Non-Perturbative Collider Phenomenology of Stealth Dark Matter

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Lattice Strong Dynamics Collaboration

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Strongly-coupled composite dark matter

- Our focus: composite DM as a strongly-bound state of some more fundamental objects (think of the neutron)

- Non-Abelian $\text{SU}(N_D)$ gauge sector, with some fermions in the fundamental rep. Not the only possibility (e.g. “dark atoms”, other non-Abelian theories) but a well-motivated, somewhat familiar foundation.

- Constituents can carry SM charges, and charged excited states active in early universe. Composite DM relic interacts via SM particles (photon, Higgs) but with form factor suppression!
Symmetries of stealth DM

- Start with SU($N_D$) gauge theory and $N_F$ Dirac fermions, in the fundamental rep, and impose some conditions.

- **First requirement**: *baryons are bosons* - even $N_D$. No magnetic moment. $N_D \geq 4$ gives automatic DM stability from Planck-scale violations.

- **Second requirement**: *couplings to electroweak and Higgs* - one EW doublet and one singlet, $N_F \geq 3$. Ensures meson decay as well.

- **Third requirement**: *custodial SU(2) for electroweak precision* - $N_F=4$. As a bonus, charge radius is eliminated $\rightarrow$ **stealth DM!**
Stealth dark matter: model details

<table>
<thead>
<tr>
<th>Field</th>
<th>$SU(N_D)$</th>
<th>$(SU(2)_L, Y)$</th>
<th>$Q$</th>
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</thead>
<tbody>
<tr>
<td>$F_1 = \begin{pmatrix} F_1^u \ F_1^d \end{pmatrix}$</td>
<td>N</td>
<td>(2, 0)</td>
<td>$+1/2$</td>
</tr>
<tr>
<td>$F_2 = \begin{pmatrix} F_2^u \ F_2^d \end{pmatrix}$</td>
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<td>(2, 0)</td>
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<tr>
<td>$F_3^u$</td>
<td>N</td>
<td>(1, +1/2)</td>
<td>$+1/2$</td>
</tr>
<tr>
<td>$F_3^d$</td>
<td>N</td>
<td>(1, −1/2)</td>
<td>$-1/2$</td>
</tr>
<tr>
<td>$F_4^u$</td>
<td>N</td>
<td>(1, +1/2)</td>
<td>$+1/2$</td>
</tr>
<tr>
<td>$F_4^d$</td>
<td>N</td>
<td>(1, −1/2)</td>
<td>$-1/2$</td>
</tr>
</tbody>
</table>

- **SU(4) gauge group with 4 Dirac fermions ($SU(2)_L$ and $SU(2)_R$ doublets)**
- **Two sources of mass allowed: vector-like and Higgs-Yukawa**
- **EW-preserving mass:**
  \[ \mathcal{L} \supset M_{12} \epsilon_{ij} F_1^i F_2^j - M_{34}^F F_3^u F_4^d + M_{34}^d F_3^d F_4^u + h.c., \]
- **EW-breaking mass:**
  \[ \mathcal{L} \supset y_{14}^u \epsilon_{ij} F_1^i H^j F_4^d + y_{14}^d F_1 \cdot H^\dagger F_4^u 
  - y_{23}^d \epsilon_{ij} F_2^i H^j F_3^d 
  - y_{23}^u F_2 \cdot H^\dagger F_3^u + h.c., \]
Mass eigenstates

- Two sources of mass, electroweak breaking and preserving:

\[ M = \frac{M_{12} + M_{34}}{2}, \quad \Delta = \left| \frac{M_{12} - M_{34}}{2} \right|, \quad y_{14} = y + \epsilon_y, \quad y_{23} = y - \epsilon_y, \quad |\epsilon_y| \ll |y|. \]

- Assume \( yv \ll M \), to avoid vacuum alignment issues w/EWSB. Then two regimes arise, depending on the origin of the mass splitting:

**Linear Case:** \( yv \gg \Delta \)

\[ y\Psi = \frac{y}{\sqrt{2}} \]

**Quadratic Case:** \( yv \ll \Delta \)

\[ y\Psi = \frac{y^2 v}{2\Delta} \]

(linear/quadratic effect observed before, see Hill and Solon 1401.3339)
Stealth dark matter on the lattice

- **The model:** SU(4) gauge theory at moderately heavy fermion mass
- **On the lattice:** plaquette gauge action, Wilson fermions *(quenched)*
- **Spectrum shown to the right**
Stealth dark matter: lattice results so far

- Spectrum and scalar current calculation: mass generation from Higgs strongly constrained.

