

Leptonic B and D Decays on the Lattice

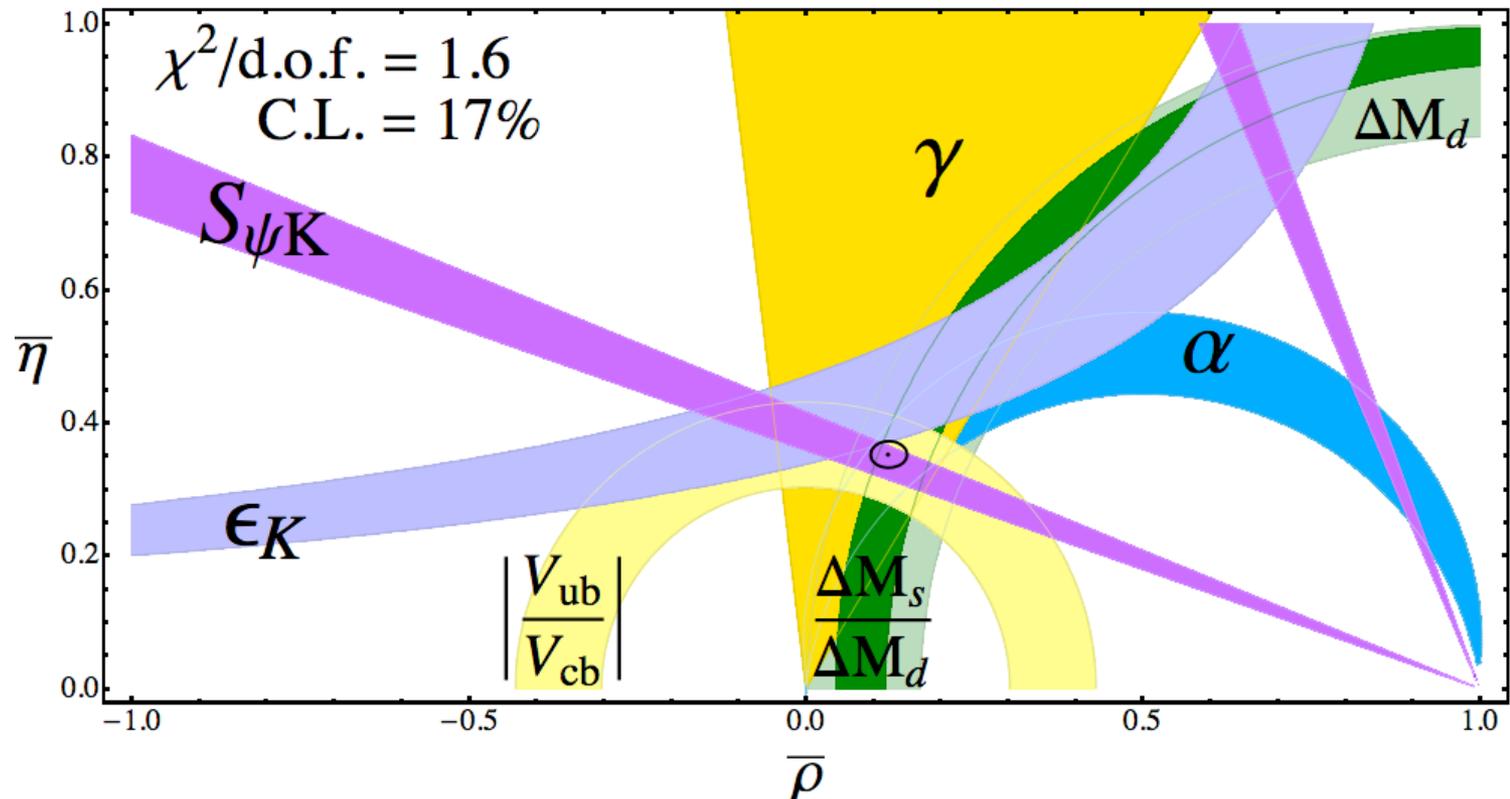
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Washington University
St. Louis

(Fermilab Lattice/MILC Collaboration)

Lattice QCD Meets Experiment Workshop 2010
Fermilab, April 26-27

CKM FIT

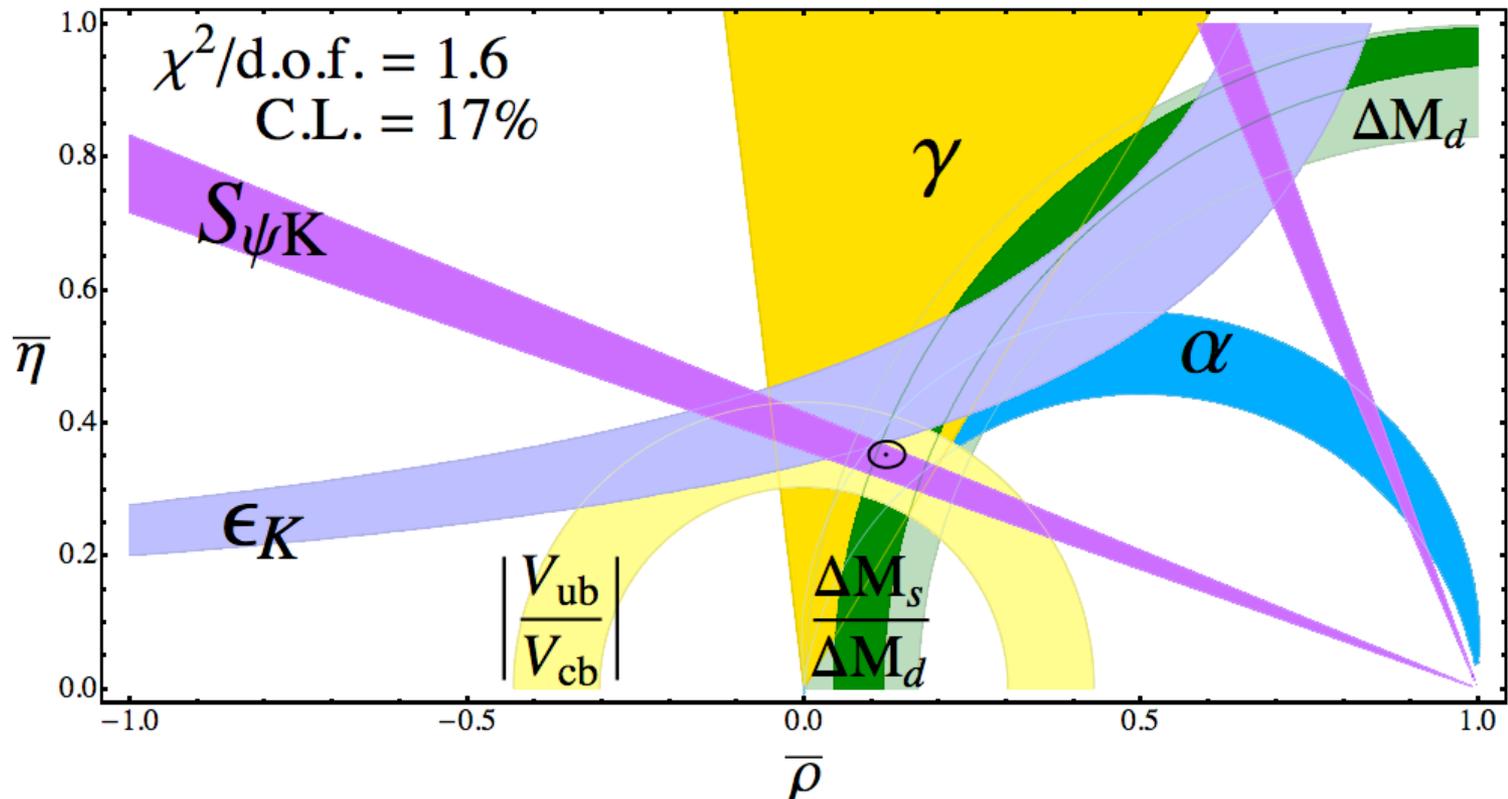
- Experimental errors < 1%
- Remaining significant errors are from lattice QCD.



