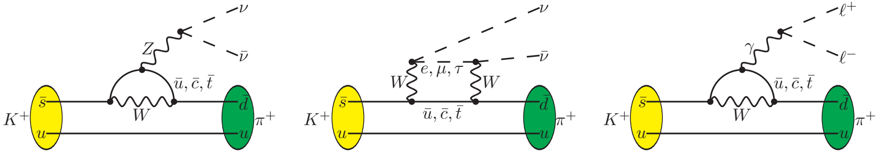
Exploratory lattice QCD study of $K \to \pi\nu\bar{\nu}$ and $K \to \pi\ell^+\ell^-$



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Collaborators

- on behalf of RBC-UKQCD collaboration

- people involved in this proposal

UKQCD
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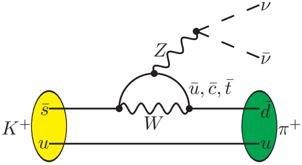
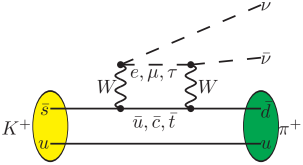
Flavor changing neutral current

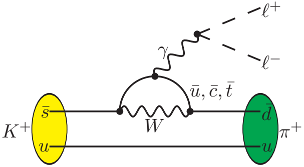
$K \to \pi\nu\bar{\nu}$ and $K \to \pi\ell^+\ell^-$ are flavor changing neutral current processes



Z-exchange



W-*W*



γ-exchange

SM effects highly suppressed in the second order → ideal probes for NP

$K \to \pi\nu\bar{\nu}$ vs $K \to \pi\ell^+\ell^-$

$K \to \pi\nu\bar{\nu}$

- in 1981, Inami & Lim found loop function in Z-exchange and W-W diagrams $\propto m_q^2$, $q = u, c, t$
- in 1995, top quark was discovered by CDF and D∅ at Fermilab

m_t(173 GeV) $\gg m_c$(1.3 GeV) $\Rightarrow$ top quark contribution dominates

$K \to \pi\ell^+\ell^-$

- loop function in γ-exchange diagram $\propto \ln m_q^2$ for $m_q \to 0$
- this is known as logarithmic or soft GIM mechanism

unsupressed sensitivity to m_u $\Rightarrow$ long-distance dominance

$K \to \pi\nu\bar{\nu}$: Theory vs Experiment

top-quark (short-distance) dominance ⇒ theoretically very clean

- dominated SM uncertainty from CKM matrix V_{td}, $V_{ts} \Leftarrow V_{ub}$, V_{cb}, γ
- two ways to determine V_{ub}, V_{cb}, γ
  - by tree-level measurement of $b \to c$, $b \to u$ transitions
  - by loop-level observables, ε_K, $\Delta M_{d,s}$, $S_{\psi K_S}$
- recent SM prediction [Buras, et.al. arXiv:1503.02693]

$$\mathrm{Br}(K^+ \to \pi^+\nu\bar{\nu})_{\mathrm{SM}} = (9.11 \pm 0.72) \times 10^{-11} \quad \text{loop-level input}$$
$$\mathrm{Br}(K^+ \to \pi^+\nu\bar{\nu})_{\mathrm{SM}} = (8.4 \pm 1.0) \times 10^{-11} \quad \text{tree-level input}$$

super small branching ratio ⇒ experimentally very diffcult

- search for 1 candidate among every 10 billion events of K^+ decays
- from 1969 to 2008, 40 years search leads to observation of 7 events

$$\mathrm{Br}(K^+ \to \pi^+\nu\bar{\nu})_{\mathrm{exp}} = (1.73^{+1.15}_{-1.05}) \times 10^{-10} \quad > 60\%\ \text{err}$$

$\mathrm{Br}_{\mathrm{exp}}$ is 2 times larger than $\mathrm{Br}_{\mathrm{SM}}$, but still consistent with $> 60\%$ error

New experiments

sometimes theory can lead experiments and $K \to \pi\nu\bar{\nu}$ are such examples

- $K^+ \to \pi^+\nu\bar{\nu}$: new generation of experiment NA62 at CERN aims at
  - observation of $O(100)$ events in two years
  - 10%-precision measurement of $\mathrm{Br}(K^+ \to \pi^+\nu\bar{\nu})$

