The Nucleon Axial-Vector Form Factor at the Physical Point with the HISQ Ensembles

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Fermilab Lattice/MILC Collaborations
+ Richard Hill (UChicago) + Ruizi Li (Wuppertal)
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Calculation Overview

We will use MILC HISQ 2+1+1 ensembles at $M_\pi = 135$ MeV with 3 lattice spacings.

Our primary objectives are to calculate:

- $g_A, F_A(q^2) \rightarrow$ Neutrino CCQE/Oscillation Studies
- $g_V, F_V(q^2) \rightarrow$ Validation Check/Proton Radius Puzzle

As a byproduct, we will also get:

- $F_P(q^2) \rightarrow \nu_\tau$ interaction cross section
- $g_S \rightarrow$ Dark matter searches and $\mu$ to $e$ conversion
- $g_T \rightarrow$ New physics in neutron $\beta$ decay
Neutrino physics needs a better understanding of axial form factor:

- Model-dependent shape parameterization introduces systematic uncertainties and underestimates errors
- Nuclear effects entangled with nucleon cross sections
- Measurement of oscillation parameters depends on nuclear models and nucleon form factors
Nucleon-level/Nucleus-level effects entangled
Measurements of observables are **model-dependent**
LQCD acts as **disruptive technology** to break the cycle
Discrepancies in the Axial-Vector Form Factor

Most analyses assume the “Dipole form factor”:

\[ F_A^{\text{dipole}}(q^2) = g_A \frac{1}{\left(1 - \frac{q^2}{m^2_A}\right)^2} \]  (1)

Dipole is an ansatz:
- unmotivated in interesting energy range
- \text{uncontrolled systematics and underestimated uncertainties}

Essential to replace ansatz with \text{model-independent}
\text{ab-initio calculation from Lattice QCD}

MiniBooNE Collab., PHYS REV D 81, 092005 (2010)
Calculation Method

\[ A_\perp^\mu(q^2) = A^\mu(q^2) - \frac{q^\mu}{q^2} q \cdot A \]  

\[ F_A(q^2) \bar{u}_p(q) \gamma_\perp^\mu \gamma^5 u_n(0) = \]

\[ \frac{\langle N' | Z_A A_\perp^a(q^2) | N \rangle}{\langle 0 | Z_A A_0^a(0) | \pi^a \rangle \omega^2} \left| \begin{array}{c} \text{this work} \\ \text{(ref)} \end{array} \right| \]

Calculation applies for \( q^2 = 0 \) without issue
Renormalization cancels

Will fit form factor using a model-independent parameterization:

\[ z(t; t_0, t_c) = \frac{\sqrt{t_c - t} - \sqrt{t_c - t_0}}{\sqrt{t_c - t} + \sqrt{t_c - t_0}} \]

\[ F_A(z) = \sum_{n=0}^{\infty} a_n z^n \]  

As validated by B meson physics, only a few coefficients necessary to accurately represent data
Other collaborations have at most one ensemble for one lattice spacing at physical pion mass

(See backup slides for references)
Advantages of HISQ

- No explicit chiral symmetry breaking in $m \to 0$ limit
- No exceptional configurations
- No chiral extrapolation (physical $\pi$ mass only)
- Several lattice spacings (true continuum extrapolation)
- Can go to high statistics easily (HISQ is fast)
Taste Mixing

\[ SU(2)_I \times GTS \text{ irrep} \quad \#N \quad \#\Delta \]

<table>
<thead>
<tr>
<th>Irrep</th>
<th>#N</th>
<th>#\Delta</th>
</tr>
</thead>
<tbody>
<tr>
<td>((\frac{3}{2}, 8))</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>((\frac{3}{2}, 8'))</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>((\frac{3}{2}, 16))</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>((\frac{1}{2}, 8))</td>
<td>5</td>
<td>1</td>
</tr>
<tr>
<td>((\frac{1}{2}, 8'))</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>((\frac{1}{2}, 16))</td>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>

(J. A. Bailey)