From [Laiho, Lunghi, & Van de Water, arXiv:0910.2928](#)

CKM FIT

- Decay constants are important because f_{B_s}/f_B enters $\xi \equiv f_{B_s} \sqrt{B_{B_s}}/f_B \sqrt{B_B}$
 - relevant to $\Delta M_s/\Delta M_d$
 - See E. Gamiz talk this afternoon



From [Laiho, Lunghi, & Van de Water, arXiv:0910.2928](#)

More Motivation

- D system decay constants generally viewed as a good place to test lattice QCD.
 - but also possible (though unexpected) place for new physics.
 - f_{D_s} saga.
- BSM decay $B_s \rightarrow \mu^+ \mu^-$ depends on hadronic matrix element f_{B_s} :
 - decay comes from effective Higgs coupling to $\bar{b}s$, usually through $\langle 0 | \bar{b} \gamma_5 s | B_s \rangle$
 - but this is proportional to f_{B_s} using equations of motion to relate pseudoscalar density to axial current.
 - Experimental status: Tevatron talk to follow by W. Hopkins.

Computations

- Simulations in full QCD with all systematics controlled have been done by 2 collaborations:
 - Fermilab Lattice/MILC
 - HPQCD
- Both use lattice configurations generated by MILC, with 3 flavors of dynamical light sea quarks, using the “rooted staggered” action for the quarks.
- There is also an evaluation with only 2 light flavors, but all other systematics controlled:
 - European Twisted Mass (ETM) Collaboration
- Finally, some results for the B system with 3 light flavors and static B quarks, but only at one lattice spacing so far:
 - RBC/UKQCD

Staggered Quarks

- Increasingly precise results from staggered simulations in past decade:
 - revived old concerns that a technical step in the simulations, “rooting,” could be invalid and lead to incorrect results, even in continuum limit.
 - (rooting is the way staggered simulations deal with the lattice doubling problem for fermions)
 - issue has now been looked at in detail theoretically:
 - Shamir, CB, Golterman, Sharpe, Adams
 - and numerically:
 - Durr, Hoelbling, & Wegner; Follana, Hart, & Davies; MILC
- Conclusion: rooting step is valid.
 - but no proofs, of course, in nonperturbative QFT.
 - See reviews: Sharpe (06), Kronfeld (07), Golterman (08), MILC (09).

Analysis (Valence Quarks)

- Fermilab/MILC:
 - “Fermilab” action for bottom and charm valence quarks.
 - staggered (“asqtad”) action for light valence quarks.
- HPQCD:
 - NRQCD action for valence bottom.
 - HISQ action (a highly improved version of staggered) for both charm and light valence quarks.
 - HISQ charm helps a lot to reduce heavy quark discretization errors: $O(a^2)$ instead of $O(a)$; light errors also reduced.
 - using same action for light & heavy valence has further technical advantages that also reduce errors.
 - Note: at the moment HISQ bottom not possible except possibly by extrapolation from lower masses.

Lattice Scale

- Lattice computations need one dimensionful experimental input to set scale of lattice spacing.
- Splittings of Υ (determined by HPQCD with NRQCD) were the most precise and have often been used.
- Scale traditionally converted to a value for r_1 (conveniently determined on lattice, but not directly experimentally accessible)
- HPQCD Υ + MILC $r_1/a \rightarrow$

$r_1 = 0.318(7)$ fm	MILC
$r_1 = 0.321(5)$ fm	HPQCD

Lattice Scale

$$r_1 = 0.318(7) \text{ fm} \quad \text{MILC} \quad (\text{from } \Upsilon)$$
$$r_1 = 0.321(5) \text{ fm} \quad \text{HPQCD}$$

- But MILC f_π has consistently given a higher energy scale (lower r_1) ~ 0.311 fm.
- Precision of f_π has improved over time, & it doesn't suffer from NRQCD truncation systematics of Υ scale.
- Summer '09, we switched to f_π scale.
- Fall '09, HPQCD redid Υ analysis with improved methods & combined with their f_π
- New scales:
 $r_1 = 0.3117(6)(22) \text{ fm} \quad \text{MILC} \quad (\text{from } f_\pi)$
 $r_1 = 0.3133(23)(3) \text{ fm} \quad \text{HPQCD} \quad (\text{from } \Upsilon, f_\pi)$

MILC Ensembles

- MILC has generated ensembles of 2+1 flavors of rooted staggered quarks, with improved “asqtad” action:

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Ensemble Lattice Spacing
0.15 fm
0.12 fm
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Ensemble Lattice Spacing	Fermilab/MILC analysis
0.15 fm	√
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0.06 fm	partly done
0.045 fm	in progress

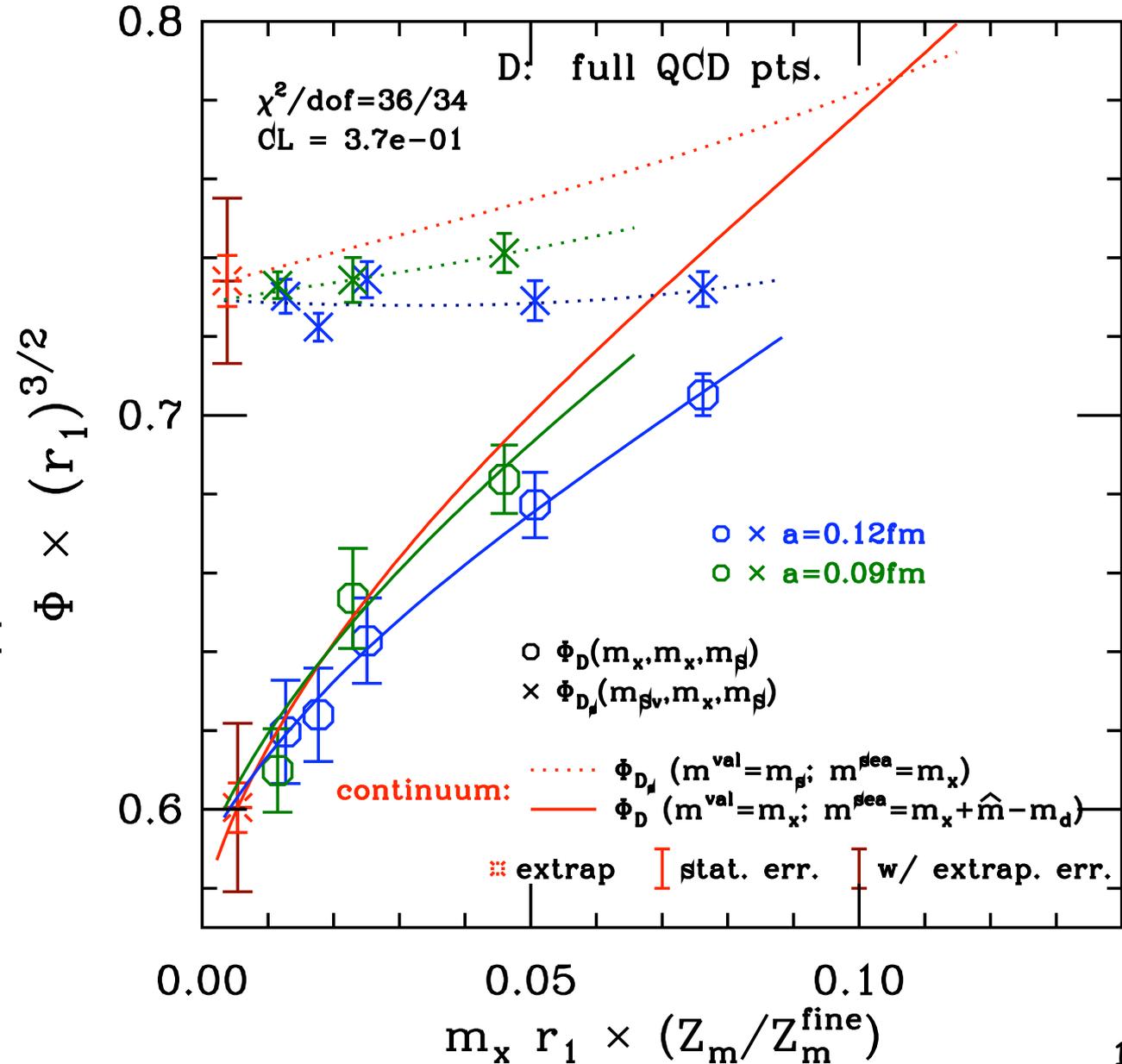
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Ensemble Lattice Spacing	Fermilab/MILC analysis	HPQCD analysis
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0.12 fm	✓	✓
0.09 fm	✓	✓
0.06 fm	partly done	prelim. result
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D system

