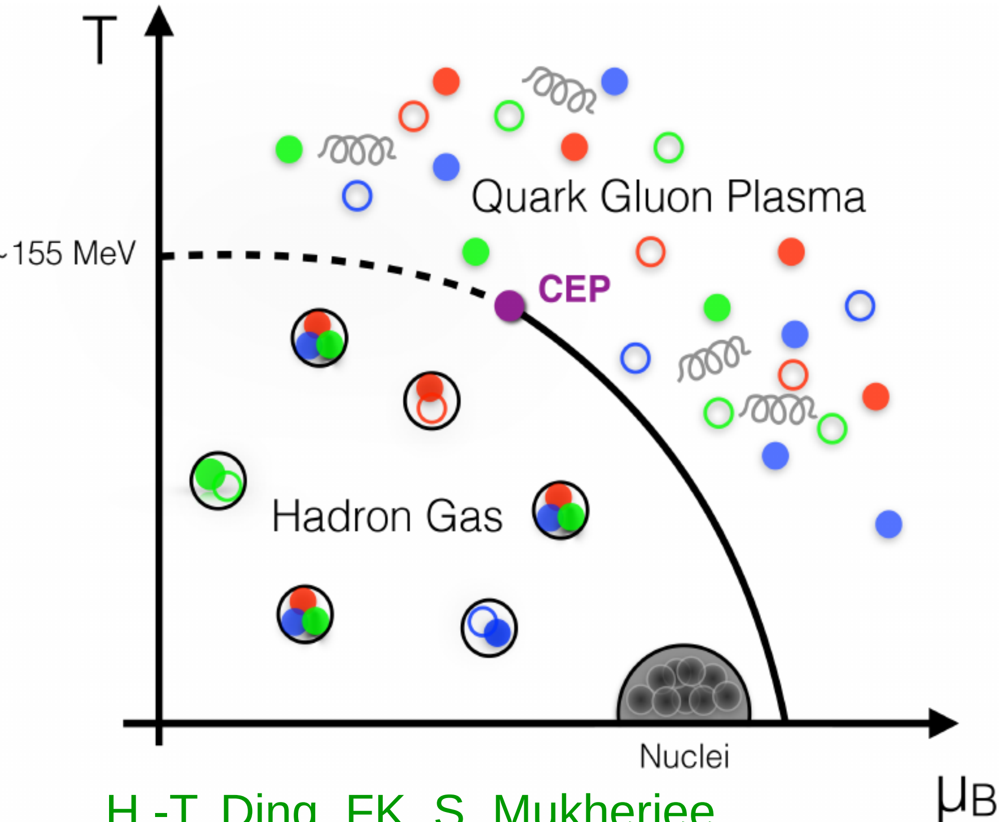
QCD thermodynamics

Frithjof Karsch, BNL/Bielefeld

USQCD All Hands Meeting
Fermilab, May 1-2, 2015



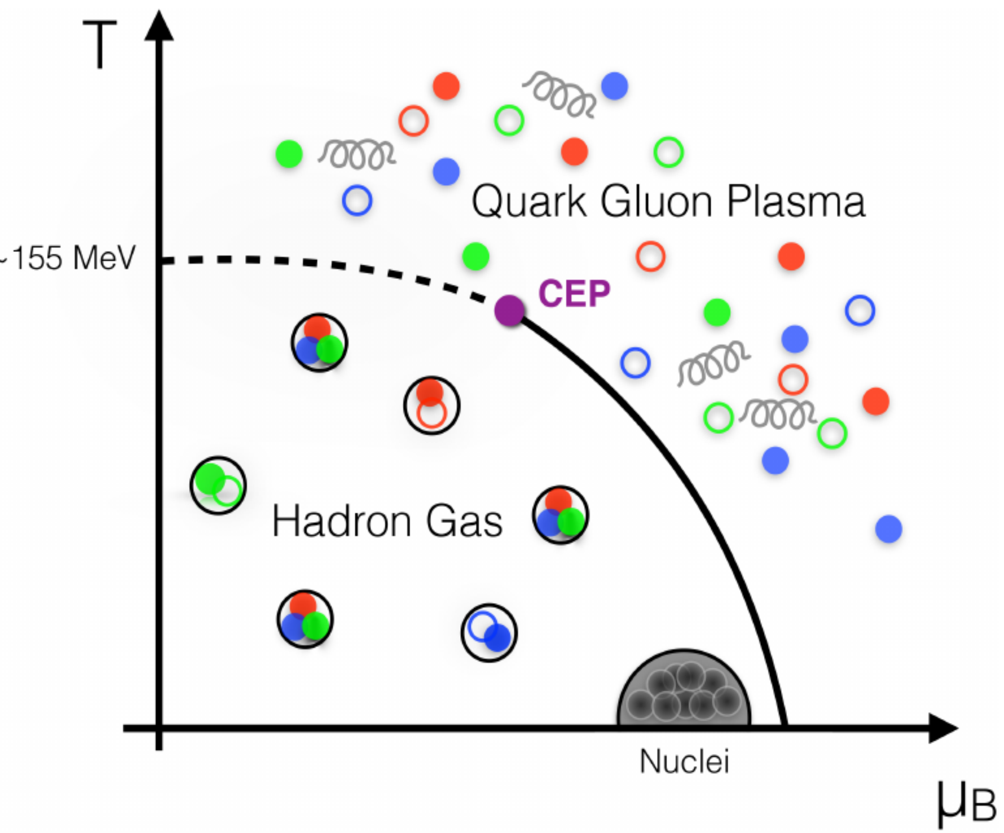
H.-T. Ding, FK, S. Mukherjee,
Thermodynamics of strong-interaction matter from Lattice QCD arXiv:1504.05274

Project Proposals:

Electric charge fluctuations in 2+1 flavor QCD
PI: F. Karsch (HotQCD Collaboration)

Non-Gaussian cumulants of conserved charges up to 6th order
PI: S. Mukherjee (BNL-Bielefeld-CCNU)

QCD thermodynamics and the Heavy Ion Collision Program at RHIC



Key Questions

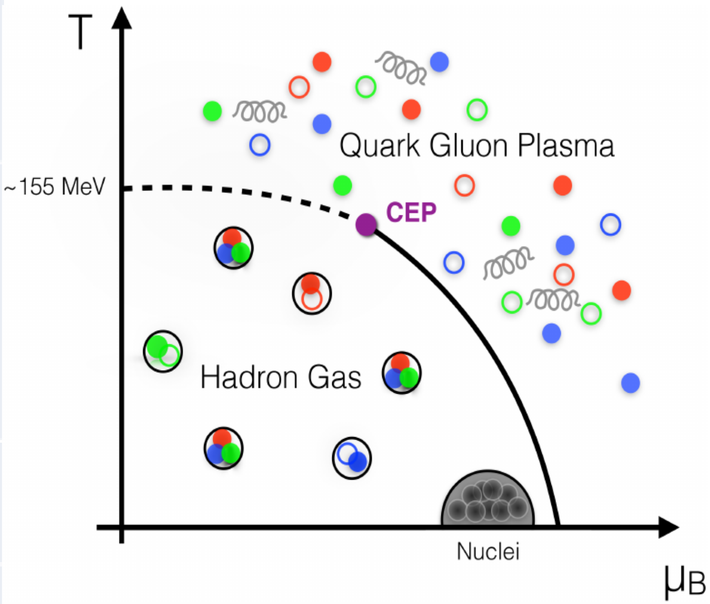
NSAC Long Range Plan 2007

- What are the phases of strongly interacting matter, and what role do they play in the cosmos?
- What does QCD predict for the properties of strongly interacting matter?
- What governs the transition of quarks and gluons into pions and nucleons?

NSAC Long Range Plan 2015

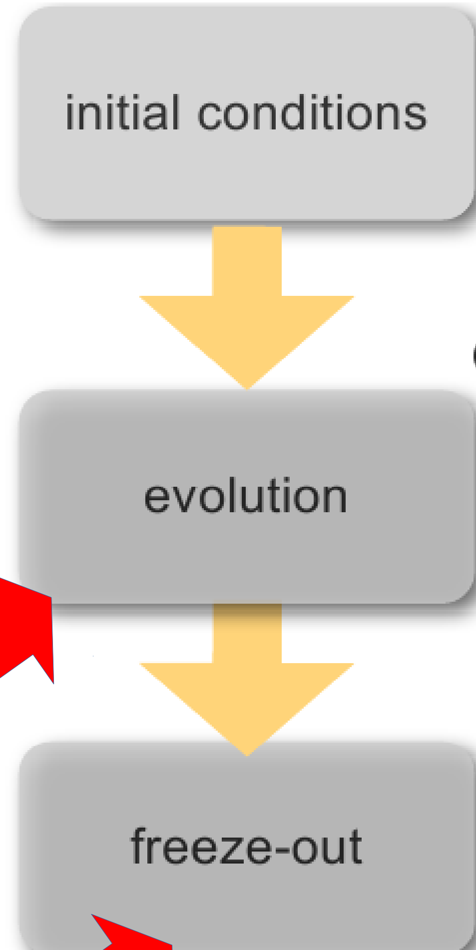
Exploring the QCD phase diagram in Theory and Experiment

