Muon $g-2$ Hadronic Vacuum Polarization from 2+1+1 flavors of sea quarks using the HISQ action

Jack Laiho

Syracuse University

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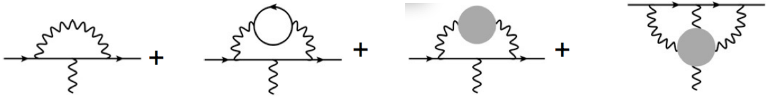
Motivation

The muon anomalous magnetic moment is currently measured to a precision of around a half a part per million, with a similar error quoted for the theory prediction.

$$a_\mu^{\mathrm{exp}} = 116\,592\,089(54)(33) \times 10^{-11}, \quad a_\mu^{\mathrm{exp}} - a_\mu^{\mathrm{SM}} = 287(80) \times 10^{-11}[3.6\sigma] \quad (1)$$

The experimental value deviates from the Standard Model prediction by 3-4 σ making it an interesting thing to study. Errors are completely dominated by hadronic contributions. There are two types: Hadronic Vacuum Polarization (HVP) and Hadronic Light-by-Light (HLbL).

Muon $g-2$



QED(4 loops) + EW (2 loops) + HVP + HLbL

Contribution	Result ($\times 10^{11}$)		Error
QED (leptons)	$116\ 584\ 718 \pm 0.14$	$\pm\ 0.04_{\alpha}$	0.00 ppm
HVP(lo) [1]	$6\ 923 \pm 42$		0.36 ppm
HVP(ho)	$-98 \pm 0.9_{\text{exp}}$	$\pm\ 0.3_{\text{rad}}$	0.01 ppm
HLbL [2]	105 ± 26		0.22 ppm
EW	154 ± 2	$\pm\ 1$	0.02 ppm
Total SM	$116\ 591\ 802 \pm 49$		0.42 ppm

Hadronic Vacuum Polarization

The HVP contribution can be obtained experimentally via the inclusive cross-section of $e^+ e^-$ scattering into hadrons and via decays of the τ lepton into hadrons. There is some tension between these two experimental determinations, though the τ decay determination has isospin corrections that are difficult to control.

The current precision in HVP from experiment is 0.6%. Thus a lattice calculation at the 1% precision would already be interesting. Current lattice calculations have errors quoted at the $\sim 5\%$ level, with important systematics not yet included. We believe we can achieve the $\sim 1\%$ precision with the new HPQCD method and with extended HISQ ensembles at the physical light quark masses including all systematic errors.

Improvements to sub-percent precision would require QED effects to be included.

HVP on the lattice

Method of Blum, '02:

$$a_{\mu,\mathrm{HVP}} = \left(\frac{\alpha}{\pi}\right)^2 \int dq^2 f(q^2) 4\pi^2 [\Pi(0) - \Pi_V(q^2)] \tag{2}$$

One must calculate on the lattice the renormalized vacuum polarization function $\hat{\Pi}(q^2) \equiv \Pi(q^2) - \Pi(0)$.

The integrand peaks at $q^2 \sim \mathscr{O}(m_\mu^2)$.

The standard method requires a calculation of $\hat{\Pi}(q^2)$ at $q^2 > 0$ and an extrapolation to zero. This leads to large uncertainties.

HPQCD Method

For spatial currents at zero spatial momentum:

$$G_{2n} \equiv a^4 \sum_t \sum_{\vec{x}} t^{2n} Z_V^2 \langle j^i(\vec{x}, t) j^i(0) \rangle = (-1)^n \frac{\partial^{2n}}{\partial q^{2n}} q^2 \hat{\Pi}(q^2)|_{q^2=0}. \qquad (3)$$

$$\hat{\Pi}(q^2) = \sum_{j=1}^{\infty} q^{2j} \Pi_j, \qquad \Pi_j = (-1)^{j+1} \frac{G_{2j+2}}{(2j+2)!}. \qquad (4)$$

Time moments of the correlator give the derivatives of $q^2 = 0$ of $\hat{\Pi}$. $\hat{\Pi}(q^2)$ is replaced with its [2,2] Padé approximant derived from Π_j. This allows one to reach high momenta for the q^2 integration, which is done numerically. The result converges rapidly as one includes more Padé terms. (Chakraborty, 2014)

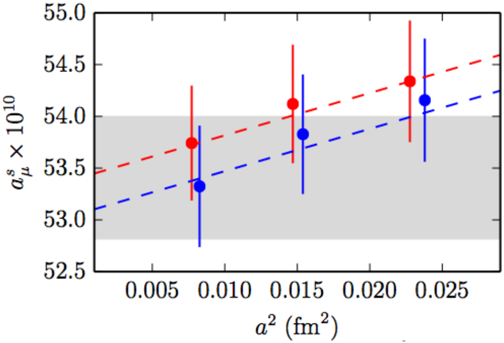
HPQCD Method

The vector current renormalization Z_V must be determined non-perturbatively. This will be done as part of the heavy-light project using our standard approach, as it is needed there anyway.

The HPQCD method trades a difficult to estimate (and forecast) systematic error for a statistical error. It is then a matter of computing to beat down the errors. It is helpful to have a reliable estimate of the cost for a given target precision. This is especially important when making promises to funding agencies.