Group theory for staggered baryons is under control

3-point functions use same operator basis as 2-point functions

Set of 4 operators to generate a \(4 \times 4\) matrix of correlators

for variational analysis

Irrep \((\frac{3}{2}, 16)\) chosen because of number of \(N/\Delta\) states

Can get priors from fitting different taste \(\Delta\) states

from \((\frac{3}{2}, 8')\) and \((\frac{1}{2}, 8')\) operators
Finite Size Effects

Doing as well as other calculations at physical masses
MILC $g - 2$ proposal to generate $a = 0.15$ fm ensemble at larger $L$
$\rightarrow$ can use for finite volume study
Estimate finite-size effects with $\chi$PT and Lüscher methodology
Excited State Removal

Will employ a number of techniques to remove excited states:

- N-state excited state analysis using Gaussian priors
- Random wall sources
- Gaussian smearing at source and sink
- Multiple source/sink separations
- Simultaneous fitting of 2-point/3-point functions
- Signal to noise optimization
- Data will be analyzed with a blinding factor applied to the 3-point function

MILC Collaboration, R. Li offer expertise for many of these methods
Effective Mass Plots

\[ a \approx 0.12 \text{ fm}, \frac{m_\ell}{m_s} \approx 0.16 \]

\[ a \approx 0.09 \text{ fm}, \frac{m_\ell}{m_s} \approx 0.22 \]

Variational method has been tested/verified on HISQ Plateau after 3-4/4-5 timeslices

→ 3-point source/sink separation approximately 2× larger

→ Scaled with physical dimensions

Excited states are under control

(R. Li, Ph.D. Thesis)
Resource Request

<table>
<thead>
<tr>
<th>≈ a</th>
<th>( N_{S}^{3} \times N_{T} )</th>
<th>( N_{\text{confs}} )</th>
<th>8( N_{\zeta}(N_{p} + N_{\text{sink}}) ) M Jpsi-core-hr</th>
</tr>
</thead>
<tbody>
<tr>
<td>fm</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.15</td>
<td>32^3 \times 48</td>
<td>1000</td>
<td>0.57</td>
</tr>
<tr>
<td>0.12</td>
<td>48^3 \times 64</td>
<td>1000</td>
<td>4.71</td>
</tr>
<tr>
<td>0.09</td>
<td>64^3 \times 96</td>
<td>1047</td>
<td>23.07</td>
</tr>
<tr>
<td>total</td>
<td></td>
<td></td>
<td>28.35</td>
</tr>
</tbody>
</table>

8 staggered baryon cube corners
\( N_{\zeta} \) color vectors (using 3)
\( N_{p} \) momenta (using 3)
\( N_{\text{sink}} \) sinks (source/sink separations) (using 2)
= 120 inversions total per gauge configuration
Conclusions

Neutrino physics is subject to underestimated and model-dependent systematics

→ To reduce systematics from modeling, need to understand nuclear physics
→ To understand nuclear physics, need to understand nucleon-level cross sections

HISQ ensembles will produce a high-statistics calculation of the axial form factor at physical pion mass and provide a model-independent description of nucleon-level physics

Other areas of study to address in the future to further the Fermilab neutrino program:

• $\nu_\ell N \rightarrow \nu_\ell N'$
• N-$\Delta$ transition currents
• $\nu_\ell N \rightarrow \pi_\ell N'$
• $\nu_\ell N \rightarrow \pi_\ell \Sigma$
Backup Slides
Nuclear Effects

Nuclear effects not well understood
→ Models which are best for one measurement are worst for another
Need to break $F_A$/nuclear model entanglement

(assumed $m_A = 0.99$ GeV, reference hyperlinks online)