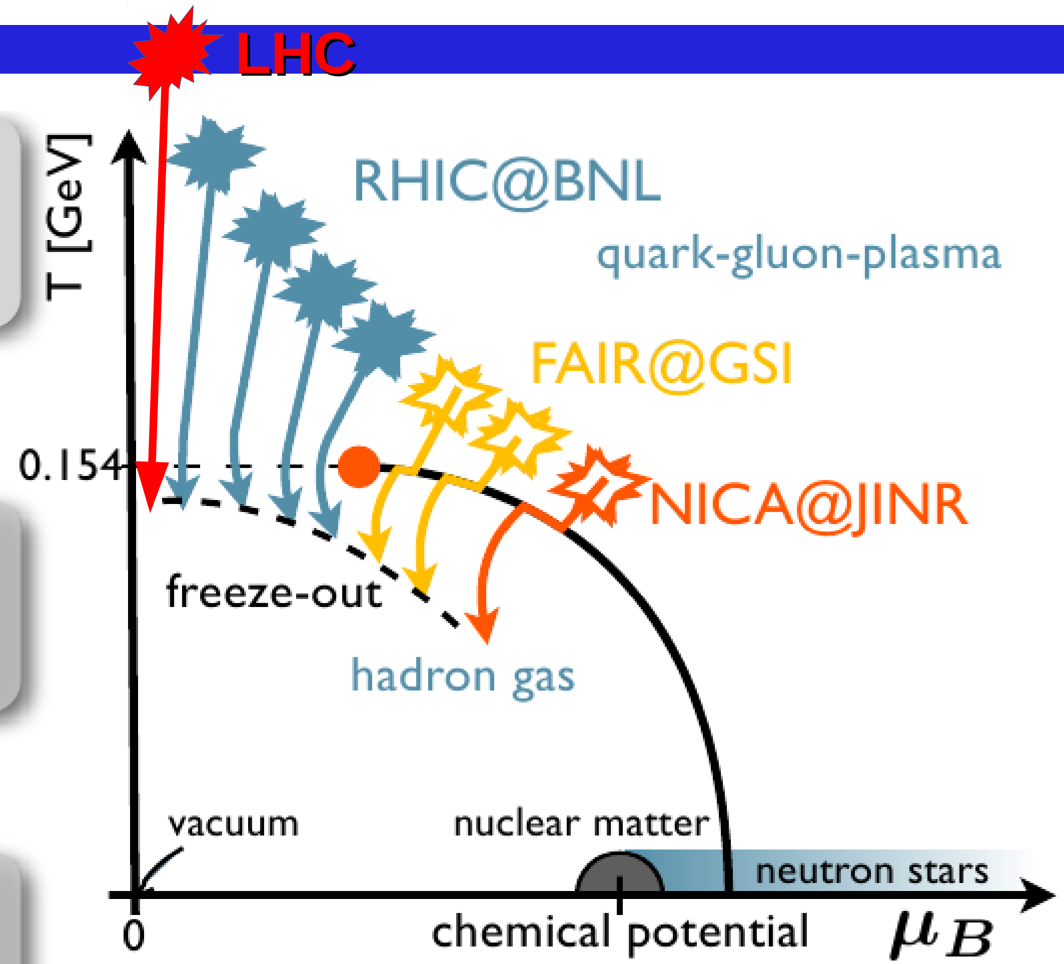
Phase Structure	Theory	Experiment Signatures
QGP Phase	EOS, densities, temperature, transport properties, Novel symmetries: LQCD + QCD based models	Jet quenching, Azimuthal anisotropy, strangeness enhancement, Quarkonia suppression, etc
Hadronic Phase	Hadron mass, spectral functions, freeze-out properties (temperature, volume, lifetime etc). LQCD + QCD models	Freeze-out properties (yields, ratios and spectra) source size and life time (HBT)
Cross over	Susceptibility volume dependence. LQCD	direct signature ? (indirect through EOS)
Critical point	Correlation lengths, Susceptibilities. LQCD + QCD based models	Fluctuations, Higher moments of conserved number distributions
New phases	Quarkyonic - QCD Models (lattice ?)	Baryon correlations
Transition line	LQCD, QCD models	Some signatures



Bedanga Mohanty (STAR), opening talk given at "Critical Point and Onset of Deconfinement 2013", Napa
PoS CPOD 2013 (2013) 001

Exploring the QCD phase diagram





controlled by the QCD equation of state, controlled by

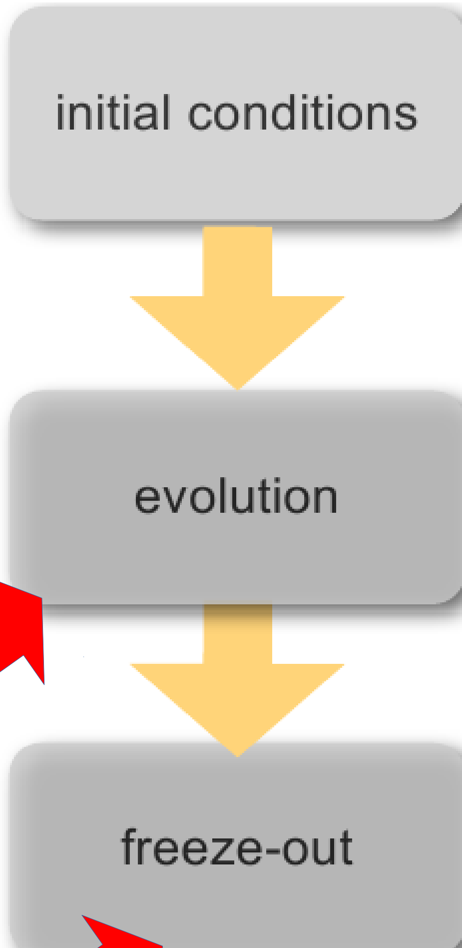
$$T, \mu_B, \mu_S$$

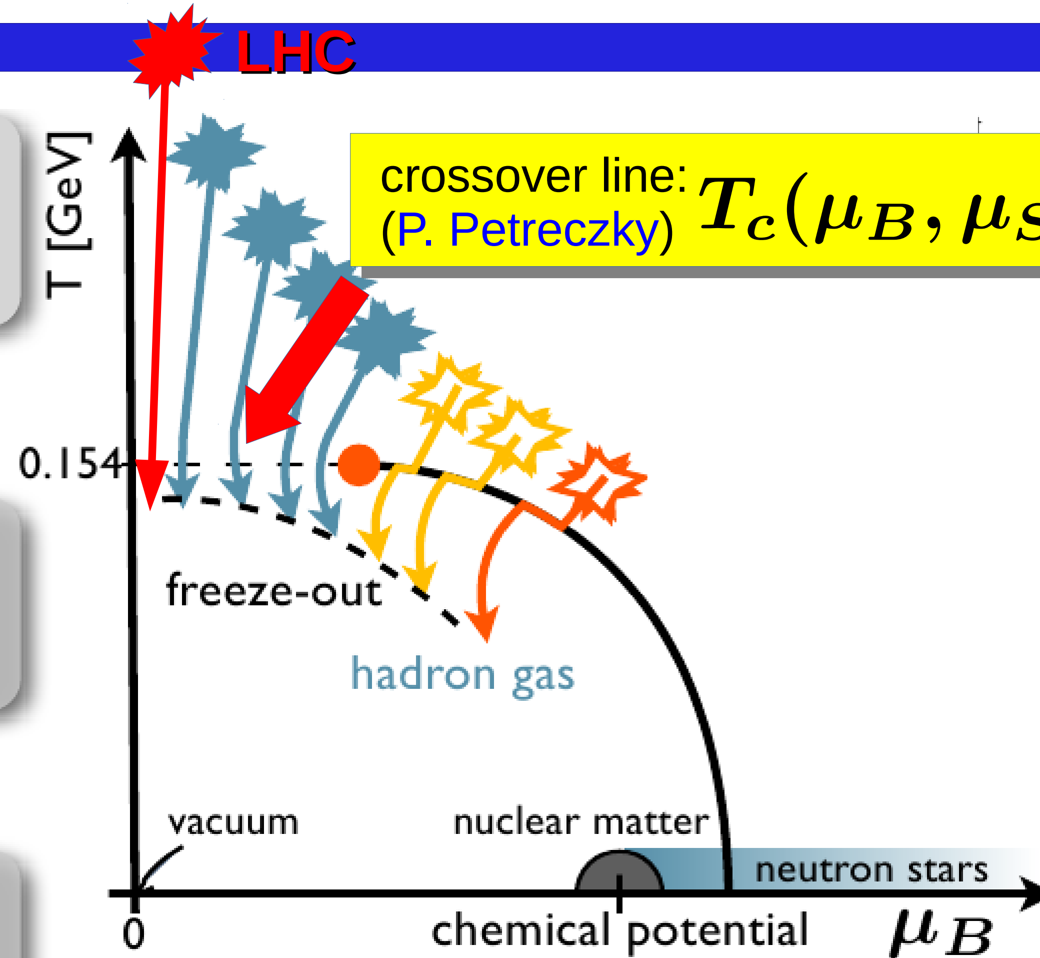
observable consequences: freeze-out/hadronization pattern of mesons and baryons, controlled by

$$T_f, \mu_B, \mu_S$$

- LHC: establish contact with the QCD PHASE transition
- RHIC: establish contact with the QCD critical point

Exploring the QCD phase diagram





crossover line: (P. Petreczky) $T_c(\mu_B, \mu_S)$

controlled by the QCD equation of state, controlled by

T, μ_B, μ_S

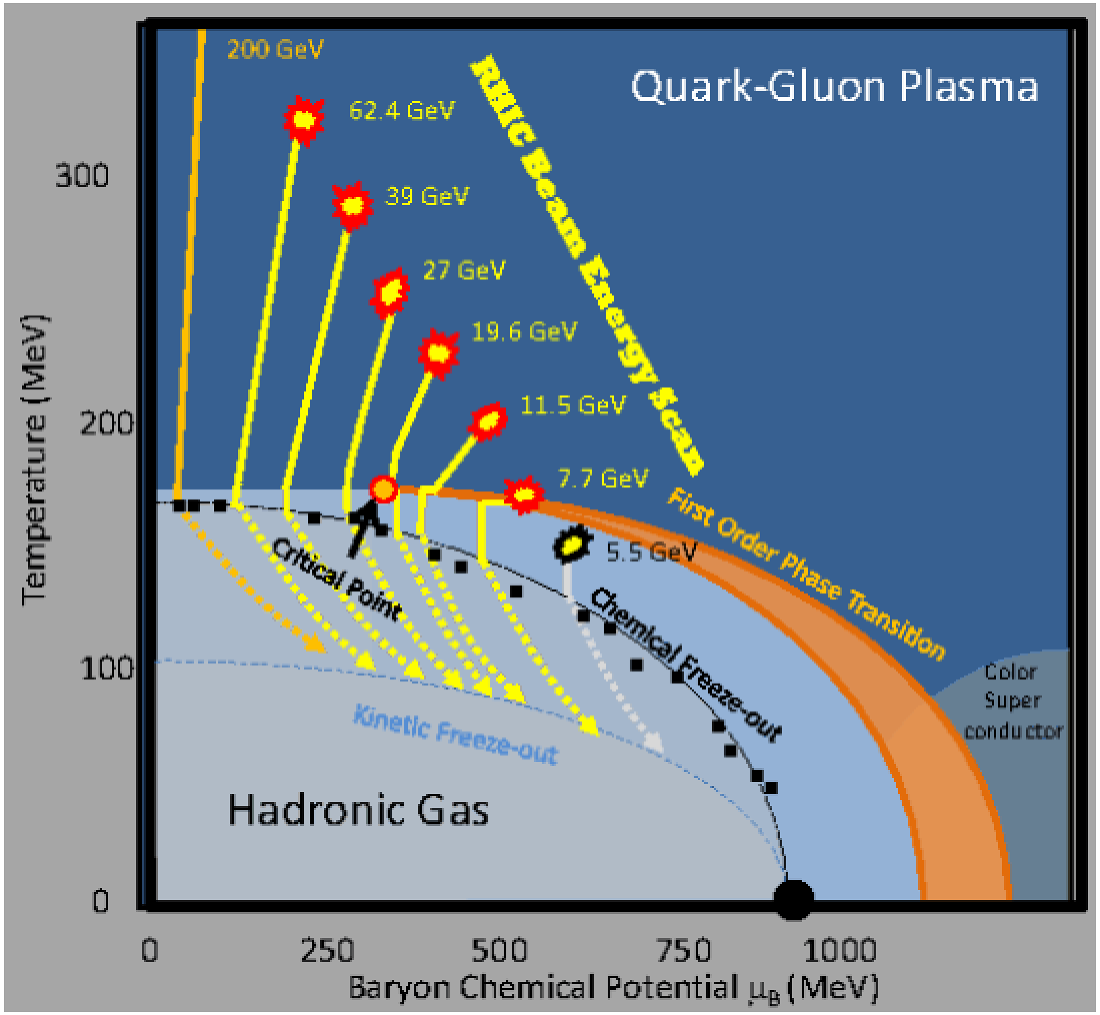
observable consequences:
freeze-out/hadronization pattern of mesons and baryons, controlled by T_f, μ_B, μ_S

