hierarchical probing
  - low-eigenmode deflation
Nucleon Excited States and SNR

Stochastic noise grows rapidly with $T$, especially with light pions [Lepage’89]:

$$\text{Signal} \quad \langle N(T)\bar{N}(0) \rangle$$
$$\text{Noise} \quad \langle |N(T)\bar{N}(0)|^2 \rangle - |\langle N(T)\bar{N}(0) \rangle|^2$$

$$\text{Signal/Noise} \quad \sim e^{-M_N T} \quad \sim e^{-3m_\pi T} \quad \sim e^{-(M_N - \frac{3}{2}m_\pi) T}$$

Physical point: SNR $\sim x(1/2)$ every (2a)

Multi-exp. fits of $T$-dependence : determined by the largest $T$

Variational method: $(-)$expensive $\sim (N_{op})^2$, $(+)$greatly extend plateaus [CSSM]

Proposal: explore and compare cost / benefit variational vs traditional
Disconnected Quark Contractions

Hierarchical probing [K.Orginos, A.Stathopoulos, ’13]:
In sum over $2^{dk+1}$ vectors (d=3),
dist(x,y) ≤ $2^k$ terms cancel exactly:

$$1 \leq \sum_{a} |x_a - y_a| \leq 2^k : \quad \frac{1}{N} \sum_{i} z_i(x)z_i(y)^\dagger \equiv 0$$

$$z_i \xrightarrow{a} z_i \odot \xi , \quad \xi(x) = \text{random } \mathbb{Z}_2\text{-vector}$$

NEW: reduce variance by treating low modes of $(\hat{D}^\dagger \hat{D})$ exactly [K.Orginos et al]

Disconnected diagrams with JLab isotropic Clover [S.Meinel’s USQCD project ’13; in prep.]
Total Request for the Joint Proposal

Computing resources request was updated to reflect non-overlapping goals in the proposals:

- LHP requested 43M
- NME requested 47M

Computing resources request was updated to reflect non-overlapping goals in the proposals:

- [common] connected and disconnected 3pt correlators on the lightest pion ensemble $m_\pi=190$ MeV: **32.8M**
- [NMEp] calculations with the heavier pion masses: **+8.2M**
- [LHPp] additional contractions (GFFs, TMDs) the lightest pion ensemble: **+9.5M**
- [common] exploration of variational method and source tuning: **+6M**

Total combined request: **56.5M**
Summary

- High-statistics, high precision nucleon structure calculations with very wide scope
  - proton form factors and charge radius
  - proton spin puzzle
  - applications to BSM and CPV searches
  - parton distributions

- Exploration of new techniques crucial for calculations at the physical point

- Equal emphasis on Connected andDisconnected (Light and Strange) contributions to the nucleon structure

*We are hopeful that the USQCD will support not only this proposal, but also generation of physical point Wilson-clover lattices*
Even though we are not requesting resources for lattice generation, we show the performance of the Multi-Grid library on NCSA BlueWaters in Fig. 7 (left) using QDP-JIT+QUDA. We show the performance of the Multi-Grid library on NCSA BlueWaters in Fig. 7 (left) using QDP-JIT+QUDA. The strong-scaling performance of the Algebraic Multi-Grid algorithm from the QDP-MG library on NCSA BlueWaters and comparison to performance on GPUs. (right) Improvement in lattice generation cost on Titan Nodes (GPUs).