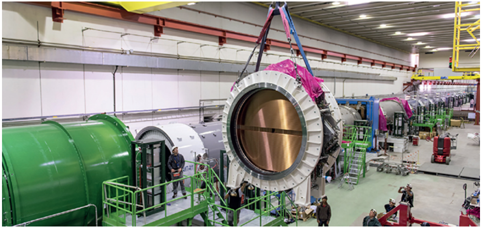
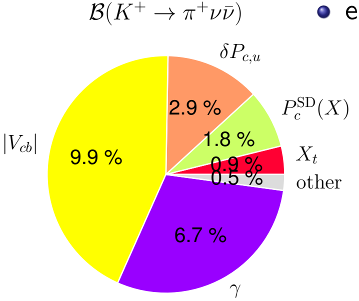


Fig: 09/2014, the final straw-tracker module is lowered into position in NA62

- $K_L \to \pi^0\nu\bar{\nu}$
  - even more challenging since π^0 decays quickly to two photons
  - only upper bound was set by KEK E391a in 2010
  - new KOTO experiment at J-PARC designed to observe K_L decays

SM error budget for $K^+ \to \pi^+\nu\bar{\nu}$

can we do better? ⇒ continue tradition to lead exp, further reduce SM err



- error budget from A.J. Buras et. al.
  - $|V_{cb}|$, γ: CKM inputs for $|V_{td}|$, $|V_{ts}|$
  - $\delta P_{c,u}$: LD contribution
  - P_c: c-quark contribution (SD part)
  - X_t: t-quark contribution
  - other: remaining SM parameters

- most important thing is to reduce the error from CKM inputs
- long-distance contribution yields the sub-dominated uncertainty
- phenomenological ansatz invovling χPT and OPE [hep-ph/0503107] yields $\delta P_{c,u} = 0.04 \pm 0.02$ ⇒ branching ratio enhanced by 6%
  - 50% err in $\delta P_{c,u}$ is a guess rather than a controlled error
  - $\delta P_{c,u}$ may be much larger or even smaller

can lattice QCD do a better job?

Lattice methodology

(use $K^+ \to \pi^+\nu\bar{\nu}$ as example)

Three-step LQCD development strategy for $K^+ \to \pi^+\nu\bar{\nu}$

1) we start with domain wall fermion configs. generated by RBC-UKQCD

- $16^3 \times 32$, $m_\pi = 420$ MeV, $m_c = 860$ MeV, $a^{-1} = 1.73$ GeV
- set up the calculation at unphysical kinematics

2) USQCD proposal this year

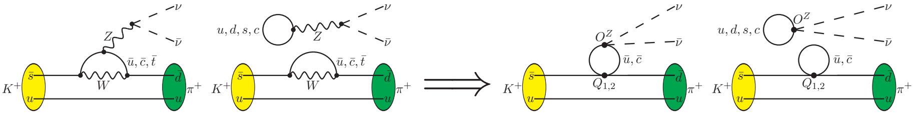
- $32^3 \times 64$, $m_\pi = 170$ MeV, $m_c = 750$ MeV, $a^{-1} = 1.37$ GeV
- further control the unphysical effects from pion mass
- $a^{-1} = 1.37$ GeV $\Rightarrow$ expect large lattice cutoff effect from m_c

3) for the future

- $80^2 \times 96 \times 192$, $m_\pi = 140$ MeV, $m_c = 1.3$ GeV, $a^{-1} = 3$ GeV
- control all systematic effect, produce $\delta P_{c,u}$ with 20% err $\Rightarrow$ 1% in Br

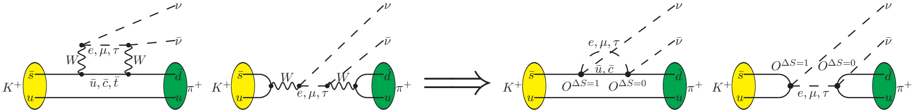
Non-local structure

Z-exchange diagrams:



$$\int d^4x \, \langle \pi^+ \nu \bar{\nu} | T\{Q_{1,2}(x) O^Z(0)\} | K^+ \rangle$$

W-W diagrams:



$$\int d^4x \, \langle \pi^+ \nu \bar{\nu} | T\{O^{\Delta S=1}(x) O^{\Delta S=0}(0)\} | K^+ \rangle$$

