The Nucleon Axial-Vector Form Factor at the Physical Point with the HISQ Ensembles

Aaron Meyer (asmeyer2012@uchicago.edu)
University of Chicago/Fermilab

May 1-2, 2015

USQCD All-Hands Meeting

Fermilab Lattice/MILC Collaborations
+ Richard Hill (UChicago) + Ruizi Li (Wuppertal)

with support from URA Visiting Scholars Program, DOE SCGSR

Calculation Overview

We will use MILC HISQ 2+1+1 ensembles at $M_\pi = 135$ MeV with 3 lattice spacings

Our primary objectives are to calculate:

- $g_A, F_A(q^2) \rightarrow$ Neutrino CCQE/Oscillation Studies
- $g_V, F_V(q^2) \rightarrow$ Validation Check/Proton Radius Puzzle

As a byproduct, we will also get:

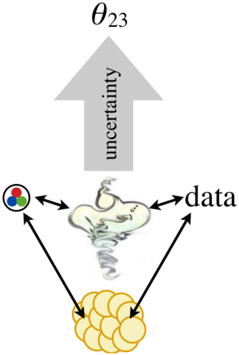
- $F_P(q^2) \rightarrow \nu_\tau$ interaction cross section
- $g_S \rightarrow$ Dark matter searches and μ to e conversion
- $g_T \rightarrow$ New physics in neutron β decay

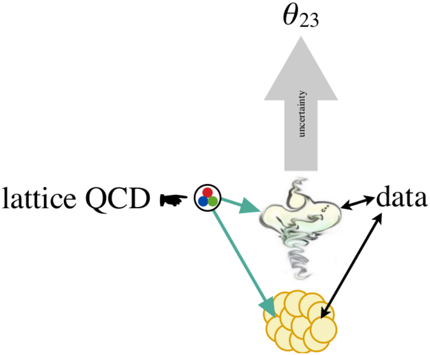
Motivation

Neutrino physics needs a better understanding of axial form factor:

- Model-dependent shape parameterization introduces systematic uncertainties and underestimates errors
- Nuclear effects entangled with nucleon cross sections
- Measurement of oscillation parameters depends on nuclear models and nucleon form factors

Circle of Uncertainty





Nucleon-level/Nucleus-level effects entangled
Measurements of observables are model-dependent
LQCD acts as disruptive techology to break the cycle

Discrepancies in the Axial-Vector Form Factor

Most analyses assume the "Dipole form factor":

$$F_A^{\text{dipole}}(q^2) = g_A \frac{1}{\left(1 - \frac{q^2}{m_A^2}\right)^2} \tag{1}$$

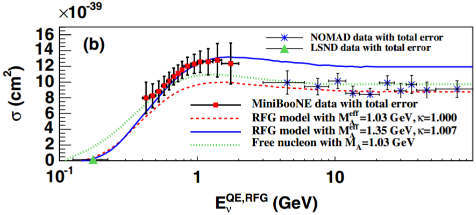
Dipole is an ansatz:

unmotivated in interesting energy range

→ uncontrolled systematics and underestimated uncertainties

Essential to replace ansatz with model-independent

ab-initio calculation from Lattice QCD



MiniBooNE Collab., PHYS REV D 81, 092005 (2010)

Calculation Method

$$A_{\perp}^{\mu}(q^2) = A^{\mu}(q^2) - \frac{q^{\mu}}{q^2} q \cdot A \tag{2}$$

$$F_A\left(q^2\right) \overline{u}_p(q) \gamma_{\perp}^{\mu} \gamma^5 u_n(0) =$$

$$\left.\frac{\langle N'|\, Z_A A^a_{\perp\,\mu}\left(q^2\right) |N\rangle}{\langle 0|\, Z_A A_0^a\,(0)\,|\pi^a\rangle\, \omega^2}\right|_{\text{this work}} \left.\frac{\langle 0|\, 2\hat{m} P^a\,(0)\,|\pi^a\rangle}{M_\pi}\right|_{\text{(ref)}} \omega^2\Big|_{a\to 0} \tag{3}$$