- EM polarizability: lower bound on direct detection for theories with charged constituents. Stealth DM visible below a TeV or so.
DM is far from lightest particle in the new sector! Much harder to produce directly in colliders, so MET signals are greatly suppressed.

On the other hand, presence of the much lighter and charged Π states gives strong bounds from complementary searches.
Meson decay

- Important consequence of electroweak coupling: allow mesons to decay, especially the charged ones!

\[ \langle 0 | j_{\text{axial}}^\mu | \Pi^\pm \rangle = i f_\Pi p_\mu \]

- Mass flip in final state, due to decay of pseudoscalar bound state (same for QCD pions.) Gives preferred decay to heaviest SM states:

\[ \Gamma(\Pi^+ \to f\bar{f}') = \frac{G_F^2}{4\pi} f_\Pi^2 m_f^2 m_\Pi c_{\text{axial}}^2 \left( 1 - \frac{m_f^2}{m_\Pi^2} \right) \]
Meson production

- First signature expected: Drell-Yan photon production of charged Π

- To calculate rate, pion form factor needed at threshold: \( F_v(Q^2 = 4m_\Pi^2) \)

- Hard to access at this momentum on lattice. In QCD, “vector meson dominance” does pretty well…

![Meson production diagram](image)
How the picture changes for $m_\rho$ below threshold:

\[ \pi\pi \text{ scattering amplitude with } m_\pi=140 \text{ MeV, for QCD (} m_\pi/m_\rho\sim0.18) \]
and \text{ for a stealth-DM-like theory (} m_\pi/m_\rho\sim0.55) \]

• Here, “rho” resonance is below $2\pi$ threshold - but it’s also much closer to the threshold. Vector-meson dominance should be reliable, but further study is needed.

• The “dark rho” is very narrow, since decay to $\pi\pi\pi$ is closed. Another (TeV-scale) state to look for in colliders!
Our plan

- Determine “stealth $\rho$” decay constant and calculate decay width
- Measure “stealth $\pi$” $F_V(Q^2)$ at space-like momenta from three-point function (pion charge radius)
- Combine with vector-meson dominance model to predict $F_V(4m_\pi^2)$ for collider production

$$F_V(Q^2) = \exp \left( \frac{\langle r^2_\Pi \rangle Q^2}{6} + \frac{Q^4}{\pi} \int_{4m_\pi^2}^{\infty} ds \frac{\delta_{11}(s)}{s^2(s - Q^2 - i\epsilon)} \right)$$

(arXiv:0812.3270)
Electroweak precision

• No T parameter by construction (custodial symm), but S parameter is an important constraint! Two asymptotic forms of S contribution:

\[ \Pi_{3Y}(q^2) \approx \frac{y^2 v^2}{4M^2} \Pi_{LR}(q^2) \]

• Calculation of strong-coupling part yields direct bounds on Yukawa couplings (important for asymmetric relic density)

(PDG 2014)
Lattice calculation details

- **Form factor:** calculate $<\pi(t)V_\mu(t')\pi(0)>$ and $<\pi(t)\pi(0)>$. Construct appropriate ratio to extract vector-current matrix element.

- Three vector-current insertion locations, four sources per config $\rightarrow$ 12 Wilson propagators; 500 (pure-gauge) configurations.

- **S-parameter:** calculate conserved-local correlators $<V_C(x)V_L(0)>$ and $<A_C(x)A_L(0)>$. Two source positions, $L_s=8$.