- LHC: establish contact with the QCD PHASE transition
- RHIC: establish contact with the QCD critical point

Exploring the QCD phase diagram with net charge fluctuations

RHIC beam energy scan: $\sqrt{s} = (7.7 - 200)\mathrm{GeV}/A$

search for the critical point



generalized susceptibilities

$$\chi_n^q = \frac{\partial^n p/T^4}{\partial(\mu_q/T)^n}$$

$$q = B,\ Q,\ S$$

Exploring the QCD phase diagram with net charge fluctuations

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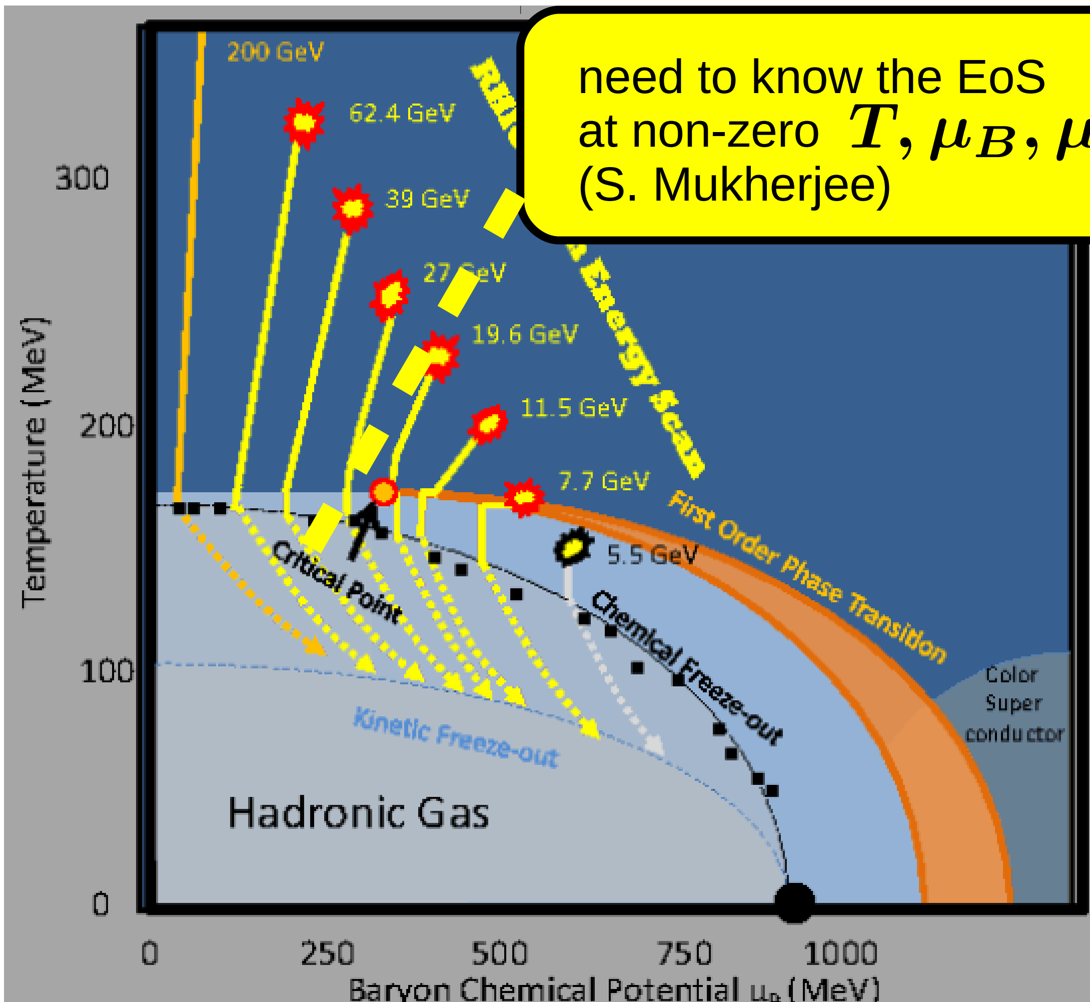

need to know the EoS at non-zero T, μ_B, μ_S (S. Mukherjee)

generalized susceptibilities

$$\chi_n^q = \frac{\partial^n p/T^4}{\partial(\mu_q/T)^n}$$

$$q = B, Q, S$$

– coefficients in the Taylor expansion of the Equation of State

Exploring the QCD phase diagram with net charge fluctuations

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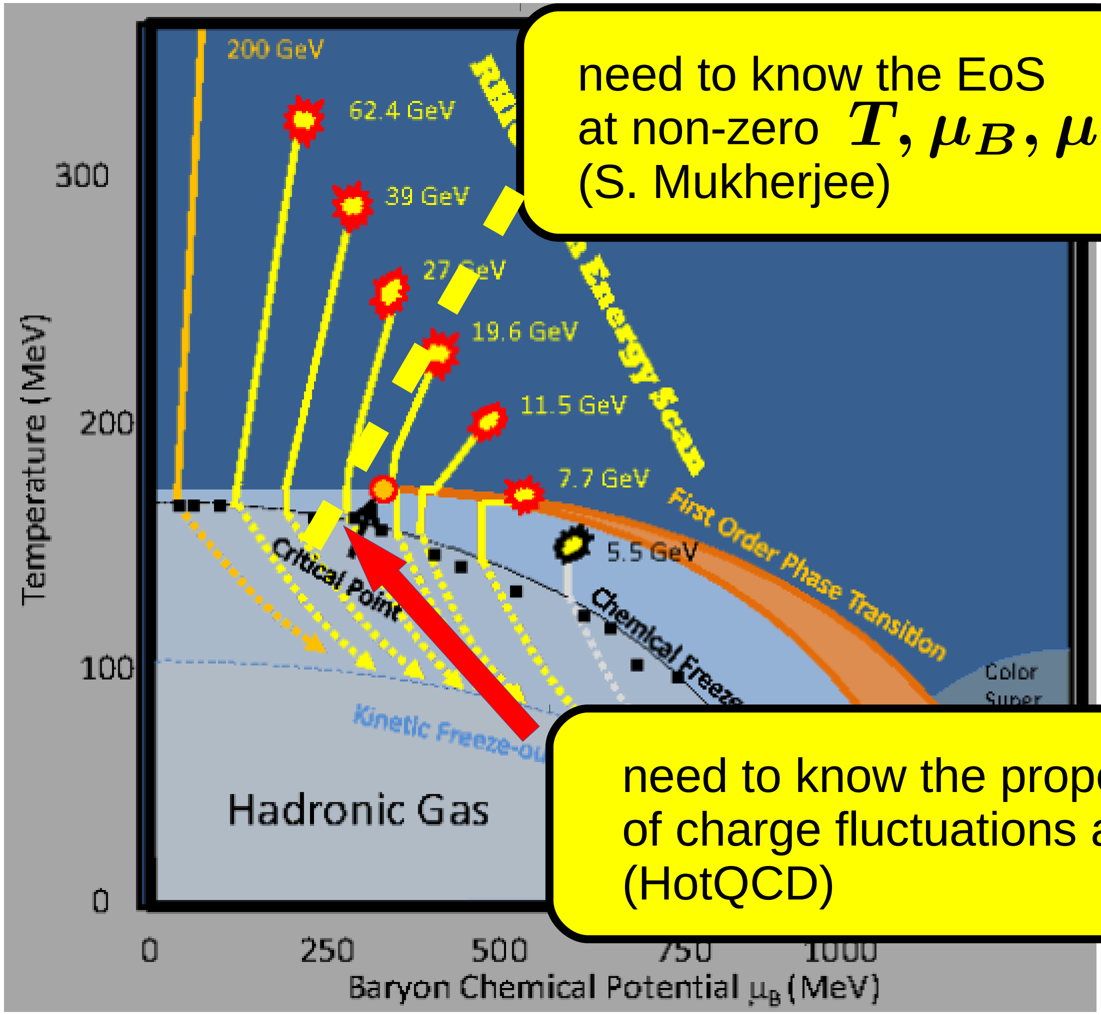


need to know the EoS at non-zero T, μ_B, μ_S (S. Mukherjee)

need to know the properties of charge fluctuations at T_f (HotQCD)