- Fermilab/MILC
- Quark mass & continuum extrapolations
- Heavy quark discretization errors are largest systematic

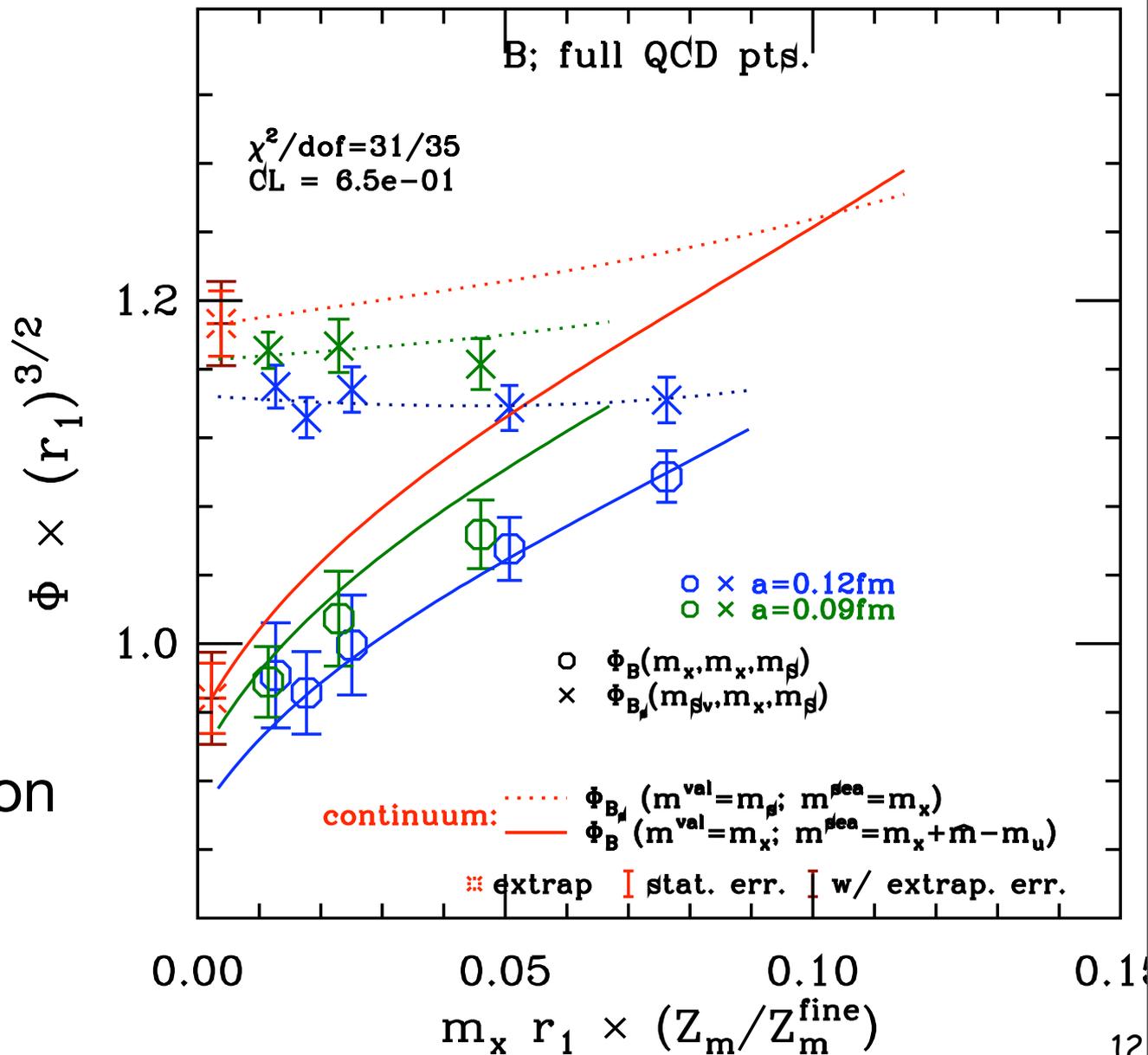


$$(\Phi \equiv f\sqrt{M})$$

B system

- Fermilab/MILC
- Quark mass & continuum extrapolations
- Statistics give biggest error
- Heavy quark discretization & mass extrapolation are comparable errors.

$$(\Phi \equiv f\sqrt{M})$$



Fermilab/MILC Results

$$f_{D_s} = 261.4 \pm 7.7 \pm 5.0 \text{ MeV}$$

$$f_{D^+} = 220.3 \pm 8.0 \pm 4.8 \text{ MeV}$$

$$f_{D_s}/f_{D^+} = 1.187 \pm 0.013 \pm 0.015$$

$$f_{B_s} = 256.3 \pm 5.9 \pm 5.5 \text{ MeV}$$

$$f_{B_d} = 211.8 \pm 6.3 \pm 5.5 \text{ MeV}$$

$$f_{B_s}/f_{B_d} = 1.210 \pm 0.014 \pm 0.015$$

- first error: statistics+discretization errors; second error: other systematics.
- slightly more than 1 (old) sigma increase in dimensionful numbers: half from scale change; rest from improved heavy quark tuning, chiral fits, statistics, & treatment of discretization errors.
- still preliminary; paper expected in summer.