Minkowski vs Euclidean

given a non-local matrix element in Minkowski space

$$\mathcal{T}^M = i\int dt\,\langle f|T[O^{\Delta S=1}(t)O^{\Delta S=0}(0)]|K\rangle$$
$$= \sum_{n_s}\frac{\langle f|O^{\Delta S=1}|n_s\rangle\langle n_s|O^{\Delta S=0}|K\rangle}{E_{n_s}-E_f+i\epsilon}-\sum_n\frac{\langle f|O^{\Delta S=0}|n\rangle\langle n|O^{\Delta S=1}|K\rangle}{E_K-E_n+i\epsilon}$$

in Euclidean space

$$\mathcal{T}^E = \sum_{t=-T_a}^{T_b}\langle f|T[O^{\Delta S=1}(t)O^{\Delta S=0}(0)]|K\rangle$$
$$= \sum_{n_s}\frac{\langle f|O^{\Delta S=1}|n_s\rangle\langle n_s|O^{\Delta S=0}|K\rangle}{E_{n_s}-E_f}\left(1-e^{(E_f-E_{n_s})T_b}\right)$$
$$-\sum_n\frac{\langle f|O^{\Delta S=0}|n\rangle\langle n|O^{\Delta S=1}|K\rangle}{E_K-E_n}\left(1-e^{(E_K-E_n)T_a}\right)$$

if $E_n < E_K$, remove exp growing contamination, $\mathcal{T}^E \Rightarrow \mathcal{T}^M$

Infinite volume vs finite volume

- above two-pion threshold, $\sum_n$ and $\sum_{n_s}$ shall be replaced by $\displaystyle{\not}\!\!\int_n$ and $\displaystyle{\not}\!\!\int_{n_s}$
- for infinite volume, integral is well defined using principal value

$$\mathcal{I}^\infty = \mathcal{P} \sum\!\!\!\!\!\!\int_n \frac{\langle f|O^{\Delta S=0}|n\rangle^{\infty\,\infty}\langle n|O^{\Delta S=1}|K\rangle}{E_K - E_n}$$

- for finite volume, energy states are always discrete, we still have

$$\mathcal{I}^L = \sum_n \frac{\langle f|O^{\Delta S=0}|n\rangle^{LL}\langle n|O^{\Delta S=1}|K\rangle}{E_K - E_n}$$

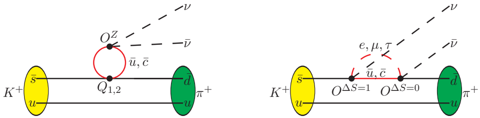
- finite-volume correction $\mathcal{I}^\infty = \mathcal{I}^L - \delta\mathcal{I}$

$$\delta\mathcal{I} = \cot(\phi(E)+\delta(E))(\phi'(E)+\delta'(E))\langle f|O^{\Delta S=0}|\pi\pi, E\rangle^{LL}\langle \pi\pi, E|O^{\Delta S=1}|K\rangle\Big|_{E=m_K}$$

  - $\phi(E)$: a known function depending on L; $\delta(E)$: $\pi\pi$ scattering phase
  - $\cot(\phi(E)+\delta(E))$ is singular at $E = E_n$; it cancels the singularity of $\mathcal{I}^L$
- for a complete derivation of $\delta\mathcal{I}$, see [N. Christ, XF, G. Martinelli, C. Sachrajda, arXiv:1504.01170]

Physical cutoff vs lattice cutoff

- by dimensional counting the loop integrals are quadratically divergent



- GIM mechanism reduces the divergence to logarithmic
- in the physical world, the SD divergence is cut off by physical M_W
- in the lattice calculation it is cut off by an energy scale $\Lambda_{lat} \sim \frac{1}{a}$
- correction can be made through $A - A_{SD}^{lat} + A_{SD}^{cont} =$

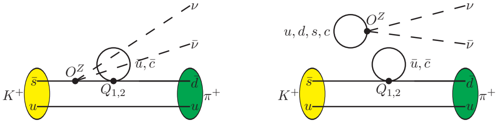
$$\int d^4x \langle f|T\{O_1(x)O_2(0)\}|K\rangle - \langle f|C^{lat}(\mu)O_{SD}|K\rangle + \langle f|C^{cont}(\mu)O_{SD}|K\rangle$$