generalized susceptibilities

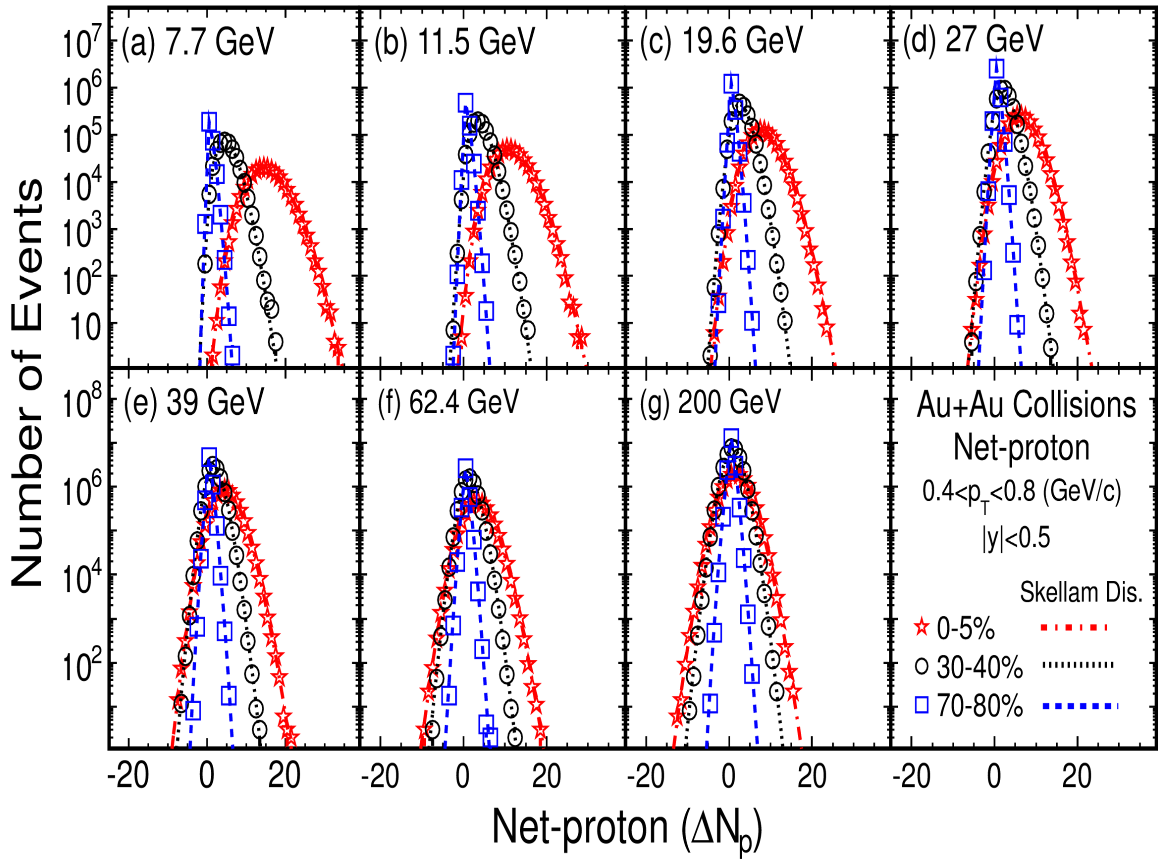
$$\chi_n^q = \frac{\partial^n p/T^4}{\partial(\mu_q/T)^n}$$

$$q = B,\ Q,\ S$$

– coefficients in the Taylor expansion of the Equation of State

– higher order cumulants of net-charge (B,Q,S) fluctuations measured in the BES at RHIC

Net proton number fluctuations at RHIC



mean $\langle \delta N_q \rangle \equiv \langle N_q - N_{\bar{q}} \rangle$

variance

$$\sigma_q^2 \equiv \langle (\delta N_q)^2 \rangle - \langle \delta N_q \rangle^2$$

skewness

$$S_q \equiv \langle (\delta N_q)^3 \rangle / \sigma_q^3$$

kurtosis

$$\kappa_q \equiv \langle (\delta N_q)^4 \rangle / \sigma_q^4 - 3$$

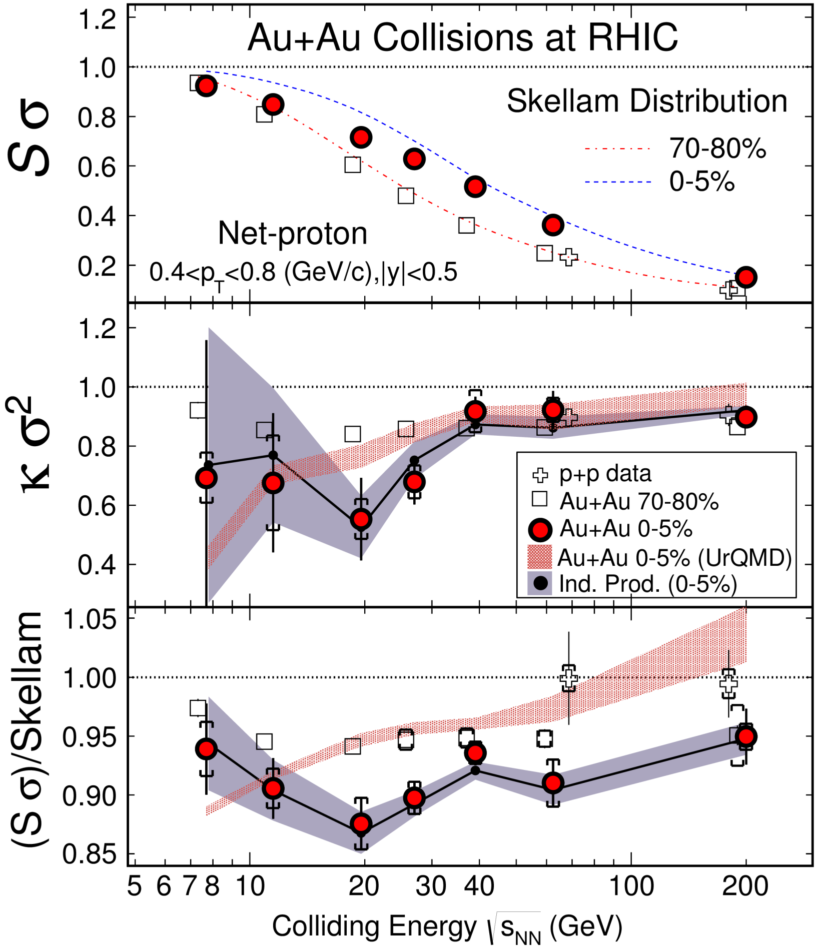
STAR Collaboration, PRL 112, 032302 (2014)

higher order cumulants characterize shape of conserved charge distributions

$$S_q \sigma_q = \frac{\chi_3^q}{\chi_2^q} \ , \ \kappa_q \sigma_q^2 = \frac{\chi_4^q}{\chi_2^q} \ \dots \qquad \frac{S_Q \sigma_Q^3}{M_Q} = \frac{\chi_3^Q}{\chi_1^Q} \qquad \frac{M_Q}{\sigma_Q^2} = \frac{\chi_1^Q}{\chi_2^Q}$$

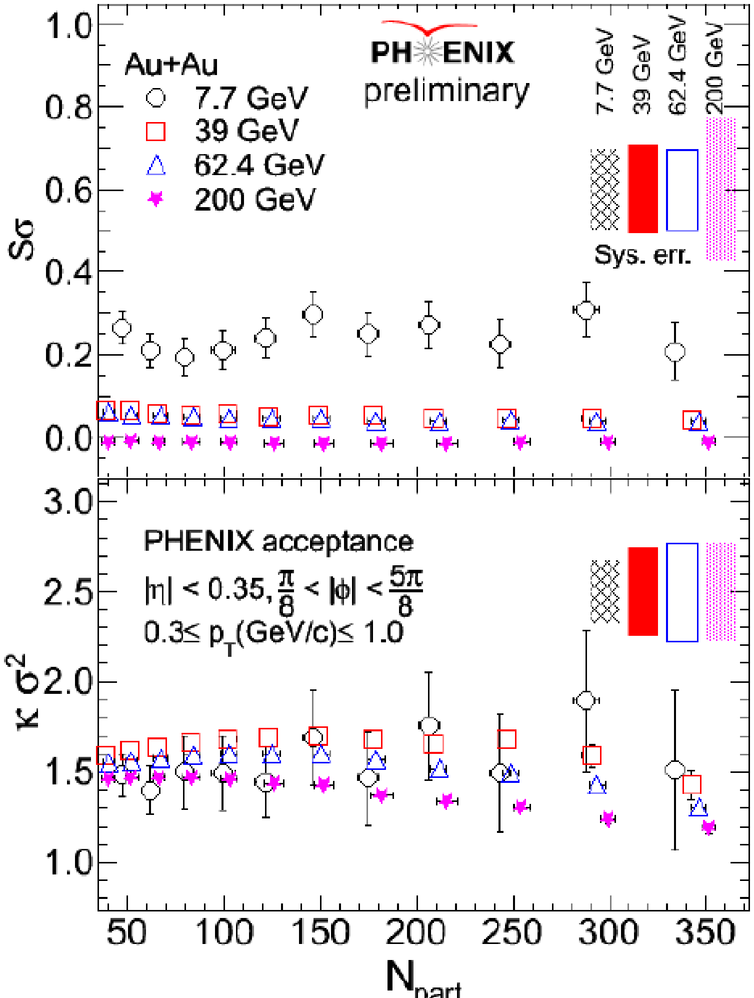
Cumulants of conserved charge fluctuations

net baryon number fluctuations



STAR Collaboration, PRL 112, 032302 (2014)

net electric charge fluctuations



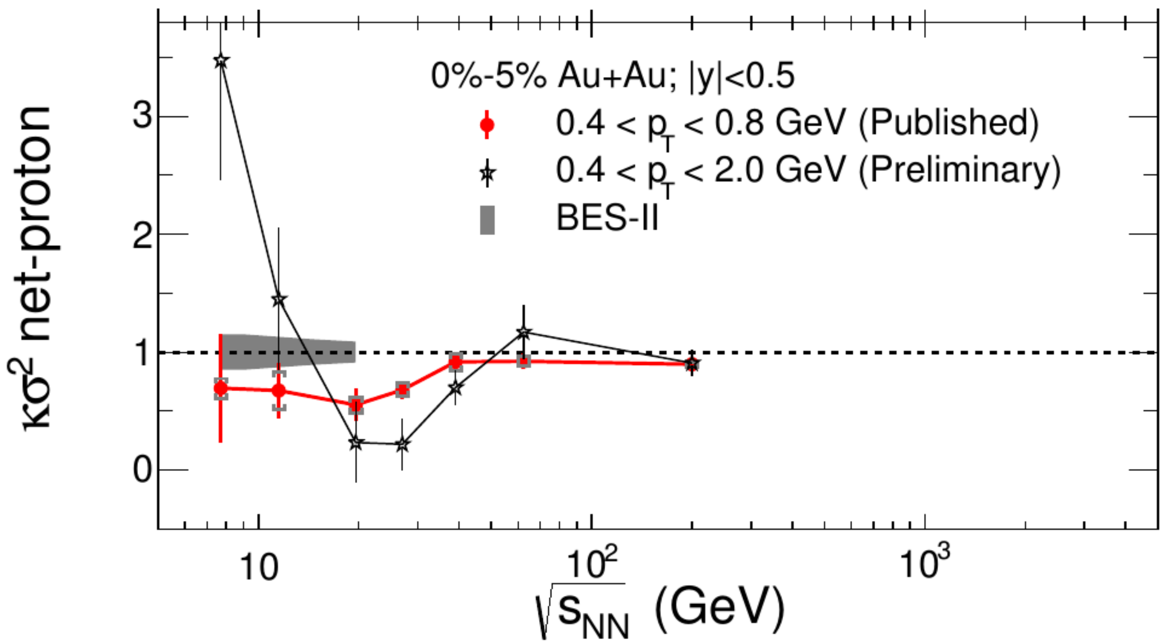
J. Mitchell (for PHENIX), CPOD2014 (2015) 075