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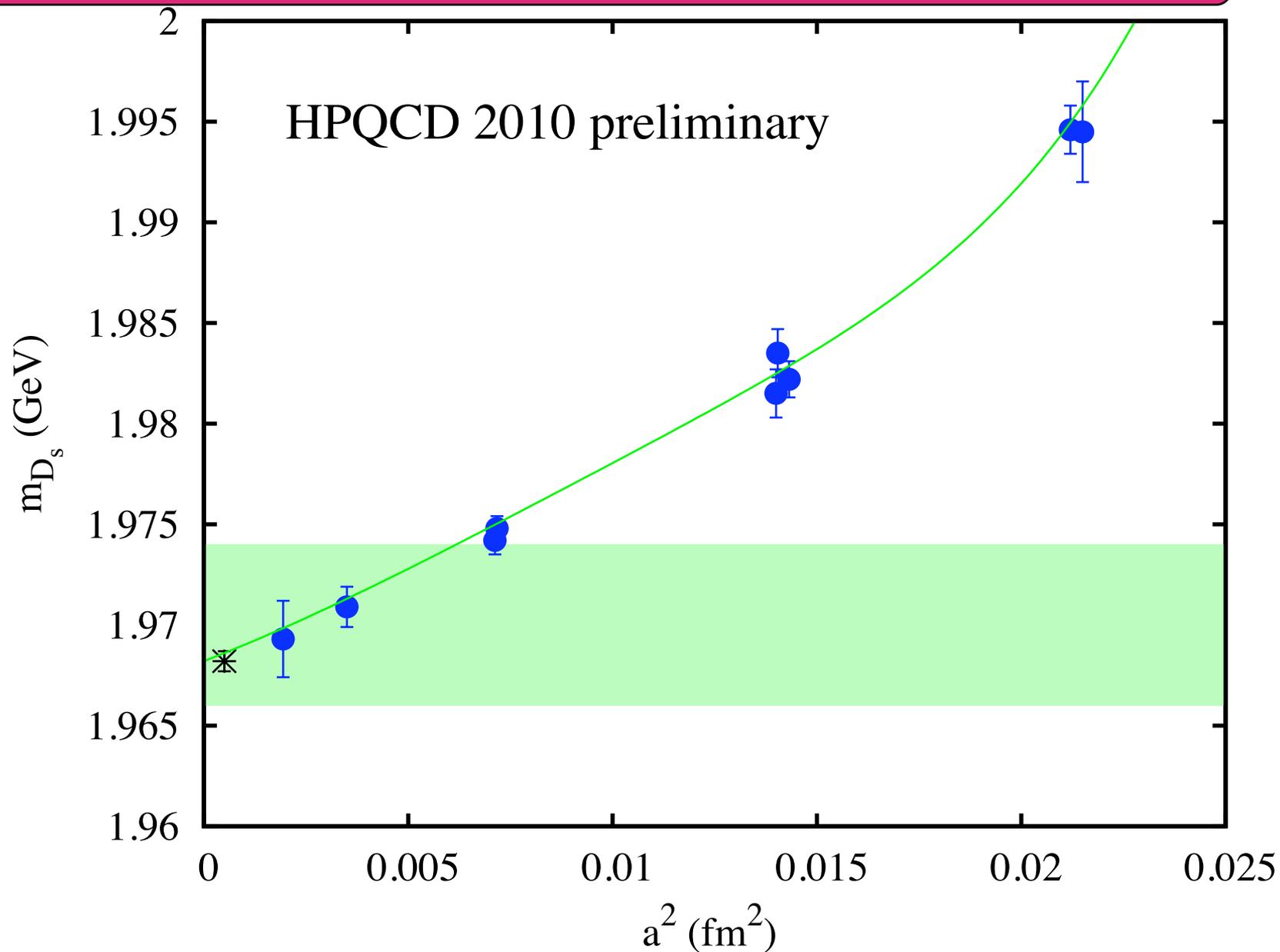
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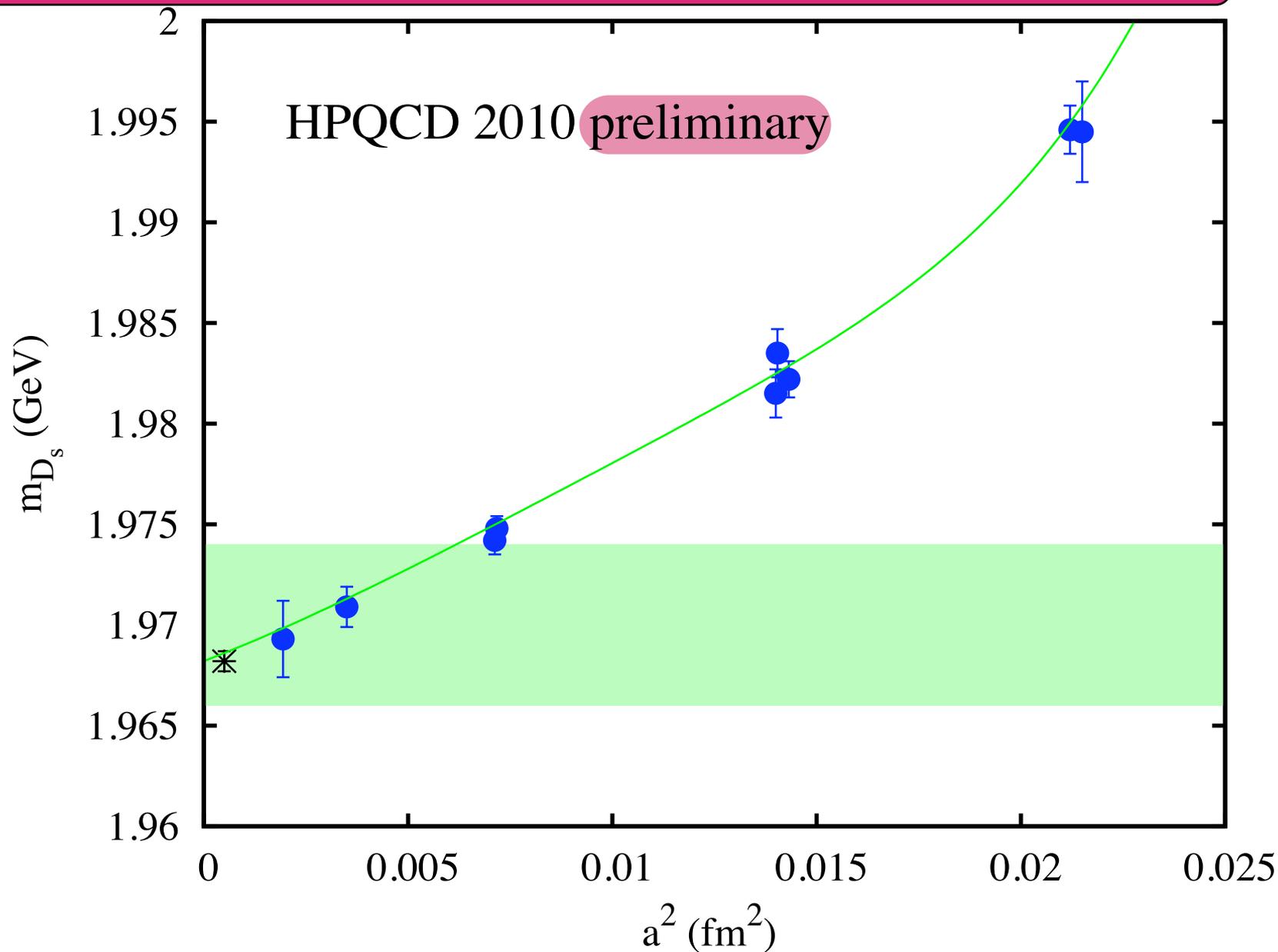
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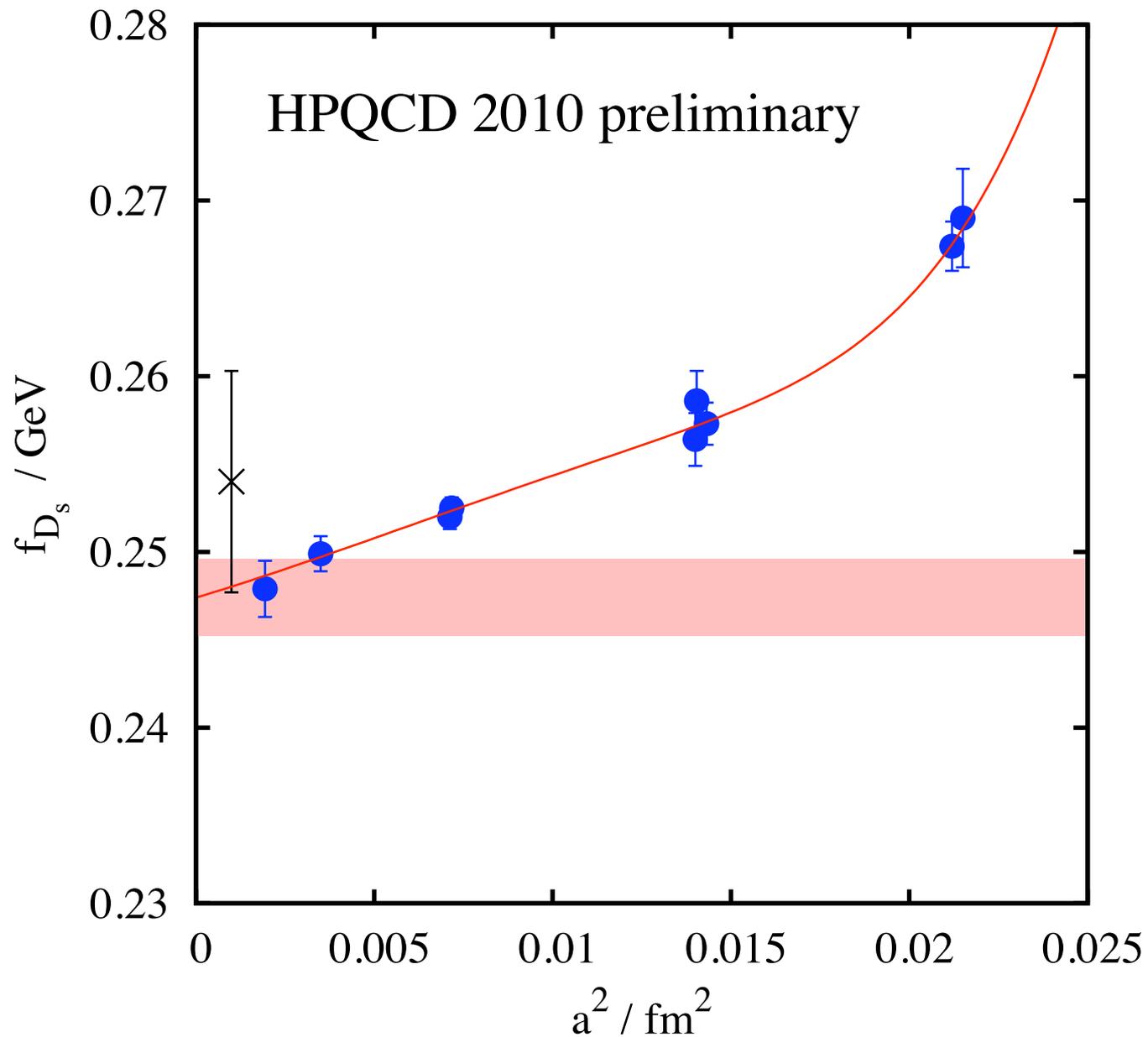
HPQCD: m_{D_s}