- $C^{lat}(\mu)$ is determined non-perturbatively using RI/MOM approach
- $C^{cont}(\mu)$ can be calculated perturbatively [C. Lehner's PhySyHCAl]

Evaluation of non-local matrix element

$$\mathcal{T} = \int dt \, \langle \pi^+ \nu \bar{\nu} | T\{O^{\Delta S=1}(t) O^{\Delta S=0}(0)\} | K^+ \rangle$$

- construct 4-point correlator $\langle \phi_\pi(t_\pi) O^{\Delta S=1}(t_1) O^{\Delta S=0}(t_0) \phi_K^\dagger(t_K) \rangle$
- wall source for ϕ_π and ϕ_K $\Rightarrow$ better overlap with ground state
- $O^{\Delta S=1}$ and $O^{\Delta S=0}$: point source for one operator, the other is sink
- perform time translation average $\rightarrow$ statistical error reduced by $\sqrt{T}$
  - low-mode deflation to reduce the time required by light quark CG
- random source propagator for self-loop and disconnected diagrams

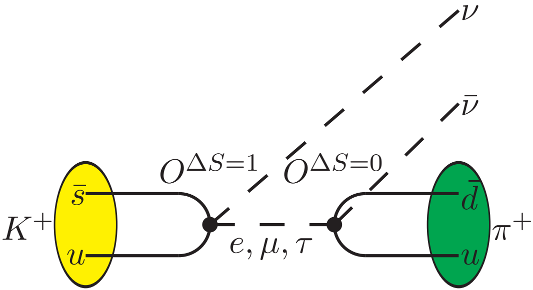


- extract the form factor

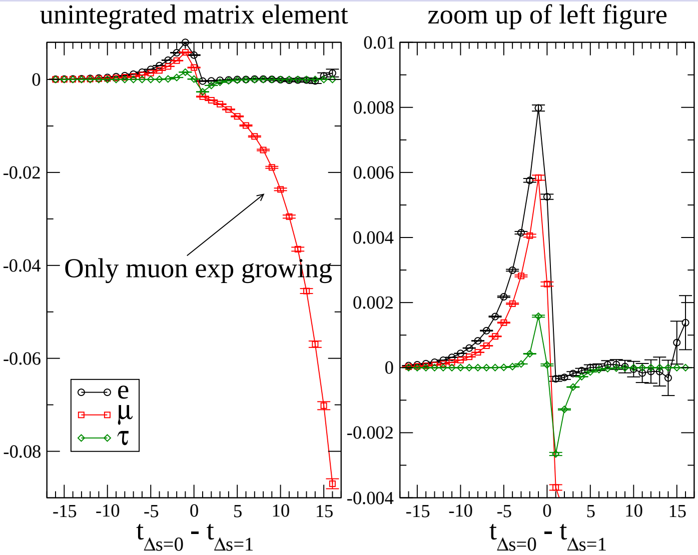
$$\mathcal{T} = F(p_K, p_\nu, p_{\bar{\nu}}) \bar{u}(p_\nu) \not{p}_K (1 - \gamma_5) v(p_{\bar{\nu}})$$

Preliminary results for W-W diagrams

Type 1 diagram

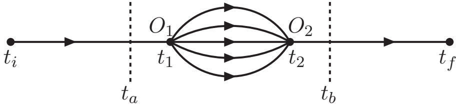


Unintegrated matrix element



- e mode: no exp growing contamination due to helicity suppression
- τ mode: no exp growing contamination since τ is heavy

Double integration

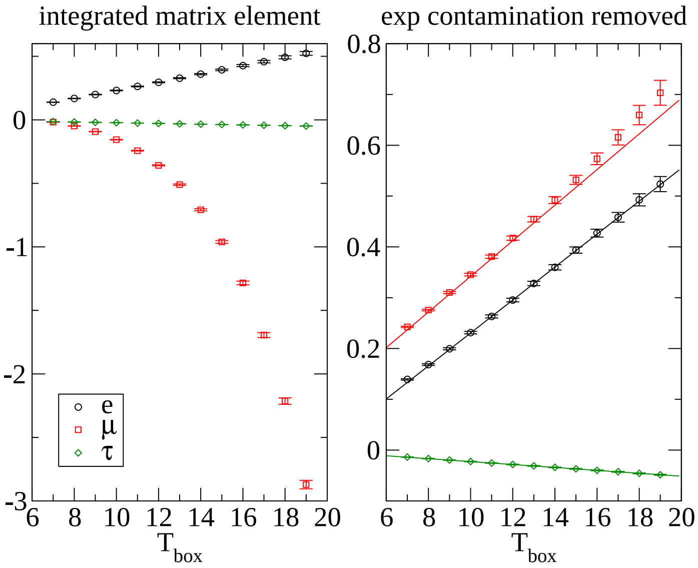


perform the double integration to gain a better precision
[N. Christ et.al., ΔM_K, arXiv:1212.5931]