Cumulants of conserved charge fluctuations

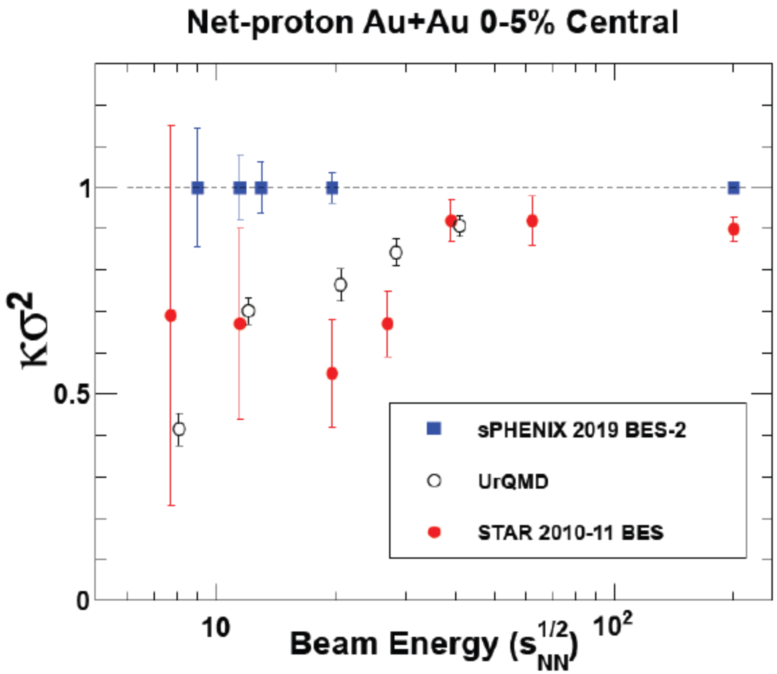
Exploring the properties of the phases of QCD matter - research opportunities and priorities for the next decade

U. Heinz et al., arXiV:1501.06477

new STAR preliminary and error projection for BES-II



X. Luo (for STAR), CPOD2014 (2015) 019



J. Mitchell (for PHENIX), CPOD2014 (2015) 075

Conserved e-Charge Cumulants in (2+1)-flavor QCD

A-project (INCITE): HotQCD

pressure: $$\frac{P}{T^4} = \sum_{i,j,k=0}^{\infty} \frac{1}{i!j!k!} \chi_{i,j,k}^{BQS}(T) \left(\frac{\mu_B}{T}\right)^i \left(\frac{\mu_Q}{T}\right)^j \left(\frac{\mu_S}{T}\right)^k$$

the simplest case: $\mu_B > 0$; $\mu_S = \mu_Q = 0$

cumulants of electric charge fluctuations

$$\chi_n^Q(T,\mu_B) = \chi_n^Q(T,0) + \frac{\chi_{2n}^{BQ}(T)}{2}\left(\frac{\mu_B}{T}\right)^2 + \frac{\chi_{4n}^{BQ}(T)}{24}\left(\frac{\mu_B}{T}\right)^4 + \ldots$$

leading order is most problematic

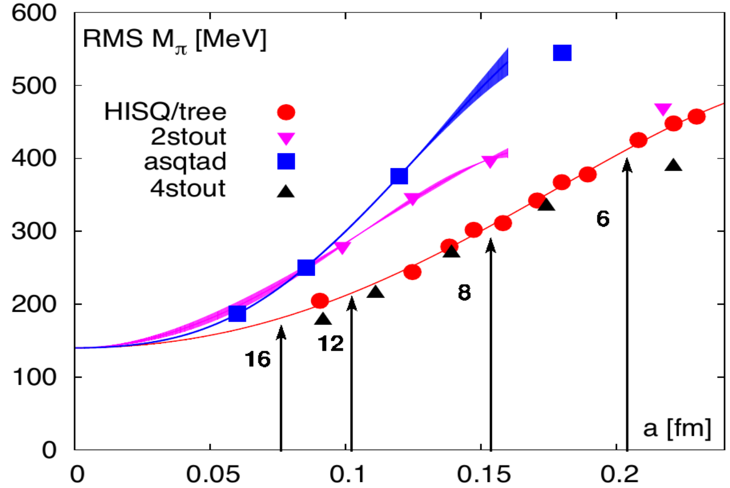
– electric charge fluctuations are dominated by pions

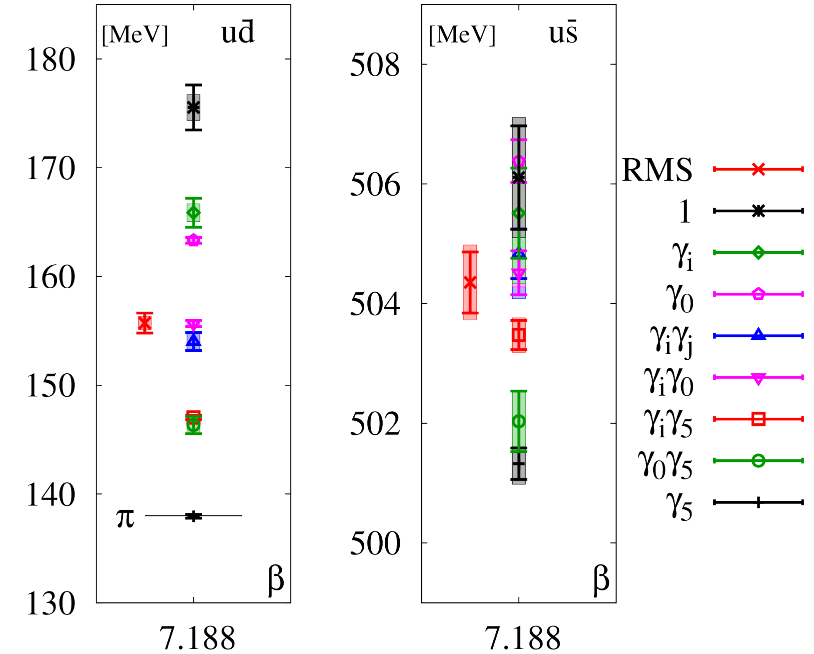
– taste violations are the largest source for systematic errors

→ large lattices are needed to control them → $64^3 \times 16$

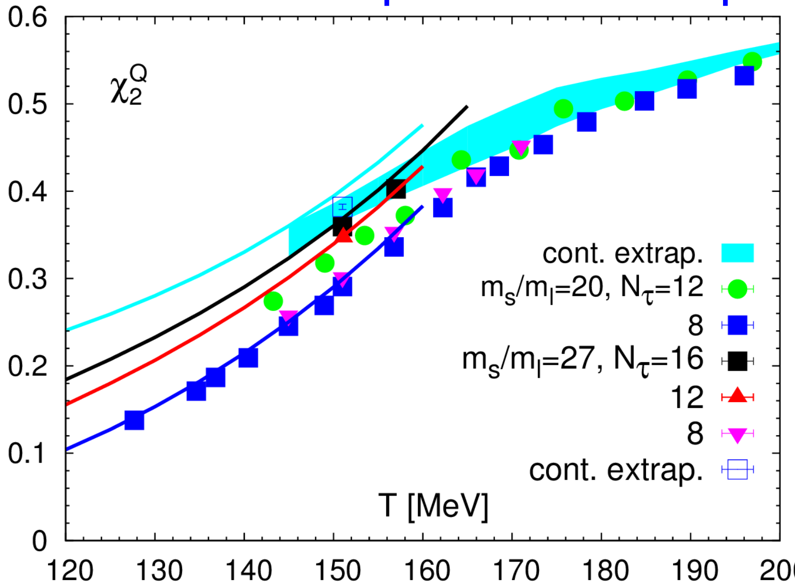
Electric charge fluctuations in (2+1)-flavor QCD

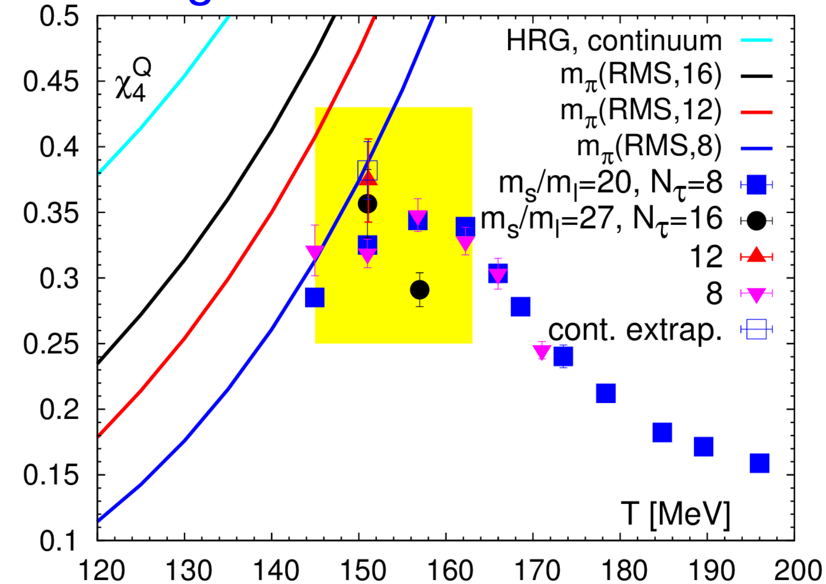
– taste violations are the largest source of systematic errors in the pion sector