<table>
<thead>
<tr>
<th>NuWro Model (χ²/DOF)</th>
<th>RFG</th>
<th>RFG+</th>
<th>assorted others</th>
</tr>
</thead>
<tbody>
<tr>
<td>leptonic(rate)</td>
<td>3.5</td>
<td>2.4</td>
<td>2.8-3.7</td>
</tr>
<tr>
<td>leptonic(shape)</td>
<td>4.1</td>
<td>1.7</td>
<td>2.1-3.8</td>
</tr>
<tr>
<td>hadronic(rate)</td>
<td>1.7[1.2]</td>
<td>3.9</td>
<td>1.9-3.7</td>
</tr>
<tr>
<td>hadronic(shape)</td>
<td>3.3[1.8]</td>
<td>5.8</td>
<td>3.6-4.8</td>
</tr>
</tbody>
</table>
Form Factor $q^2$ Interpolation

**z-Expansion** is a model-independent description of the axial form factor

\[ t = q^2 = -Q^2 \quad t_c = 9m^2_{\pi} \]

\[ z(t; t_0, t_c) = \frac{\sqrt{t_c - t} - \sqrt{t_c - t_0}}{\sqrt{t_c - t} + \sqrt{t_c - t_0}} \]

(5)

\[ F_A(z) = \sum_{n=0}^{\infty} a_n z^n \]

(6)

Maps kinematically allowed region ($t \leq 0$) to within $z = \pm 1$

From **B meson physics**, only a few coefficients necessary to accurately represent data

z-Expansion implemented in **GENIE**, to be released soon [autumn]
## Error Budgets

<table>
<thead>
<tr>
<th>Source of Uncertainty</th>
<th>MINOS Absolute/νₑ</th>
<th>T2K νₑ</th>
<th>LBNE νₑ</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam Flux after N/F extrapolation</td>
<td>3%/0.3%</td>
<td>2.9%</td>
<td>2%</td>
<td>MINOS is normalization only. LBNE normalization and shape highly correlated between νᵢ/νₑ.</td>
</tr>
<tr>
<td>Detector effects</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Energy scale (νᵢ)</td>
<td>7%/3.5%</td>
<td>included above</td>
<td>(2%)</td>
<td>Included in LBNE νᵢ sample uncertainty only in three-flavor fit. MINOS dominated by hadronic scale.</td>
</tr>
<tr>
<td>Absolute energy scale (νₑ)</td>
<td>5.7%/2.7%</td>
<td>3.4%</td>
<td>2%</td>
<td>Totally active LArTPC with calibration and test beam data lowers uncertainty.</td>
</tr>
<tr>
<td>Fiducial volume</td>
<td>2.4%/2.4%</td>
<td>1%</td>
<td>1%</td>
<td>Larger detectors = smaller uncertainty.</td>
</tr>
</tbody>
</table>

### Neutrino interaction modeling

- Simulation includes: hadronization cross sections nuclear models
  - 2.7%/2.7% 7.5% ~ 2%
  - Hadronization models are better constrained in the LBNE LArTPC. N/F cancellation larger in MINOS/LBNE. X-section uncertainties larger at T2K energies. Spectral analysis in LBNE provides extra constraint.

### Total

- 5.7% 8.8% 3.6%
- Uncorrelated νₑ uncertainty in full LBNE three-flavor fit = 1-2%.
Calculation references

**ETMC**
S. Dinter et al. [arXiv:1108.1076 [hep-lat]]

**PNDME**
T. Bhattacharya et al. [arXiv:1306.5435 [hep-lat]]

**CSSM**
B. J. Owen et al., [arXiv:1212.4668 [hep-lat]]

**χQCD**

**LHP(BMW)**
J. Green et al., [arXiv:1211.0253 [hep-lat]]
S. N. Syritsyn et al., [arXiv:0907.4194 [hep-lat]]
S. Dürr et al. [arXiv:1011.2711 [hep-lat]]

**LHP(asqtad)**

**LHP-RBC**
S. Syritsyn et al. [arXiv:1412.3175 [hep-lat]]

**RBC-UKQCD**
S. Ohta [arXiv:1309.7942 [hep-lat]]

**UKQCD-QCDSF**
M. Göckeler et al. [arXiv:1102.3407 [hep-lat]]