$$
\begin{aligned}
&\sum_{t_1=t_a}^{t_b}\sum_{t_2=t_a}^{t_b} \langle f|T[O^{\Delta S=1}(t_2)O^{\Delta S=0}(t_1)]|K\rangle\, e^{m_K t_1} e^{-m_f t_1} \\
= \quad & \sum_{n_s} \frac{\langle f|O^{\Delta S=1}|n_s\rangle\langle n_s|O^{\Delta S=0}|K\rangle}{E_{n_s}-E_f}\left(T_{\text{box}} - \frac{1-e^{(E_f-E_{n_s})T_{\text{box}}}}{E_{n_s}-E_f}\right) \\
& -\sum_{n} \frac{\langle f|O^{\Delta S=0}|n\rangle\langle n|O^{\Delta S=1}|K\rangle}{E_K-E_n}\left(T_{\text{box}} + \frac{1-e^{(E_K-E_n)T_{\text{box}}}}{E_K-E_n}\right)
\end{aligned}
$$

here $T_{\text{box}} = t_b - t_a + 1$ is defined as size of the integral window
remove the exponential growing contamination, and fit with $a + bT_{box}$, the slope b is what we want

Integrated matrix element



right figure: the slope of the curve gives $F^{\ell}(p_K, p_\nu, p_{\bar{\nu}})$

F^ℓ for type 1 diagram

F^ℓ	lattice	model
e	$3.244(90)\times 10^{-2}$	$3.352(12)\times 10^{-2}$
μ	$3.506(77)\times 10^{-2}$	$3.511(13)\times 10^{-2}$
τ	$-2.871(70)\times 10^{-3}$	$-2.836(10)\times 10^{-3}$

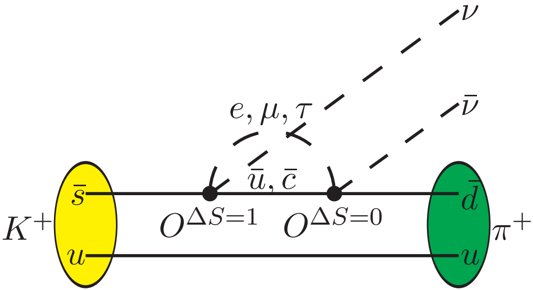
- vacuum saturation approximation assumes only single-lepton contribution in the intermediate state

$$f_K f_\pi \bar{u}(p_\nu)\not{p}_K(1-\gamma_5)\frac{\not{q}}{q^2-m_\ell^2}\not{p}_\pi(1-\gamma_5)v(p_{\bar{\nu}})$$
$$= f_K f_\pi \frac{2q^2}{q^2-m_\ell^2}\bar{u}(p_\nu)\not{p}_K(1-\gamma_5)v(p_{\bar{\nu}})$$

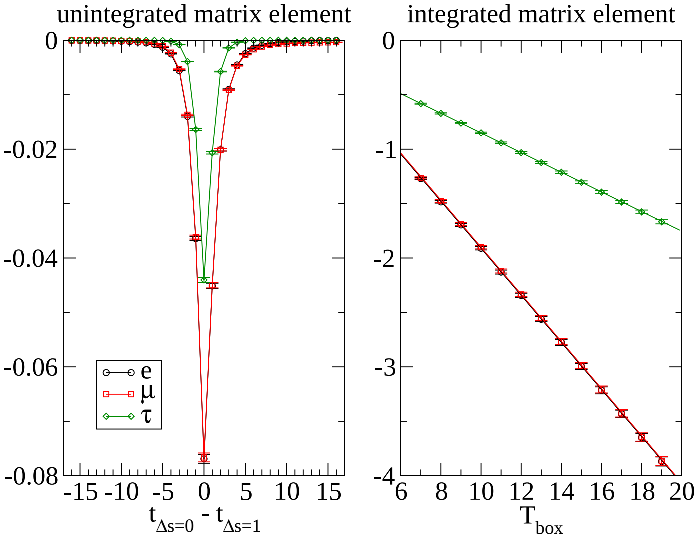
with $q = p_K - p_\nu = p_\pi + p_{\bar{\nu}}$