quadratic and quartic electric charge fluctuations





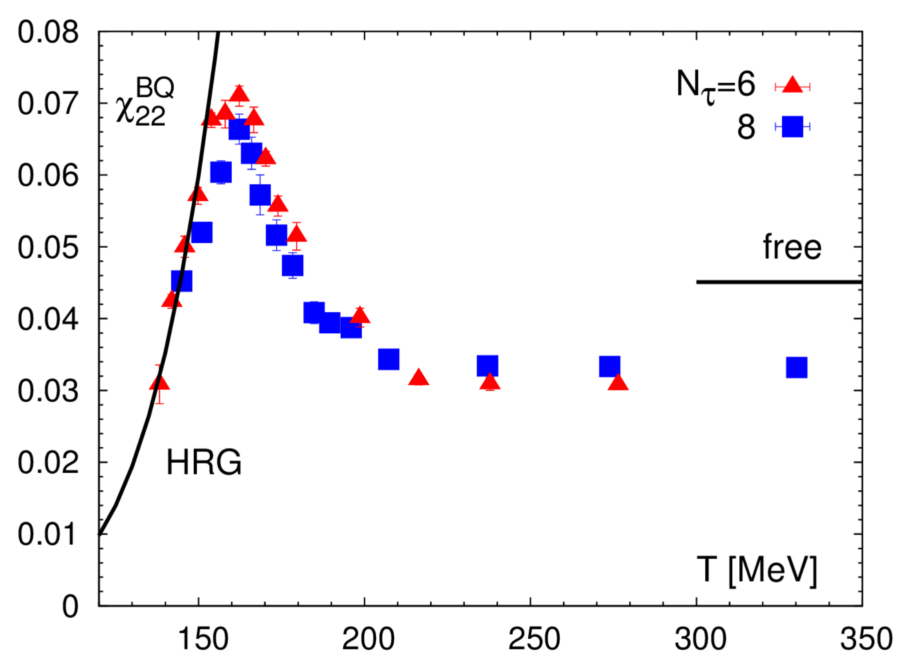
Electric charge fluctuations in (2+1)-flavor QCD

cumulants of electric charge fluctuations

$$\chi_n^Q(T,\mu_B) = \chi_n^Q(T,0) + \frac{\chi_{2n}^{BQ}(T)}{2}\left(\frac{\mu_B}{T}\right)^2 + \frac{\chi_{4n}^{BQ}(T)}{24}\left(\frac{\mu_B}{T}\right)^4 + \dots$$

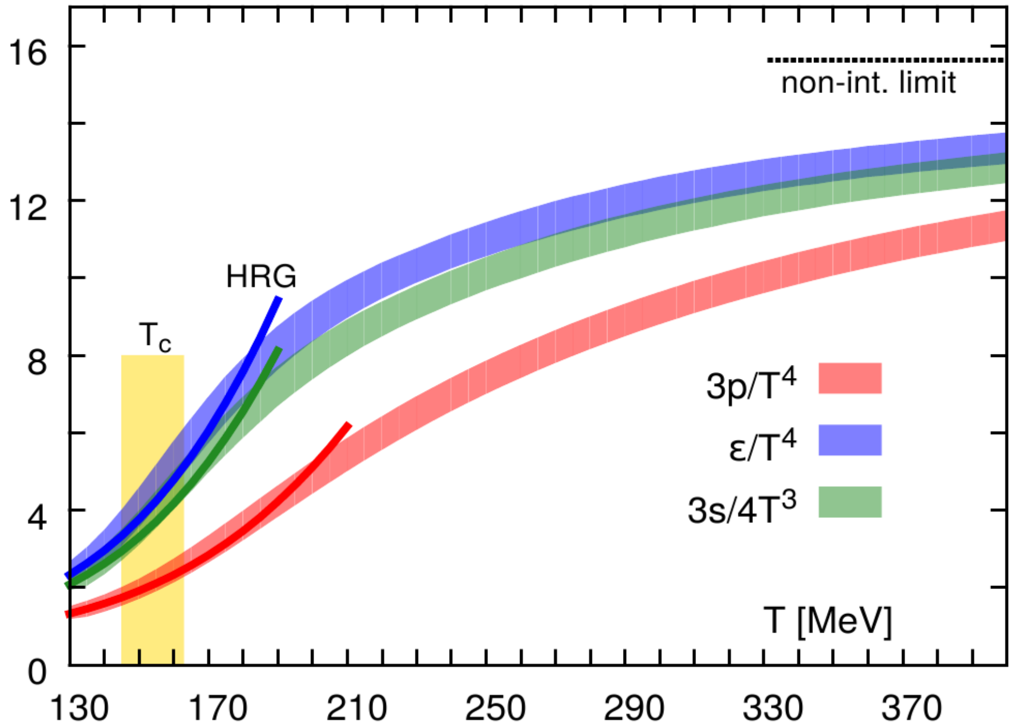
leading order is most problematic

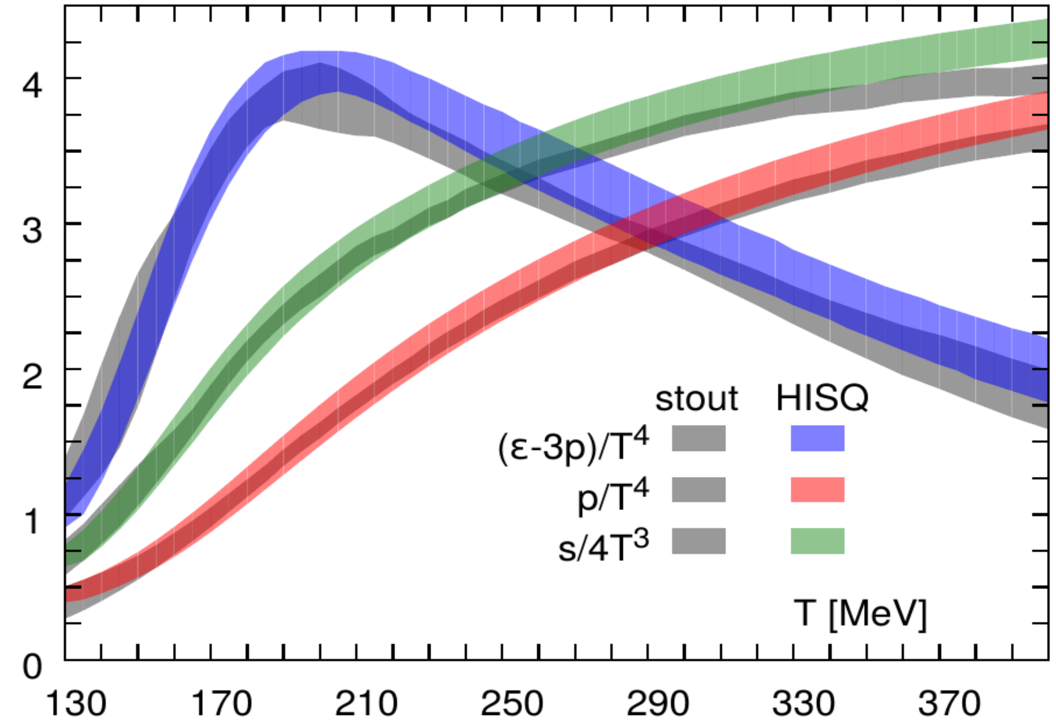
next-to-leading order (and higher) is controlled by baryons: signal-to-noise ratio is the main problem – but can be handled



Equation of state of (2+1)-flavor QCD

pressure, **entropy** & **energy** density





A. Bazavov et al. (hotQCD) ,
Phys. Rev. D90 (2014) 094503

$$\mu_B = \mu_S = \mu_Q = 0$$

– improves over earlier hotQCD calculations:
A. Bazavov et al., Phys. Rev. D80, 014504 (2009)

– consistent with results from Budapest-Wuppertal (stout): S. Borsanyi et al., PL B730, 99 (2014)

– up to the crossover region the QCD EoS agrees quite well with hadron resonance gas (HRG) model calculations; **However**, QCD results are systematically above HRG

Equation of state of (2+1)-flavor QCD: $\mu_B/T > 0$

A-project (GPUs): S. Mukherjee et al

$$\frac{P}{T^4} = \sum_{i,j,k=0}^{\infty} \frac{1}{i!j!k!} \chi_{i,j,k}^{BQS}(T) \left(\frac{\mu_B}{T}\right)^i \left(\frac{\mu_Q}{T}\right)^j \left(\frac{\mu_S}{T}\right)^k$$

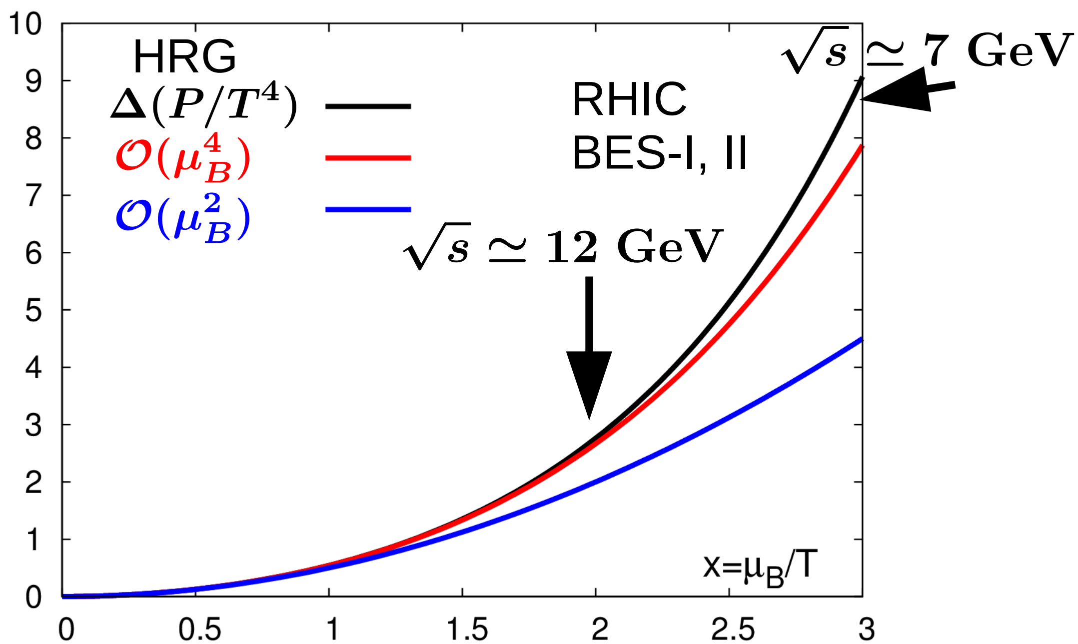
the simplest case: $\mu_B > 0$; $\mu_S = \mu_Q = 0$

$$\frac{P(T,\mu_B)}{T^4} = \frac{P(T,0)}{T^4} + \frac{\chi_2^B(T)}{2}\left(\frac{\mu_B}{T}\right)^2 + \frac{\chi_4^B(T)}{24}\left(\frac{\mu_B}{T}\right)^4 \frac{\chi_6^B(T)}{720}\left(\frac{\mu_B}{T}\right)^6 + \mathcal{O}(\mu_B^8)$$

An $\mathcal{O}((\mu_B/T)^4)$ expansion is exact in a QGP up to $\mathcal{O}(g^2)$

How good is an $\mathcal{O}((\mu_B/T)^4)$ expansion in a HRG?