Calculation applies for $q^2 = 0$ without issue
Renormalization cancels

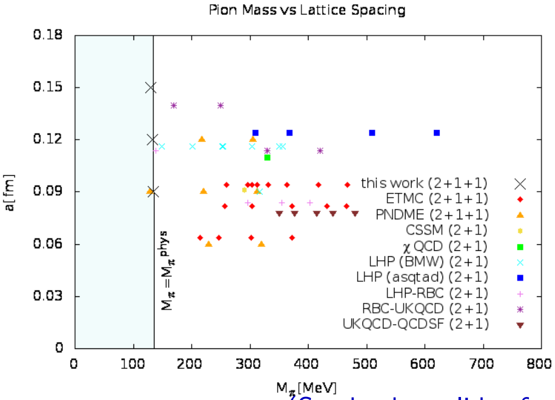
Will fit form factor using a model-independent parameterization:

$$z(t; t_0, t_c) = \frac{\sqrt{t_c - t} - \sqrt{t_c - t_0}}{\sqrt{t_c - t} + \sqrt{t_c - t_0}} \qquad F_A(z) = \sum_{n=0}^{\infty} a_n z^n \tag{4}$$

As validated by B meson physics, only a few coefficients necessary to accurately represent data

Current Calculations of g_A

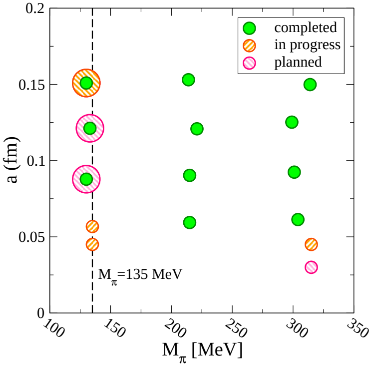
Other collaborations have at most one ensemble for one lattice spacing at physical pion mass



(See backup slides for references)

Advantages of HISQ

- No explicit chiral symmetry breaking in m→0 limit
- No exceptional configurations
- No chiral extrapolation (physical π mass only)
- Several lattice spacings (true continuum extrapolation)
- Can go to high statistics easily (HISQ is fast)



Taste Mixing

$SU(2)_I \times GTS$ irrep	#N	#Δ
$\left(\frac{3}{2}, \mathbf{8}\right)$	3	2
$\left(\frac{3}{2}, \mathbf{8'}\right)$	0	2
$\left(\frac{3}{2}, \mathbf{16}\right)$	1	3
$\left(\frac{1}{2}, \mathbf{8}\right)$	5	1
$\left(\frac{1}{2}, \mathbf{8'}\right)$	0	1
$\left(\frac{1}{2}, \mathbf{16}\right)$	3	4

(J. A. Bailey)

Group theory for staggered baryons is under control

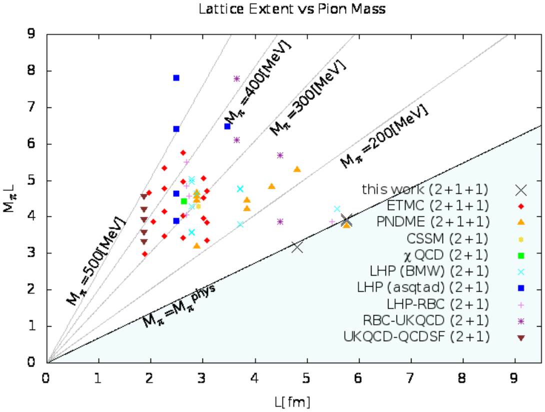
3-point functions use same operator basis as 2-point functions

Set of 4 operators to generate a 4×4 matrix of correlators
for variational analysis

Irrep $\left(\frac{3}{2}, 16\right)$ chosen because of number of N/Δ states

Can get priors from fitting different taste Δ states
from $\left(\frac{3}{2}, 8'\right)$ and $\left(\frac{1}{2}, 8'\right)$ operators