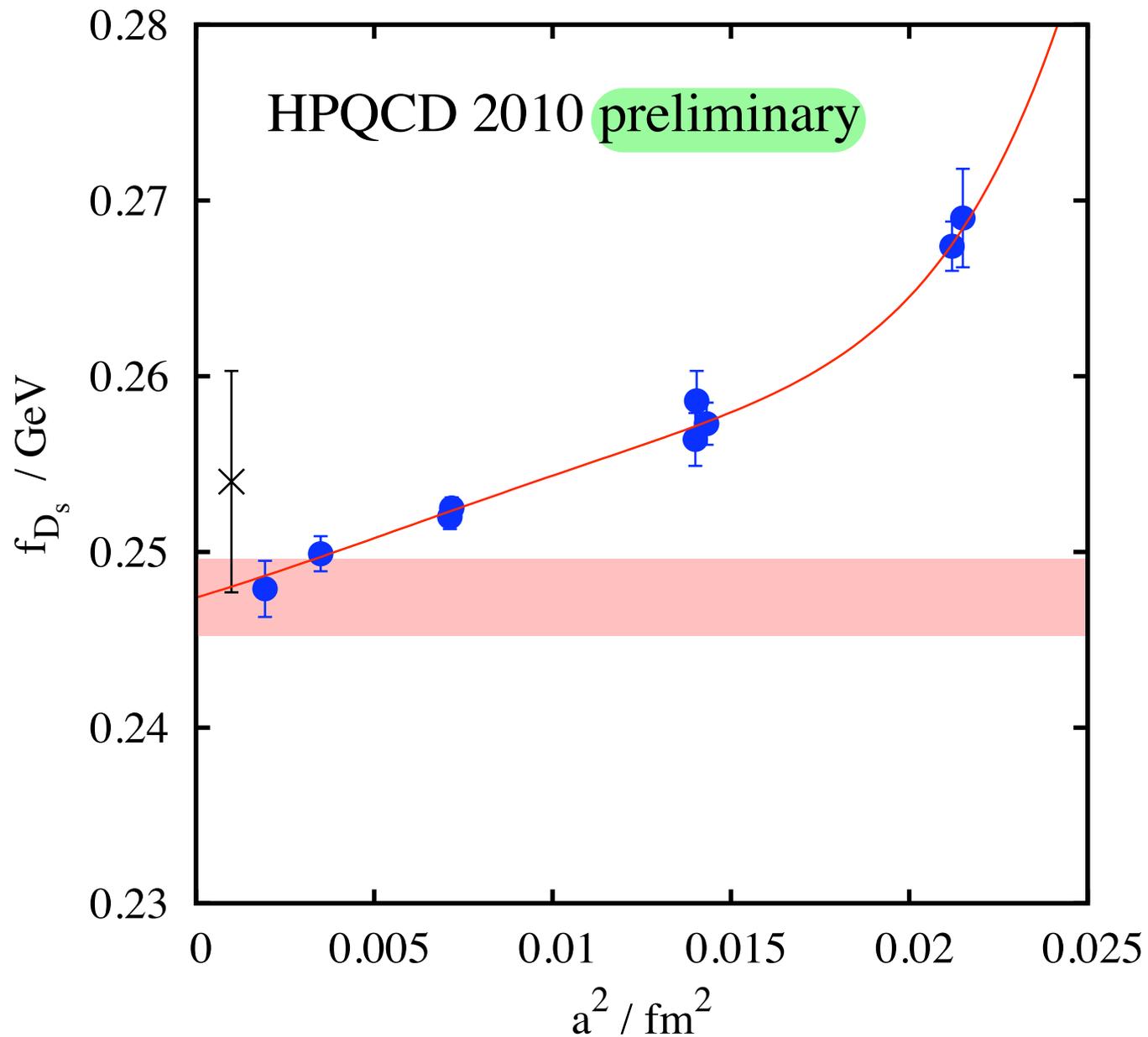
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HPQCD Results

2007:

$$f_{D_s} = 241 \pm 3 \text{ MeV}$$

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prelim. 2010:

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2009:

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$$f_{B_d} = 197 \pm 14 \text{ MeV}$$

$$f_{B_s}/f_{B_d} = 1.226 \pm 0.026$$

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- They showed scale dependence explicitly
--- I have put in their new scale.