- needed for $\mu_B/T \simeq 1.2$
– deviations less than 3% at $\mu_B/T = 2$



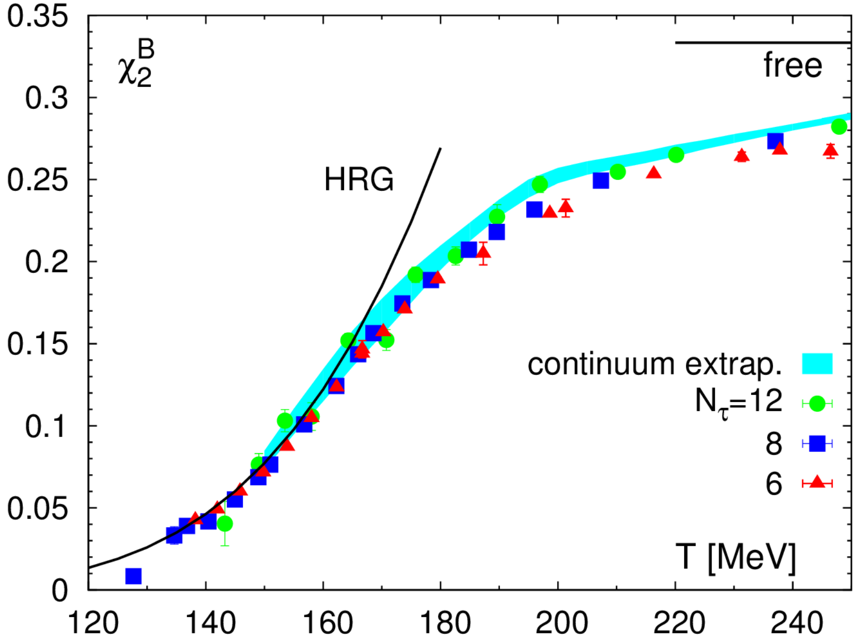
Equation of state of (2+1)-flavor QCD: $\mu_B/T > 0$

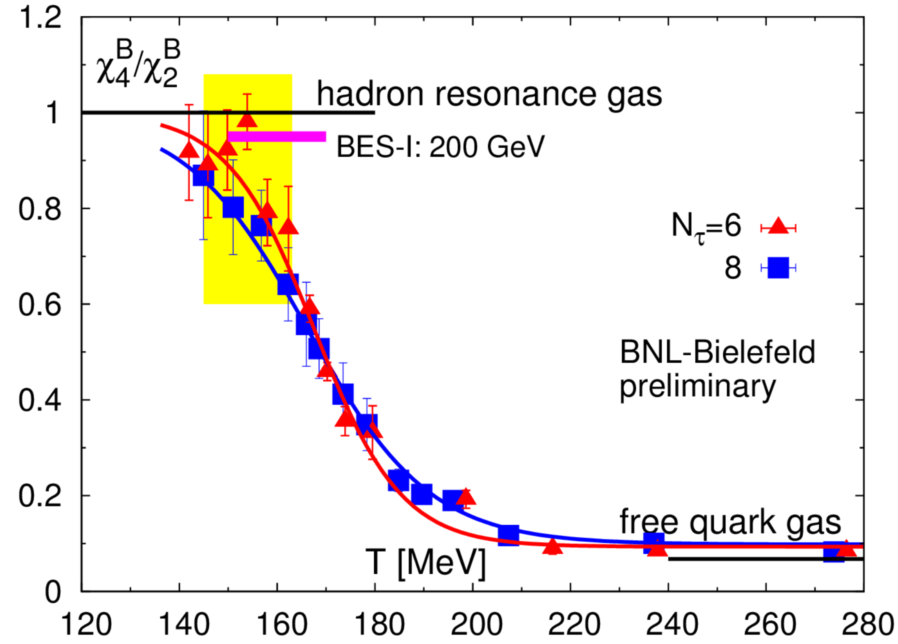
$$\frac{\Delta(T,\mu_B)}{T^4} = \frac{P(T,\mu_B) - P(T,0)}{T^4} = \frac{\chi_2^B}{2}\left(\frac{\mu_B}{T}\right)^2 \left(1 + \frac{1}{12}\frac{\chi_4^B}{\chi_2^B}\left(\frac{\mu_B}{T}\right)^2 + ...\right)$$

variance of net-baryon number distribution

kurtosis*variance

$\kappa_B \sigma_B^2$





Equation of state of (2+1)-flavor QCD: $\mu_B/T > 0$

$$\frac{P}{T^4} = \sum_{i,j,k=0}^{\infty} \frac{1}{i!j!k!} \chi_{i,j,k}^{BQS}(T) \left(\frac{\mu_B}{T}\right)^i \left(\frac{\mu_Q}{T}\right)^j \left(\frac{\mu_S}{T}\right)^k$$

the simplest case: $\mu_B > 0$; $\mu_S = \mu_Q = 0$

$$\frac{P(T,\mu_B)}{T^4} = \frac{P(T,0)}{T^4} + \frac{\chi_2^B(T)}{2}\left(\frac{\mu_B}{T}\right)^2 + \frac{\chi_4^B(T)}{24}\left(\frac{\mu_B}{T}\right)^4 + \frac{\chi_6^B(T)}{720}\left(\frac{\mu_B}{T}\right)^6$$

the general case: $\mu_B > 0, \mu_S > 0, \mu_Q > 0$

introduce constraints: (i) strangeness neutrality
(ii) fixed electric charge/baryon number ratio

$$\frac{P(T,\mu_B)}{T^4} = \frac{P(T,0)}{T^4} + \frac{\chi_2^{SN}(T)}{2}\left(\frac{\mu_B}{T}\right)^2 + \frac{\chi_4^{SN}T)}{24}\left(\frac{\mu_B}{T}\right)^4 + ...$$

$$\chi_2^{SN} = \chi_2^B + \chi_2^Q q_1^2 + \chi_2^S s_1^2 + 2\left(\chi_{11}^{BQ} q_1 + \chi_{11}^{BS} s_1 + \chi_{11}^{QS} q_1 s_1\right)$$

$$s_1 = -\frac{\chi_{11}^{BS}}{\chi_2^S} - \frac{\chi_{11}^{QS}}{\chi_2^S} q_1 \quad , \quad q_1 = ...$$

Calculating charge fluctuations on the lattice

cumulants, e.g.: $$\chi_{ijk}^{uds} = \frac{\partial^i}{\partial \mu_i^u} \frac{\partial^j}{\partial \mu_j^d} \frac{\partial^k}{\partial \mu_k^s} \ln Z(T, \mu_u, \mu_d, \mu_s)$$

for instance: $$\chi_{31}^{us} = -3\mathcal{A}_{11}^{us}\mathcal{A}_{20}^{us} + \mathcal{A}_{31}^{us}$$

$$\mathcal{A}_{31}^{us} = \langle (D_1^u)^3 D_1^s \rangle + 3\langle D_1^u D_2^u D_1^s \rangle + D_3^u D_1^s \rangle$$

$$D_n^f = \frac{1}{4} \frac{\partial^n \ln \det M(\mu_f)}{\partial \mu_f^n}$$

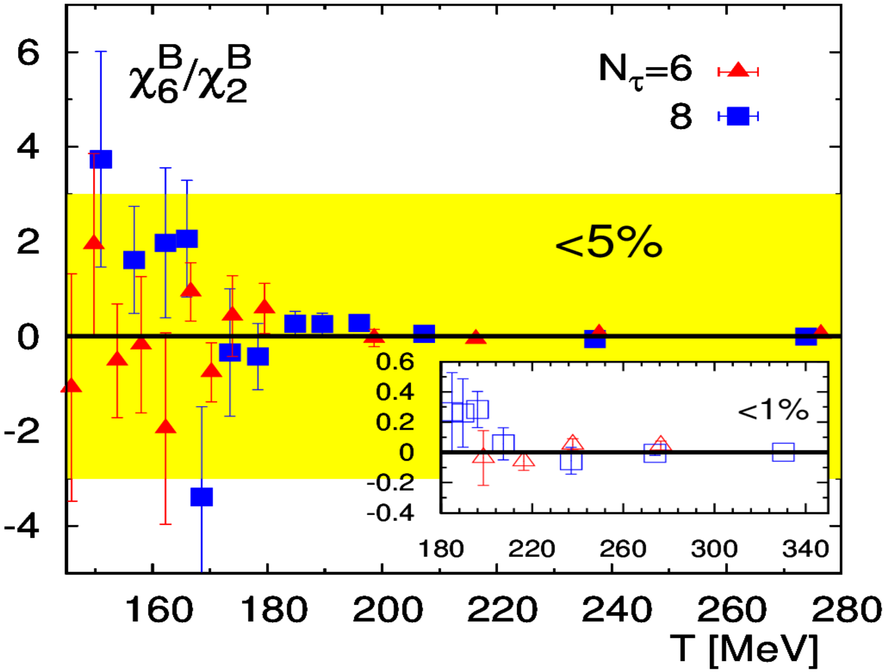
$$= \frac{1}{4} \mathrm{Tr}(M_f^{-1} \partial_\mu M)(M_f^{-1} \partial_\mu M) \dots (M_f^{-1} \partial_\mu M)$$