Finite Size Effects



Doing as well as other calculations at physical masses

MILC $g-2$ proposal to generate $a = 0.15$ fm ensemble at larger L

$\rightarrow$ can use for finite volume study

Estimate finite-size effects with χPT and Lüscher methodology

Excited State Removal

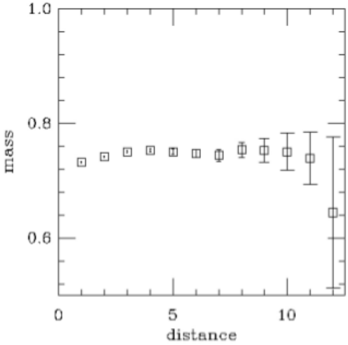
Will employ a number of techniques to remove excited states:

- N-state excited state analysis using Gaussian priors
- Variational method, cf. arXiv:1411.4676 [hep-lat] and R. Li (IU Ph.D. Thesis)
- Random wall sources
- Gaussian smearing at source and sink
- Multiple source/sink separations
- Simultaneous fitting of 2-point/3-point functions
- Signal to noise optimization
- Data will be analyzed with a blinding factor applied to the 3-point function

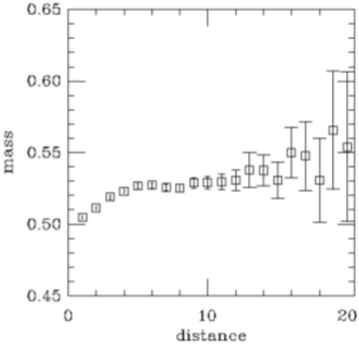
MILC Collaboration, R. Li offer expertise for many of these methods

Effective Mass Plots

$a \approx 0.12$ fm, $\frac{m_\ell}{m_s} \approx 0.16$



$a \approx 0.09$ fm, $\frac{m_\ell}{m_s} \approx 0.22$



(R. Li, Ph.D. Thesis)

Variational method has been tested/verified on HISQ

Plateau after 3-4/4-5 timeslices

→ 3-point source/sink separation approximately 2× larger

→ Scaled with physical dimensions

Excited states are under control

Resource Request

$\approx a$ fm	$N_S^3 \times N_T$	N_{confs}	$8N_\zeta(N_p + N_{\text{sink}})$ M Jpsi-core-hr
0.15	$32^3 \times 48$	1000	0.57
0.12	$48^3 \times 64$	1000	4.71
0.09	$64^3 \times 96$	1047	23.07
		total	28.35

8 staggered baryon cube corners
N_ζ color vectors (using 3)
N_p momenta (using 3)
N_{sink} sinks (source/sink separations) (using 2)
$= 120$ inversions total per gauge configuration

Conclusions

Neutrino physics is subject to
underestimated and model-dependent systematics

$\rightarrow$ To reduce systematics from modeling,
need to understand nuclear physics

$\rightarrow$ To understand nuclear physics,
need to understand nucleon-level cross sections

HISQ ensembles will produce a high-statistics calculation of the axial form factor at physical pion mass and provide a model-independent description of nucleon-level physics

Other areas of study to address in the future to further the Fermilab neutrino program:

- $\nu_\ell N \rightarrow \nu_\ell N'$
- N-Δ transition currents
- $\nu_\ell N \rightarrow \pi \ell N'$
- $\nu_\ell N \rightarrow \pi \ell \Sigma$

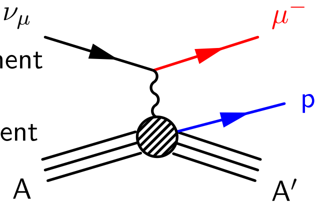
Backup Slides

Nuclear Effects

Nuclear effects not well understood

$\rightarrow$ Models which are best for one measurement are worst for another

Need to break F_A/nuclear model entanglement



(assumed $m_A = 0.99$ GeV, reference hyperlinks online)