ETMC Results

from 2009:

$$f_{D_s} = 244 \pm 8 \text{ MeV}$$

$$f_D = 197 \pm 9 \text{ MeV}$$

$$f_{D_s}/f_D = 1.24 \pm 0.011$$

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$$f_{B_d} = 191 \pm 14 \text{ MeV}$$

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- Two flavors of sea quarks!

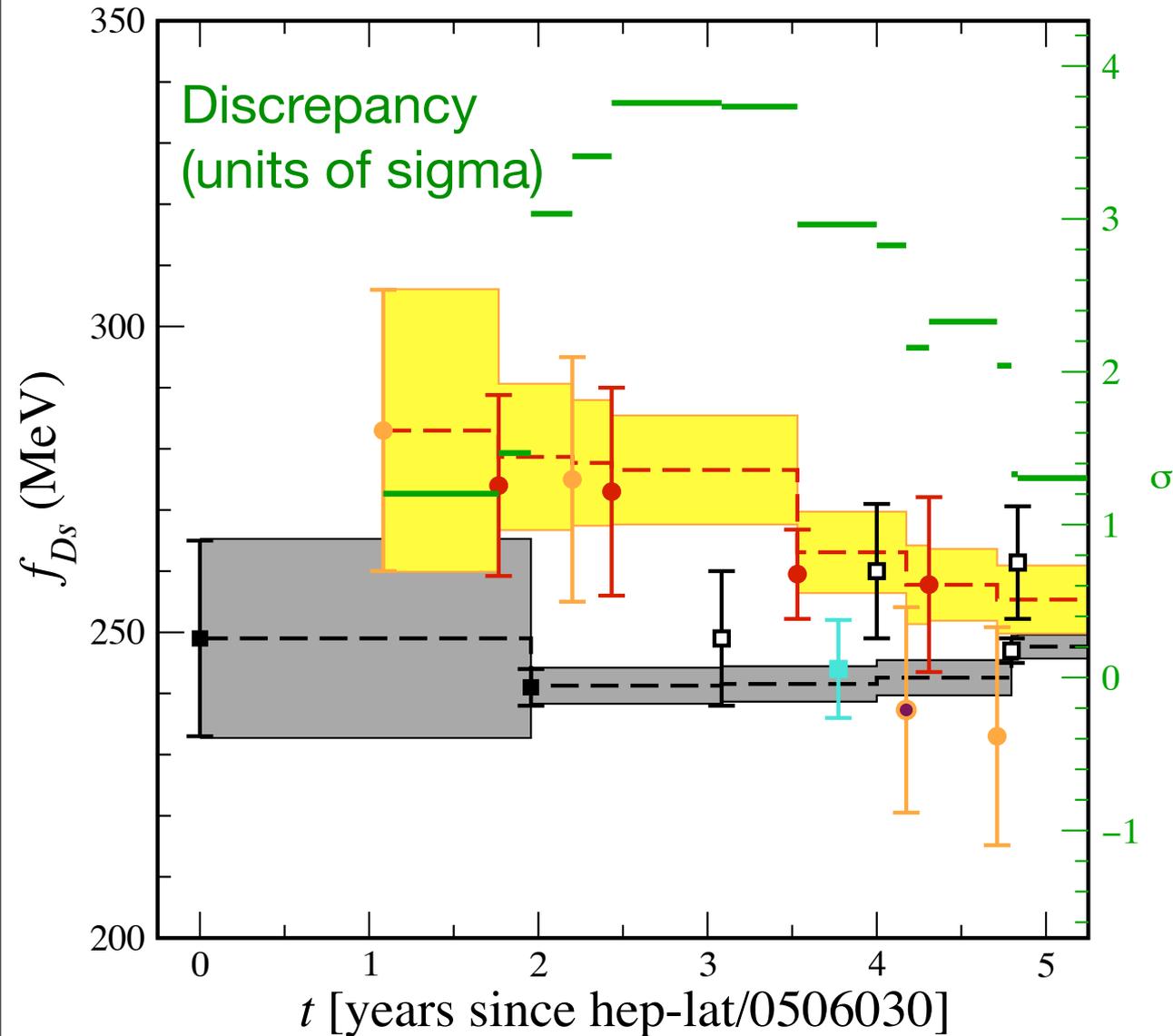
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f_{D_s} Saga

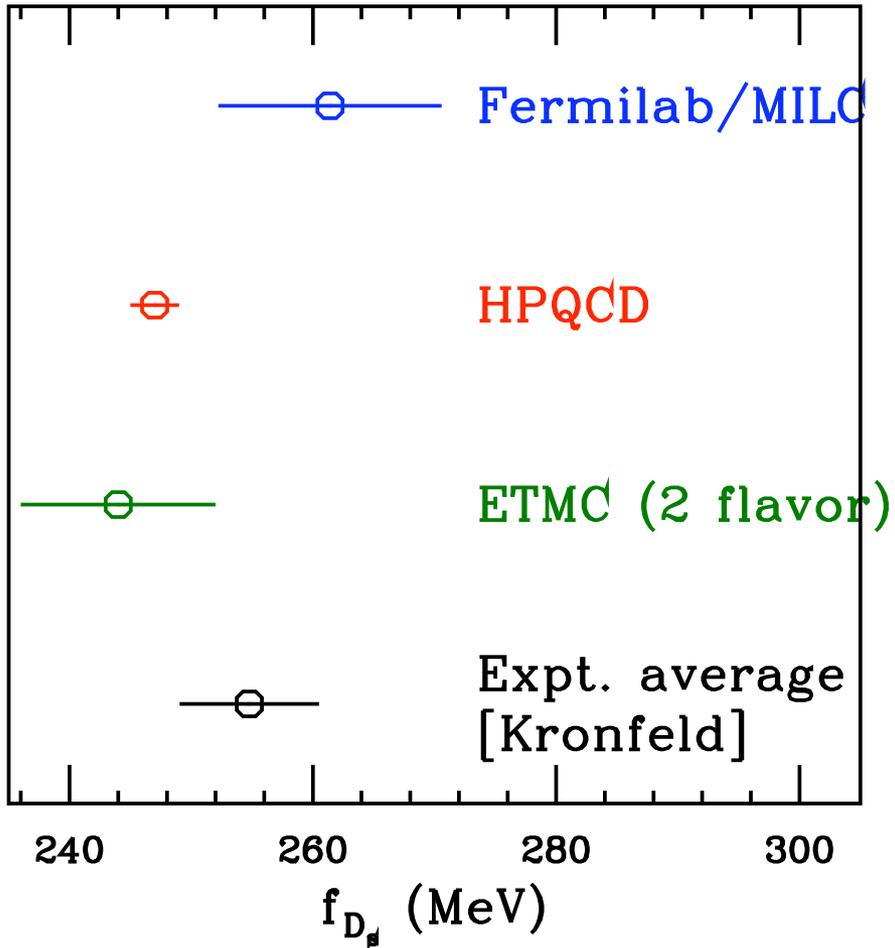


- **Yellow:** expt. average
- **Gray:** lattice average
- **Circles:** expts.:
 - orange: $\Upsilon(4S)$
 - red: $D_s^{(*)} D_s^{(*)}$ threshold
- **Squares:** lattice
 - filled: published
 - open: prelim or conference proc.
 - cyan: 2 flavors