- in the above table, model results are given by $Z_A^{-2} f_K f_\pi \frac{2q^2}{q^2-m_\ell^2}$

Type 2 diagram



Type 2 diagram, left: unintegrated, right: integrated



intermediate state is given by $\ell + \pi^0$, since pion is heavy, we don't observe significant exponential growing effects

Preliminary results

- type 1 diagram

F^ℓ	lattice	model
e	$3.244(90) \times 10^{-2}$	$3.352(12) \times 10^{-2}$
μ	$3.506(77) \times 10^{-2}$	$3.511(13) \times 10^{-2}$
τ	$-2.871(70) \times 10^{-3}$	$-2.836(10) \times 10^{-3}$

- type 2 diagram before SD subtraction

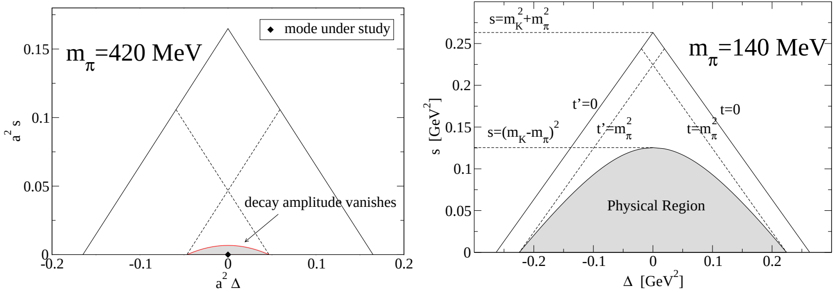
F^ℓ	lattice
e	$-2.164(31) \times 10^{-1}$
μ	$-2.164(31) \times 10^{-1}$
τ	$-9.03(14) \times 10^{-2}$

- type 2 diagram after SD subtraction, using $C^{lat}(\mu)$

F^ℓ	$\mu = 2$ GeV	$\mu = 3$ GeV
e	$-1.400(31) \times 10^{-1}$	$-1.849(31) \times 10^{-1}$
μ	$-1.402(31) \times 10^{-1}$	$-1.850(31) \times 10^{-1}$
τ	$-4.13(14) \times 10^{-2}$	$-6.68(14) \times 10^{-2}$

From $m_\pi = 420$ MeV to $m_\pi = 170$ MeV

- allowed momentum region becomes much larger in Dalitz plot



- at $m_\pi = 170$ MeV, kaon mass above two-pion threshold

$$\langle \pi\nu\bar{\nu}|O_Z(t)|\pi\pi\rangle\langle\pi\pi|Q_{1,2}(0)|K\rangle,$$

- evaluate light-quark contribution with controlled error
- however, for charm quark, we need ensemble of $80^2 \times 96 \times 192$, $a^{-1} = 3$ GeV

Required resources

- $32^3 \times 64$, $m_\pi = 170$ MeV: ~ 500 configs separated by every 4 trajs
- in our proposal, plan to use 100 configurations for exploratory study
- time estimates per config. based on a 512-node BG/Q @BNL

	Cost
600 eigenvectors for light quark (1 periodic mom + 4 twisted mom)	6h
Deflated light-quark CG (64 wall + 64 point + 128 random source + 4×64 twisted mom + 4×64 periodic mom)	16h
charm-quark CG (64 point + 128 random source)	2h
strang-quark CG (64 wall + 64 point + 128 random source)	4h
contractions for W-W diagrams	6h
contractions for Z/γ-exchange diagrams	6h

- 4 twisted mom. for W-W and 4 periodic mom. for Z-exchange
- 100 configs $\Rightarrow$ 33 million BG/Q core hours

Conclusion

$K^+ \to \pi^+ \nu\bar{\nu}$ is interesting for a lattice QCD study

- new experiment NA62 will confront SM soon
- although LD contribution is small, its exact size is not fully understood $\Rightarrow$ lattice QCD can determine it with controlled err