NuWro Model (χ^2/DOF)	RFG [GENIE]	RFG+ TEM	assorted others
leptonic(rate)	3.5	2.4	2.8-3.7
leptonic(shape)	4.1	1.7	2.1-3.8
hadronic(rate)	1.7[1.2]	3.9	1.9-3.7
hadronic(shape)	3.3[1.8]	5.8	3.6-4.8

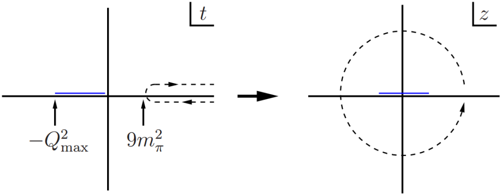
Form Factor q^2 Interpolation

z-Expansion is a model-independent description of the axial form factor

$$t = q^2 = -Q^2 \qquad t_c = 9m_\pi^2$$

$$z(t; t_0, t_c) = \frac{\sqrt{t_c - t} - \sqrt{t_c - t_0}}{\sqrt{t_c - t} + \sqrt{t_c - t_0}} \tag{5}$$

$$F_A(z) = \sum_{n=0}^{\infty} a_n z^n \tag{6}$$



Maps kinematically allowed region ($t \leq 0$) to within $z = \pm 1$

From B meson physics, only a few coefficients necessary to accurately represent data

z-Expansion implemented in GENIE, to be released soon [autumn]

Error Budgets

Source of Uncertainty	MINOS Absolute/ν_e	T2K ν_e	LBNE ν_e	Comments
Beam Flux after N/F extrapolation	3%/0.3%	2.9%	2%	MINOS is normalization only. LBNE normalization and shape highly correlated between ν_μ/ν_e.
Detector effects				
Energy scale (ν_μ)	7%/3.5%	included above	(2%)	Included in LBNE ν_μ sample uncertainty only in three-flavor fit. MINOS dominated by hadronic scale.
Absolute energy scale (ν_e)	5.7%/2.7%	3.4% includes all FD effects	2%	Totally active LArTPC with calibration and test beam data lowers uncertainty.
Fiducial volume	2.4%/2.4%	1%	1%	Larger detectors = smaller uncertainty.
Neutrino interaction modeling				
Simulation includes: hadronization cross sections nuclear models	2.7%/2.7%	7.5%	$\sim 2\%$	Hadronization models are better constrained in the LBNE LArTPC. N/F cancellation larger in MINOS/LBNE. X-section uncertainties larger at T2K energies. Spectral analysis in LBNE provides extra constraint.
Total	**5.7%**	**8.8%**	**3.6 %**	**Uncorrelated ν_e uncertainty in full LBNE three-flavor fit = 1-2%.**

LBNE Experiment

g_A Calculation references

ETMC
S. Dinter et al. arXiv:1108.1076 [hep-lat]
PNDME
T. Bhattacharya et al. arXiv:1306.5435 [hep-lat]
T. Bhattacharya, R. Gupta, and B. Yoon arXiv:1503.05975 [hep-lat]
R. Gupta, T. Bhattacharya, A. Joseph, H.-W. Lin, and B. Yoon arXiv:1501.07639 [hep-lat]
CSSM
B. J. Owen et al., arXiv:1212.4668 [hep-lat]
χ**QCD**
Y.-B. Yang, M. Gong, K.-F. Liu, and M. Sun arXiv:1504.04052 [hep-ph]
LHP(BMW)
J. Green et al., arXiv:1211.0253 [hep-lat]
S. N. Syritsyn et al., arXiv:0907.4194 [hep-lat]
S. Dürr et al. arXiv:1011.2711 [hep-lat]
LHP(asqtad)
S. N. Syritsyn, Exploration of nucleon structure in lattice QCD with chiral quarks, Ph.D. thesis, Massachusetts Institute of Technology (2010).
LHP-RBC
S. Syritsyn et al. arXiv:1412.3175 [hep-lat]
RBC-UKQCD
S. Ohta arXiv:1309.7942 [hep-lat]
UKQCD-QCDSF
M. Göckeler et al. arXiv:1102.3407 [hep-lat]