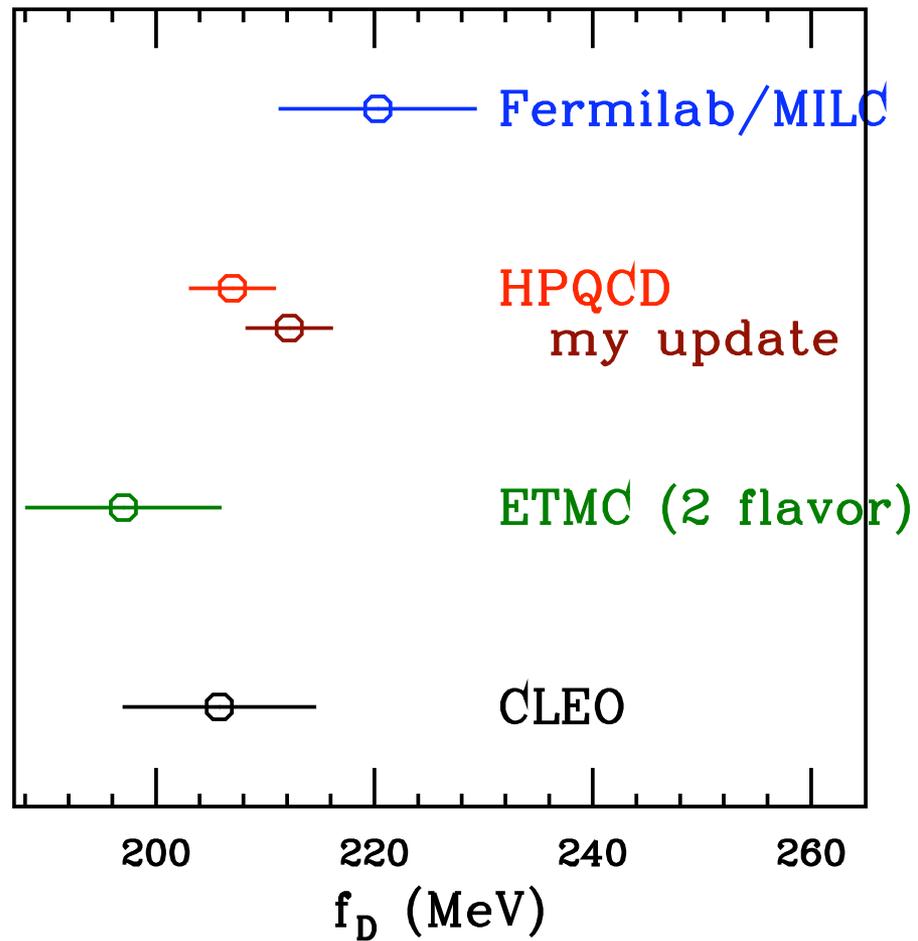
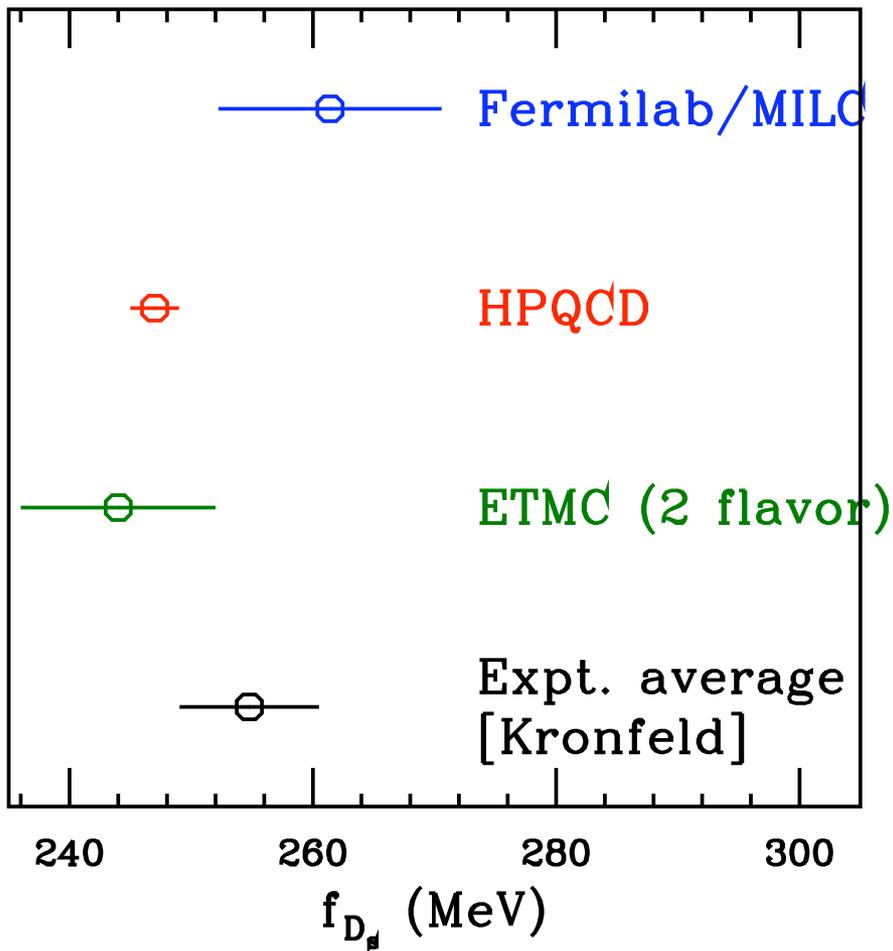
from [Kronfeld, arXiv:0912.0543](https://arxiv.org/abs/0912.0543), & his updates.

Comparison: D system

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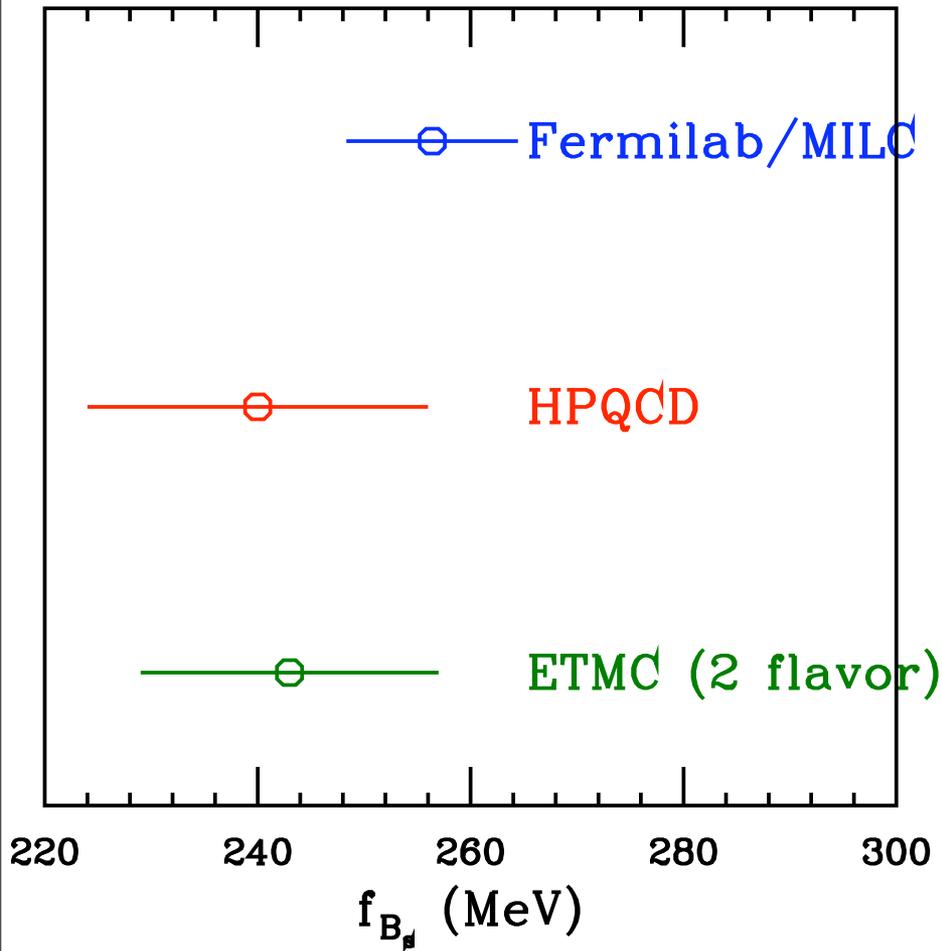