$K \to \pi \ell^+ \ell^-$ is interesting for a lattice QCD study

- long-distance dominance $\Rightarrow$ lattice QCD can play an important role

the calculation is highly non-trivial due to its non-local structure

our team is ready

Backup slides

Importance of lattice QCD calculation of $K \to \pi \ell^+ \ell^-$

Three decay modes for $K \to \pi\ell^+\ell^-$

- $K^+, K_S \to \pi\ell^+\ell^-$: CP conserving process
  - can described by a $K \to \pi\gamma^*$ transition from factor

$$\mathcal{A}_i = -\frac{G_F\alpha}{4\pi} V_i(q^2)(p_K + p_\pi)^\mu \bar{u}_l\gamma_\mu v_l, \quad q^2 = (p_K - p_\pi)^2, \quad i = +, S$$

  - form factor $V_i(q^2)$ at $q^2 = 0 \quad \Rightarrow \quad a_+,\ a_S$
  - χPT + experimental data is used to determine a_+ and a_S
  - a_+: well determined; a_S: due to lack of exp. input, its sign is unknown
  - lattice QCD determine a_+, a_S, also q^2-dependence of $V_i(q^2)$
- $K_L \to \pi^0\ell^+\ell^-$: CP violating process
  - direct CPV: $K_L \to \pi^0(\ell^+\ell^-)_{J=1}$, $\propto \mathrm{Im}\,\lambda_t$, SD dominated
  - indirect CPV: $K_L \xrightarrow{\epsilon} K_S \to \pi^0\gamma^* \to \pi^0\ell^+\ell^-$, related to a_S
  - CPC: $K_L \to \pi^0(\gamma^*\gamma^*)_{J=0,2} \to \pi^0\ell^+\ell^-$, LD dominated by 2γ-exchange
  - the three main contributions are of comparable size

Theoretical status for $K_L \to \pi^0 \ell^+ \ell^-$

- direct + indirect CPV contribution to branching ratio [arXiv:1107.6001]

$$\mathrm{Br}(K_L \to \pi^0 e^+ e^-)_{CPV} =$$
$$= 10^{-12} \times \left[15.7|a_S|^2 \pm 6.2|a_S| \left(\frac{\mathrm{Im}\,\lambda_t}{10^{-4}}\right) + 2.4 \left(\frac{\mathrm{Im}\,\lambda_t}{10^{-4}}\right)^2 \right]$$
$$\mathrm{Br}(K_L \to \pi^0 \mu^+ \mu^-)_{CPV} =$$
$$= 10^{-12} \times \left[3.7|a_S|^2 \pm 1.6|a_S| \left(\frac{\mathrm{Im}\,\lambda_t}{10^{-4}}\right) + 1.0 \left(\frac{\mathrm{Im}\,\lambda_t}{10^{-4}}\right)^2 \right]$$

  - $\mathrm{Im}\,\lambda_t/10^{-4} = 1.35$; $|a_S|$ is determined to be $O(1)$

a change in the sign of a_S can cause a large difference in the predicted Br

Preliminary results for Z-exchange diagrams

Evaluation of non-local matrix element

$$T_\mu^Z = \int dt \, \langle \pi^+ | T\{Q_{1,2}(t) J_\mu^Z(0)\} | K^+ \rangle$$

- Z-exchange diagrams do not require on-shell neutrinos
  - we use $\vec{p}_K = \vec{p}_\pi = 0$, $\quad J_\mu^Z$, $\mu = t$
- hadronic current J_μ^Z has vector and axial vector component
  - for the vector current, according to Ward identity (WI), we have

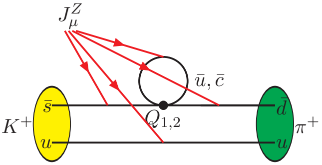
$$T_\mu^{Z,V} = F^{Z,V}(q^2)\left(q^2(p_K + p_\pi)_\mu - (m_K^2 - m_\pi^2) q_\mu\right), \quad q = p_K - p_\pi$$

  - with $\vec{p}_K = \vec{p}_\pi = 0 \quad \Rightarrow \quad q^2(p_K + p_\pi)_\mu - (m_K^2 - m_\pi^2) q_\mu = 0$
  - WI suggests $T_\mu^{Z,V} = 0$, this is confirmed by our numerical calculation
- in the following, I will present the results for axial vector current