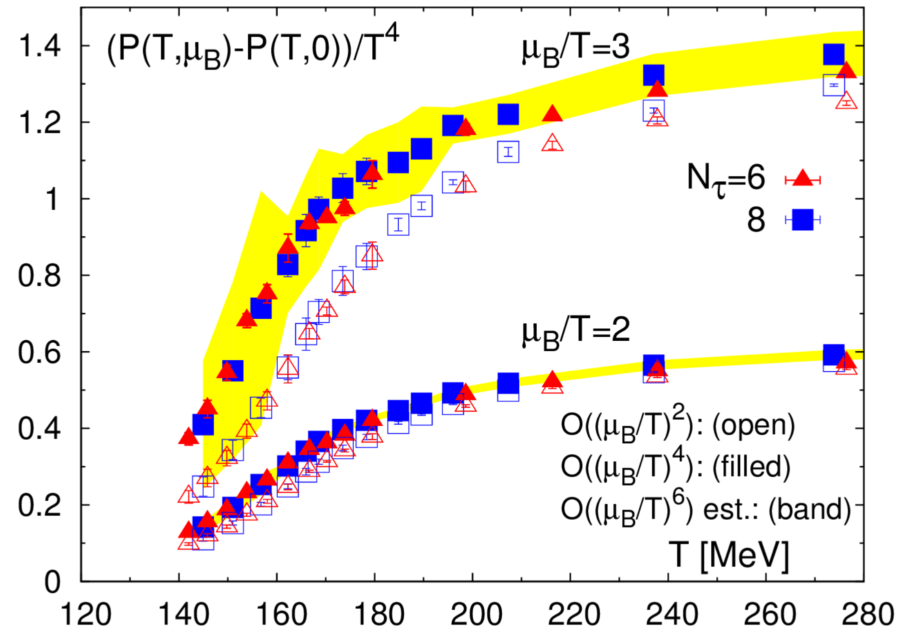
evaluate traces using stochastic estimators

EoS at $\mu_B/T > 0$: Current status

$$\frac{\Delta(T,\mu_B)}{T^4} = \frac{P(T,\mu_B) - P(T,0)}{T^4} = \frac{\chi_2^B}{2}\left(\frac{\mu_B}{T}\right)^2 \left(1 + \frac{1}{12}\frac{\chi_4^B}{\chi_2^B}\left(\frac{\mu_B}{T}\right)^2\right)$$

estimating the $\mathcal{O}((\mu_B/T)^6)$ correction: $\sim \frac{1}{720}\frac{\chi_6^B}{\chi_2^B}\left(\frac{\mu_B}{T}\right)^6$





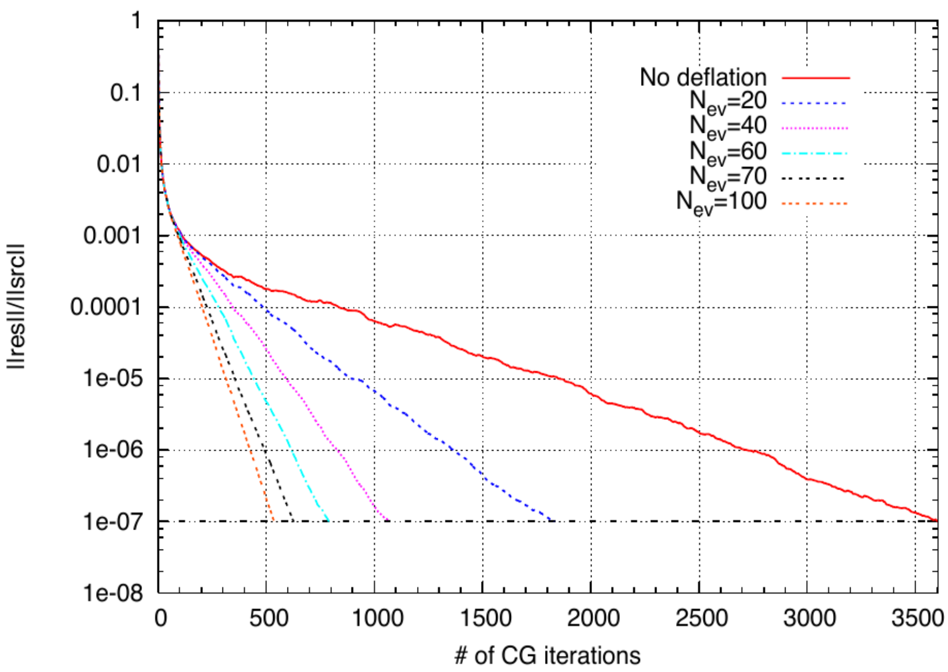
The EoS is well controlled for $\mu_B/T \leq 2$

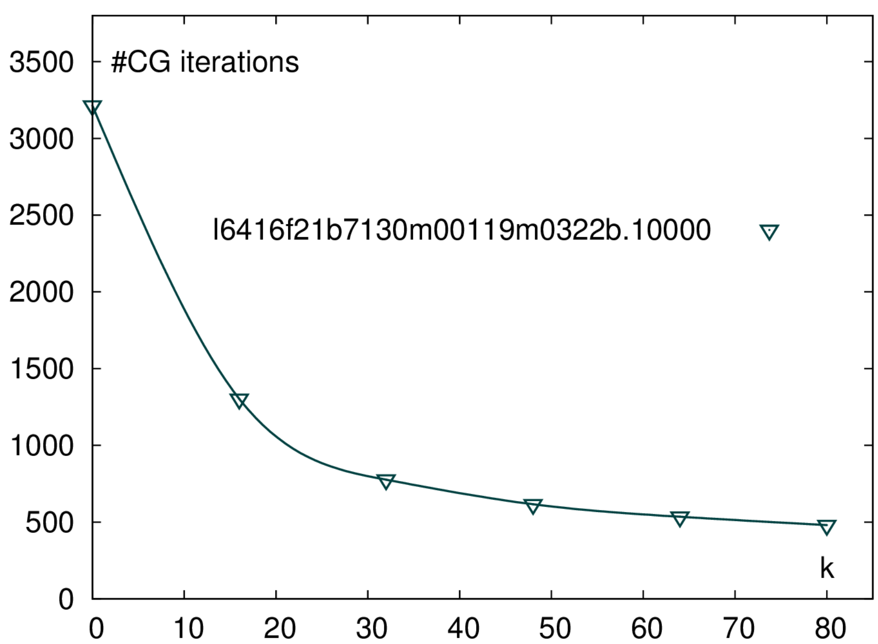
Calculating charge fluctuations on the lattice

recent advances:

– understood that higher order cumulants are free of divergences and can be calculated using the so-called linear-mu formulation:
R.V. Gavai and S. Sharma, Phys. Rev. D85 (2012) 054508
now post-doc at BNL

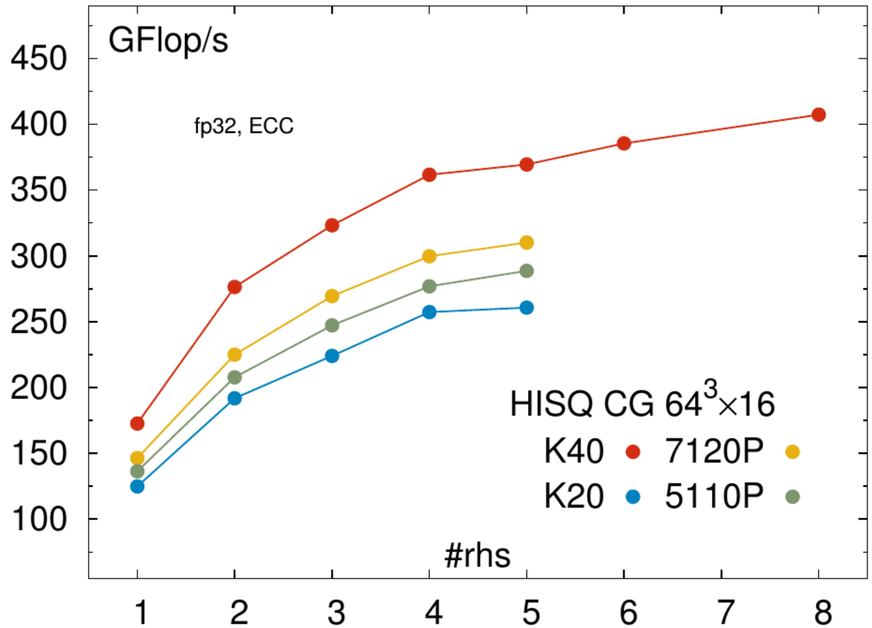
– developed efficient deflation code for the evaluation of charge fluctuation observables on GPUs

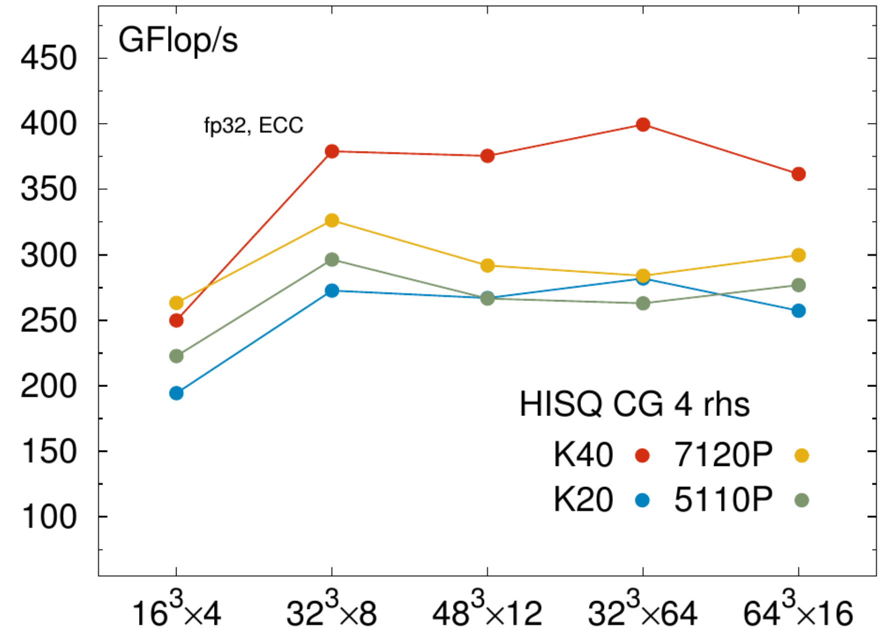




Calculating charge fluctuations on the lattice

– highly efficient CG inverter for HISQ action on GPUs (and also on MIC)





O. Kaczmarek, C. Schmidt, P. Steinbrecher, M. Wagner, arXiv:1411.4439

↳ SciDAC-3 grad.student at BNL

– gain by using multiple right hand sides – balance against #EV used
– Titan specific: shift eigenvector calculation for deflation to CPU; generation of eigenvectors "comes for free" and GPU can run with more right hand sides