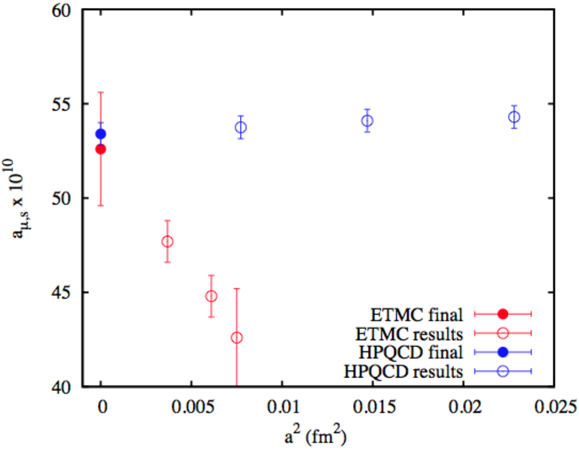
Result for a_μ^s

$$a_\mu^s = 53.41(59) \times 10^{-10}$$



Blue points: $m_\ell = m_s/5$, Red points: $m_\ell = m_\ell^{\mathrm{phys}}$. From HPQCD 2014.

Comparison to ETMC



Longer Range Plan

Table: Planned ensembles for use in this multi-year project.

a(fm)	$L^3 \times T$	# of configs	J/psi or Mira core-hours	Theory
≈ 0.15	$32^3 \times 48$	50,000	200 million J/psi core-hours	QCD
≈ 0.15	$48^3 \times 48$	3000	39 million J/psi core-hours	QCD
≈ 0.12	$48^3 \times 64$	10,000	256 million J/psi core-hours	QCD
≈ 0.09	$64^3 \times 96$	6000	330 million Mira core-hours	QCD
≈ 0.15	$32^3 \times 48$	50,000	400 million J/psi core-hours	QCD+QED
≈ 0.15	$48^3 \times 48$	3000	80 million J/psi core-hours	QCD+QED
≈ 0.12	$48^3 \times 64$	10,000	500 million J/psi core-hours	QCD+QED

USQCD Resource Request for 2015-16

Table: Computing and storage resource requests for this year's proposal. The computing time includes both configuration generation and analysis.

a(fm)	$L^3 \times T$	# of configs	J/psi or Mira core-hrs	Storage required
≈ 0.12	$48^3 \times 64$	3000	77 M J/psi core-hrs	5.7 TB
≈ 0.09	$64^3 \times 96$	2000	110 M Mira core-hrs	14 TB

The division is about 60/40 between configuration generation and measurement runs. We would make 4 light and strange quark propagators at 16 time sources per configuration (using random wall sources).

Responses to the SPC

1. Can you clarify the breakdown between the costs of configuration generation and correlator measurements?

The cost of configuration generation versus correlator measurements is 58/42, so that the cost for our $48^3 \times 64$ ensemble would be 44.6 million J/psi core-hours for configuration generation and 32.4 million J/psi core-hours for correlator measurements. The same breakdown is true for our $64^3 \times 96$ ensemble, with 63.8 million Mira core-hours going to configuration generation and 46.2 going to correlator measurements.

Responses to the SPC

2. Are there potential cost savings or additional advantages to be exploited by coordinating with Aubin proposal using the more standard method for HVP on the same or similar sets of ensembles?

The two calculations propose to use different valence actions. We are using HISQ, while Aubin et. al. are using a HISQ-like action but without the Naik improvement term. Thus, there is no possibility of potential cost savings from combining the calculations. The authors of the Aubin proposal agreed with us on this point when contacted.

Responses to the SPC

3. About half a dozen groups around the world are involved with hadronic vacuum polarization calculations. Please compare and contrast your effort with the others.

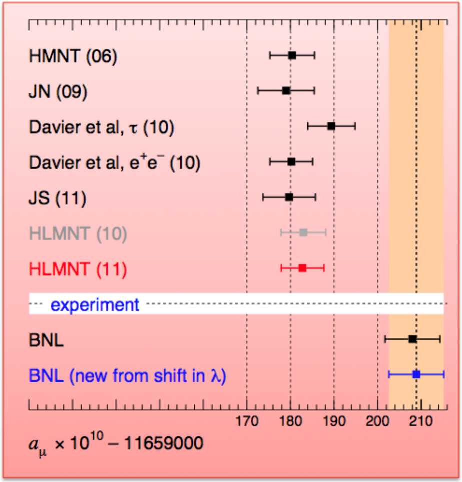
Other groups doing HVP calculations include Aubin et al. with staggered quarks, RBC/UKQCD with domain wall, the Mainz group with clover quarks, BMW with clover quarks, ETMC with twisted mass quarks. These groups (with the exception of BMW) are performing calculations at non-zero q^2 and are attempting to extrapolate the results to zero q^2 using various methods. This results in a systematic error that is large and can be difficult to estimate. By evaluating the relevant quantity directly at zero q^2, the HPQCD approach trades this systematic error for a statistical error that is well understood and can be reduced with enough computing. It is not yet clear how much computing the extrapolation approach requires to meet the same target precision. Given the importance of $g-2$ to the funding situation of US lattice QCD, it is important that we be able to deliver a result on a fixed time-scale with an error that can be predicted reliably at the same level as our historically very successful weak matrix element calculations.

Responses to the SPC

The advantages of using the HISQ action are significant; HISQ is the most highly Symanzik improved action being used in large scale simulations, and we are using 2+1+1 flavors (i.e., dynamical charm), which is important for reaching our target precision. The HISQ action is extremely fast to simulate compared to the competition, so we can amass larger statistics given similar resources. We plan to simulate directly at the physical quark masses with dynamical charm. Only the Aubin et al. calculation has both of these features, as far as we can tell from efforts that have been reported publicly.

Back-up Slides

Comparison


HMNT (06)
JN (09)
Davier et al, τ (10)
Davier et al, e+e− (10)
JS (11)
HLMNT (10)
HLMNT (11)
experiment
BNL
BNL (new from shift in λ)
170
180
190
200
210
$a_\mu \times 10^{10} - 11659000$