- By-products of DWF calculation: $F_\pi$, mass renormalization $(m_f/M_B)$. 
Resource request

- Three mass points for domain-wall S parameter calculation; we expect mild mass dependence, based on experience

- One point at $\beta=11.5$, to test discretization effects (spectroscopy here shows no significant deviations)

- We are working on a new fully threaded/vectorized code base, meant to replace QDP/C; Wilson solver in progress. Up to 2x speed-up in calculations expected, but no benchmarks available yet, and we don’t include this factor above

<table>
<thead>
<tr>
<th>$\beta$</th>
<th>$\text{vol}$ $= 32^3 \times 64$</th>
<th>$\kappa$</th>
<th>$m_{PSL}$</th>
<th>$m_{PS}/m_V$</th>
<th>Cost (DWF)</th>
<th>Cost (Wilson)</th>
<th>Total cost</th>
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<td>0.68</td>
<td>1.58</td>
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<td>0.1568</td>
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<tr>
<td>Total</td>
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<td></td>
<td>5.34</td>
<td>6.08</td>
<td>11.42</td>
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</table>

Table 1: Estimated resources required for the $S$-parameter calculation (DWF propagators) and the dark pion form factor calculation (Wilson propagators). All costs are given in units of J/Psi core-hours times $10^6$. 

Table 2: Estimated resources required for the $S$-parameter calculation (DWF propagators) and the dark pion form factor calculation (Wilson propagators). All costs are given in units of J/Psi core-hours times $10^6$. 

References


Backup slides
Stability of composite dark matter candidates

\[ \Pi \sim \bar{\Psi} \Psi \quad B \sim \bar{\Psi} \Psi \ldots \Psi \quad N_D \text{ constituents} \]

- Lightest mesons (\( \Pi \)) can be stabilized by flavor symmetries* or G-parity**, but then one has to argue against the presence of dimension-5 operators like
  \[ \frac{1}{\Lambda} \bar{\Psi} \Psi H^\dagger H \quad \rightarrow \quad \text{instability over lifetime of the universe.} \]

- Accidental dark baryon number symmetry provides automatic stability for B on very long timescales (as long as \( N_D > 2! \)) E.g. for \( N_D=4 \), decay through dimension-8(!)
  \[ \frac{1}{\Lambda^4} \bar{\Psi} \Psi \Psi \Psi H^\dagger H \]

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*nM. Buckley and EN, arXiv:1209.6054
**Y. Bai and R. Hill, arXiv:1005.0008

Abundance

**Symmetric**

\[ B \quad \quad \Pi \quad \quad (\text{more } \Pi \text{s}) \]

\[ B^* \quad \quad \Pi \]

If 2 -> 2 dominates thermal annihilation rate and saturates unitarity, expect

\[ m_B \sim 100 \text{ TeV} \]

Griest, Kamionkowski; 1990

Unfortunately, this is hard calculation to do using lattice...

**Asymmetric**

e.g., through EW sphalerons

Chivukula, Farhi, Barr; 1990

\[ n_D \sim n_B \left( \frac{y_v}{m_B} \right)^2 \exp \left[ - \frac{m_B}{T_{\text{sph}}} \right] \]

IF EW breaking comparable to EW preserving masses, expect roughly

\[ m_B \lesssim m_{\text{techni-B}} \sim 1 \text{ TeV} \]

How much less depends on several factors...

(slide from G. Kribs)
Polarizability on the lattice

- Measure response to applied background field $E$ (quadratic Stark shift)
  \[
  E_{B,4c} = m_B + 2C_F |E|^2 + \mathcal{O}(E^4)
  \]
- SU(3) case simulated for comparison; complicated by magnetic moment $\mu_B$
  \[
  E_{B,3c} = m_B + \left(2C_F - \frac{\mu_B^2}{8m_B^3}\right) |E|^2 + \mathcal{O}(E^4)
  \]
- Comparable results for SU(3) and SU(4), in units of $m_B$. 