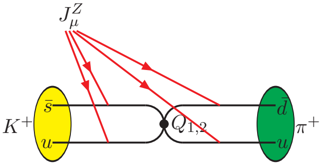
Summary of Z-exchange diagrams

four classes: Type 1 (Q_1), Type 1 (Q_2), Type 2 (Q_1), Type 2 (Q_2)

- connected diagrams, J_μ^Z can be inserted into all the possible quark line

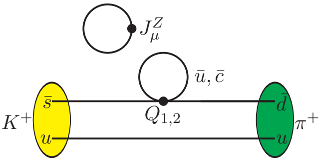


Type 1 diagram

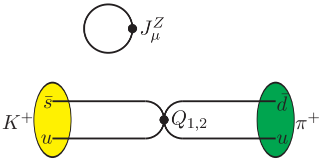


Type 2 diagram

- disconnected diagrams (usually excluded in lattice calculation since they are noisy and difficult to calculate)

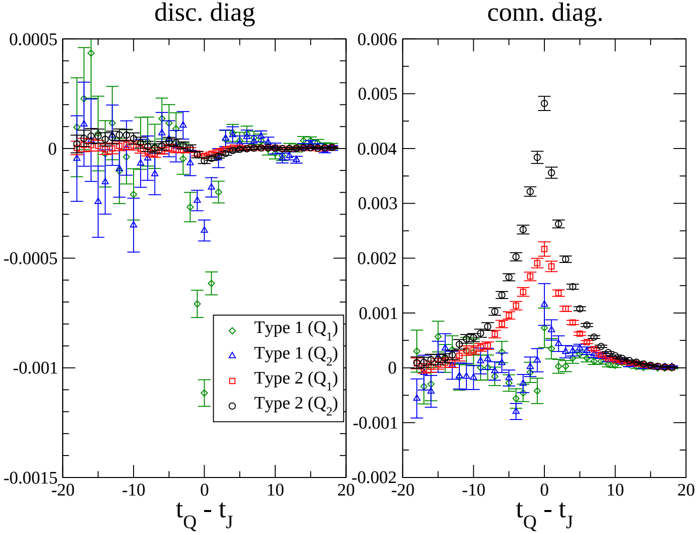


Type 1 diagram



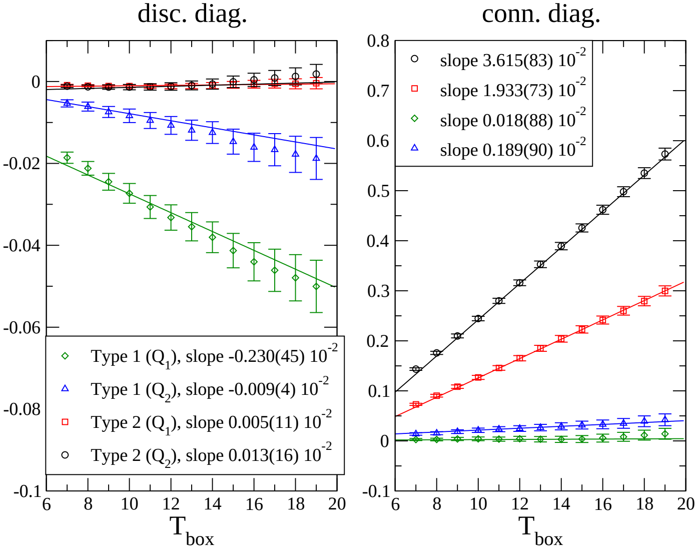
Type 2 diagram

Disc. diag. vs Conn. diag.



- although noisy, clear signal from disc. diagram
- scale of y-axis in disc. plot is 4 times smaller than that in conn. plot

Integrated matrix element



disc. diag. is relatively noisy, but its contribution is small. Adding the disc. part does not affect the conn. part significantly

Short-distance subtraction

- evaluate off-shell Green's function with $p_i^2 \gg \Lambda_{QCD}^2$
- energy scale of internal momentum, μ^2, is forced to be larger than p_i^2
- at high energy scale μ, mainly SD contribution to off-shell Green's function
- correctly represented by a SD operator multiplying with Wilson coefficient $C^{lat}(\mu)$