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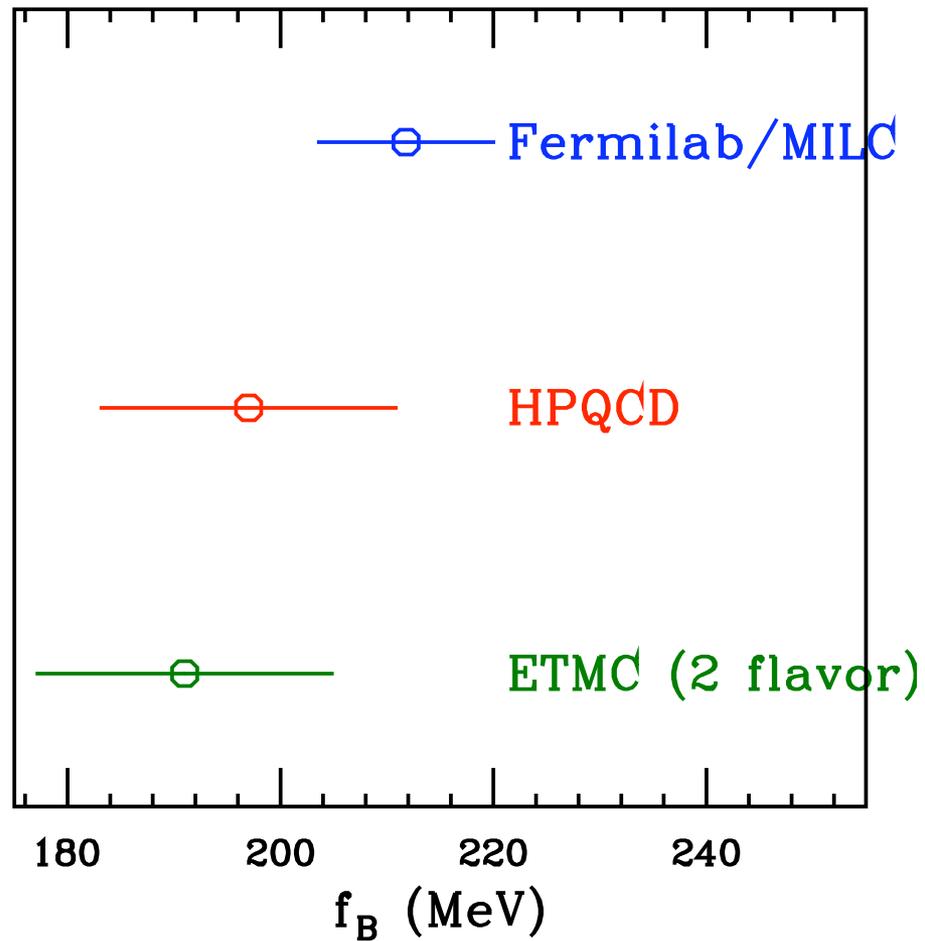
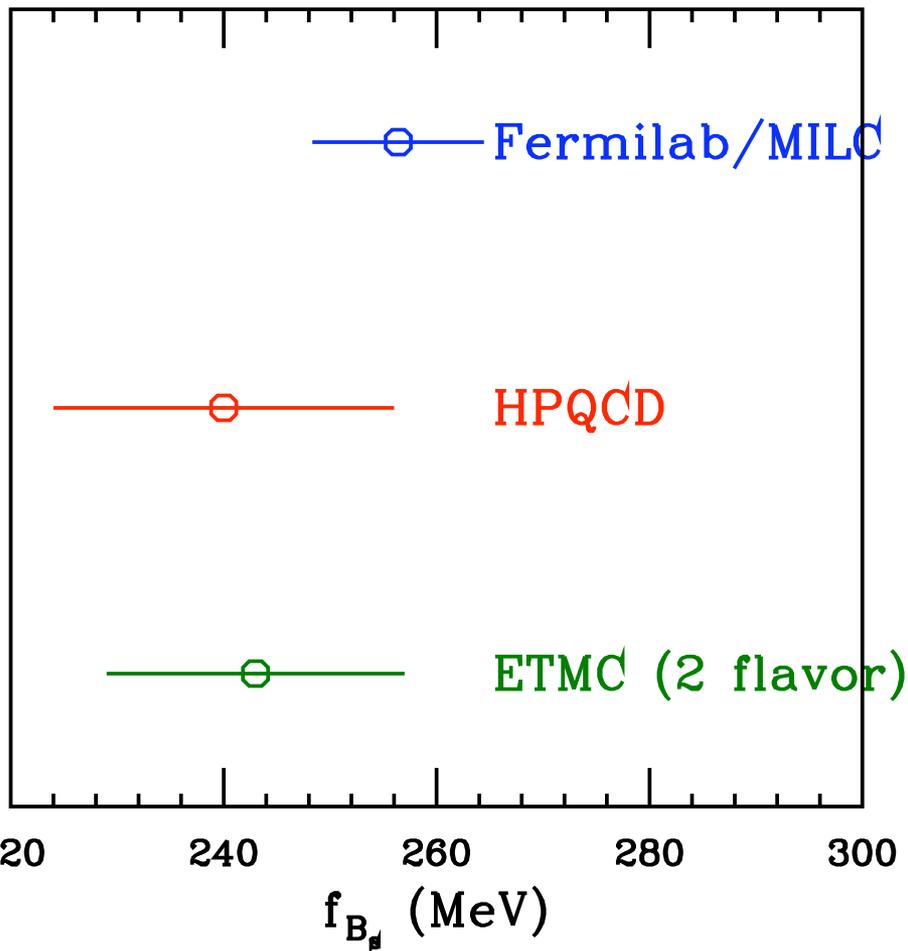


Comparison: B system

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Outlook: Fermilab/MILC

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% Errors

Quantity	Now	~1 year	~3-5 yrs.
f_{D_s}	3.5	1.8	0.6
f_D	4.3	2.2	0.7
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HISQ
valence
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HISQ valence & sea

Fermilab valence b; HISQ sea & light valence

- May do better for B with HISQ valence & extrapolation (HPQCD)
- Or doing f_{B_s}/f_{D_s} with Fermilab heavy quark + HISQ f_{D_s}

Fermilab Lattice/MILC Collaboration

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