<table>
<thead>
<tr>
<th>$N_D$</th>
<th>$m_p/m_v$</th>
<th>$\tilde{m}_B$</th>
<th>$\tilde{\alpha}_F$</th>
<th>$\tilde{\alpha}_F^2$</th>
<th>$\tilde{\mu}_B$</th>
<th>$\tilde{\mu}_B'$</th>
<th>$\chi^2$/dof</th>
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<tr>
<td>4</td>
<td>0.77</td>
<td>0.98204(93)</td>
<td>0.1420(56)</td>
<td>-0.089(29)</td>
<td>---</td>
<td>---</td>
<td>0.7/3</td>
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<tr>
<td></td>
<td>0.70</td>
<td>0.88805(113)</td>
<td>0.1514(106)</td>
<td>-0.142(68)</td>
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<td>---</td>
<td>4.8/3</td>
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<td>3</td>
<td>0.77</td>
<td>0.69812(51)</td>
<td>0.2829(127)</td>
<td>-0.177(45)</td>
<td>-6.87(26)</td>
<td>714(103)</td>
<td>3.0/7</td>
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<tr>
<td></td>
<td>0.70</td>
<td>0.61904(59)</td>
<td>0.2829(81)</td>
<td>-0.165(24)</td>
<td>-5.55(18)</td>
<td>396(78)</td>
<td>13.4/7</td>
</tr>
</tbody>
</table>
Mass scales

Could arise dynamically

\[ M_f \sim \Lambda_D \]

(plot from G. Kribs)
Study of systematic effects

Finite-volume effects

Cutoff effects
that each coefficient in Eq. (15) has corrections that go as

in odd

trum comparisons of this kind is that all the baryons were
corrections. One common theme between previous spec-
baryons with three, five, and seven colors [38, 39]; well

settings) [35], in lattice 3-color calculations where a variety
kinds of large-

need to be extracted from two initial input values. These

the rotor spectrum [31, 35, 38]

(are expected to see appreciable scaling as

states are not expected to change appreciably as

baryons spectrums are expected to have significantly differ-

ground state in each (lattice) spin channel.

flavor space, one would expect to have overlap with the

unique combination

and for four flavors (combination

combinations

One point that was emphasized in Ref. [39] is the fact

At a fixed scale (such as the string tension), mesons and

fermions in a color-antisymmetric combination,

theories to bosonic baryons in even

parameters rotor-spectrum (black diamonds) in Eq. (16) is

for the 4-color baryons using the 3-color baryon input does

what is worth noting here here is that the two-parameter

between these two theories were chosen in Ref. [28] to

3/2 baryon mass). For that reason, Eq. (15) with two free

C is the subleading

where

limit, Eq. (15) should be written as [39]

Eq. (16),where the 3-color baryon spectrum and 4-color spin-0

trum predictions for the 4-color spin-1 and spin-2 baryon masses,

spin-2 is the heaviest. Similarly, the three-paramter rotor spec-

were for

FIG. 3. Comparison of 3-color (square, dashed) and 4-color

PS

V

Solid - 4 colors

Blue - Spin 1

Purple - Spin 1/2

Black - Spin 2

spin-3/2

spin-0

spin-1/2

** :  \[ M(N_c, J) = N_c m_0 + \frac{J(J+1)}{N_c} B + \mathcal{O}(1/N_c^2) \]

\*:  \[ M(N_c, J) = N_c m_0^{(0)} + C + \frac{J(J+1)}{N_c} B + \mathcal{O}(1/N_c^2) \]
SU(3) polarizability vs. the PDG

- Our polarizability differs from the PDG convention:
  \[ \alpha_E = C_F / \pi \]

- Have to compare at very different masses! Expected scaling is
  \[ \alpha_E \sim \frac{A}{m_\pi} + B \]
  \[ m_B \sim C + Dm_\pi^2 \]

- Qualitative agreement with expected trend! (Can’t fit well - mass range too large.)

(PDG entry for neutron)
(LSD, this work)
(Detmold, Tiburzi, and Walker-Loud, PRD81 (054502), 2010)