$K \rightarrow \pi\pi$ decays on the lattice

Lattice QCD Meets Experiment Workshop

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RBC and UKQCD Collaborations

Outline

- Introduction
- Operator renormalization
- $\pi - \pi$ final states methods
  - 2008 RBC/UKQCD results using ChPT
  - Lellouch-Luscher
- $\Delta I = 3/2$
- $\Delta I = 1/2$
- Outlook

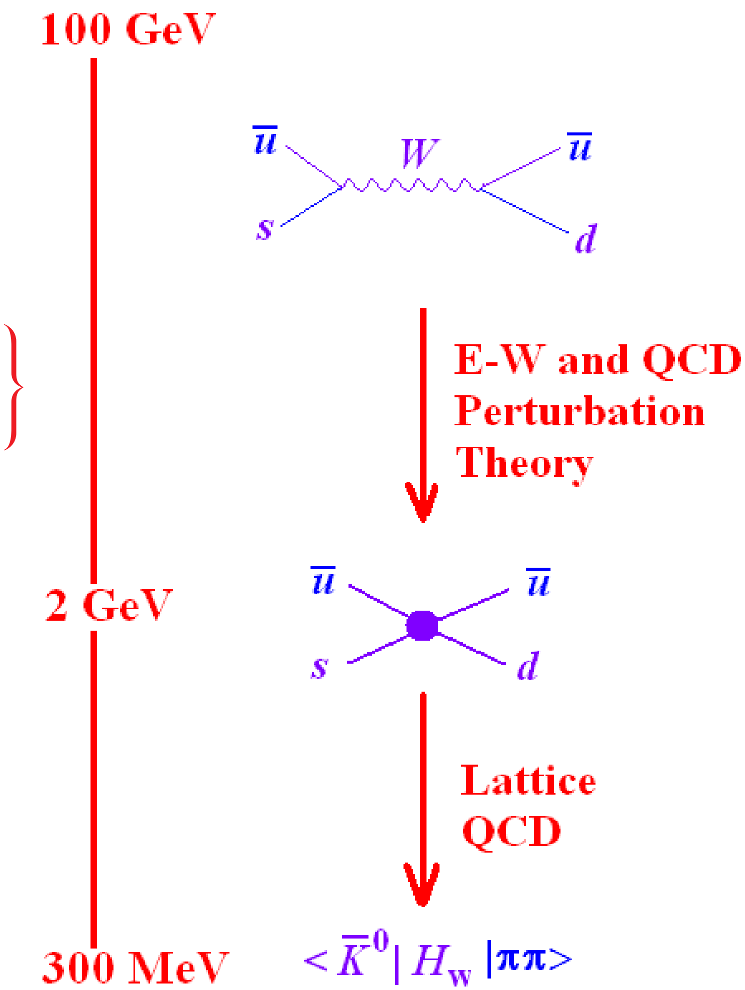
Introduction

Low Energy Effective Theory

- Represent weak interactions by local four-quark Lagrangian

$$\mathcal{H}^{(\Delta S=1)} = \frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \left\{ \sum_{i=1}^{10} \left[z_i(\mu) - \frac{V_{td}}{V_{us}^*} \frac{V_{ts}^*}{V_{ud}} y_i(\mu) \right] Q_i \right\}$$

- $V_{qq'}$ – CKM matrix elements
- z_i and y_i – Wilson Coefficients
- Q_i – four-quark operators



Four quark operators

- **Current-current operators**

$$Q_1 \equiv (\bar{s}_\alpha d_\alpha)_{V-A}(\bar{u}_\beta u_\beta)_{V-A}$$

$$Q_2 \equiv (\bar{s}_\alpha d_\beta)_{V-A}(\bar{u}_\beta u_\alpha)_{V-A}$$

- **QCD Penguins**

$$Q_3 \equiv (\bar{s}_\alpha d_\alpha)_{V-A} \sum_{q=u,d,s} (\bar{q}_\beta q_\beta)_{V-A}$$

$$Q_4 \equiv (\bar{s}_\alpha d_\beta)_{V-A} \sum_{q=u,d,s} (\bar{q}_\beta q_\alpha)_{V-A}$$

$$Q_5 \equiv (\bar{s}_\alpha d_\alpha)_{V-A} \sum_{q=u,d,s} (\bar{q}_\beta q_\beta)_{V+A}$$

$$Q_6 \equiv (\bar{s}_\alpha d_\beta)_{V-A} \sum_{q=u,d,s} (\bar{q}_\beta q_\alpha)_{V+A}$$

- **Electro-Weak Penguins**

$$Q_7 \equiv \frac{3}{2}(\bar{s}_\alpha d_\alpha)_{V-A} \sum_{q=u,d,s} e_q(\bar{q}_\beta q_\beta)_{V+A}$$

$$Q_8 \equiv \frac{3}{2}(\bar{s}_\alpha d_\beta)_{V-A} \sum_{q=u,d,s} e_q(\bar{q}_\beta q_\alpha)_{V+A}$$

$$Q_9 \equiv \frac{3}{2}(\bar{s}_\alpha d_\alpha)_{V-A} \sum_{q=u,d,s} e_q(\bar{q}_\beta q_\beta)_{V-A}$$

$$Q_{10} \equiv \frac{3}{2}(\bar{s}_\alpha d_\beta)_{V-A} \sum_{q=u,d,s} e_q(\bar{q}_\beta q_\alpha)_{V-A}$$

Status

- The $\Delta I = ½$ rule and ε'/ε are long-standing problems in particle physics.
- Accurate experimental result allows test of standard model CP violation.

  $$\mathrm{re}(\varepsilon'/\varepsilon) = 16.8\ (1.4) \times 10^{-4}$$

- Natural target for lattice QCD.
- Even 10-20% errors would be of great value.

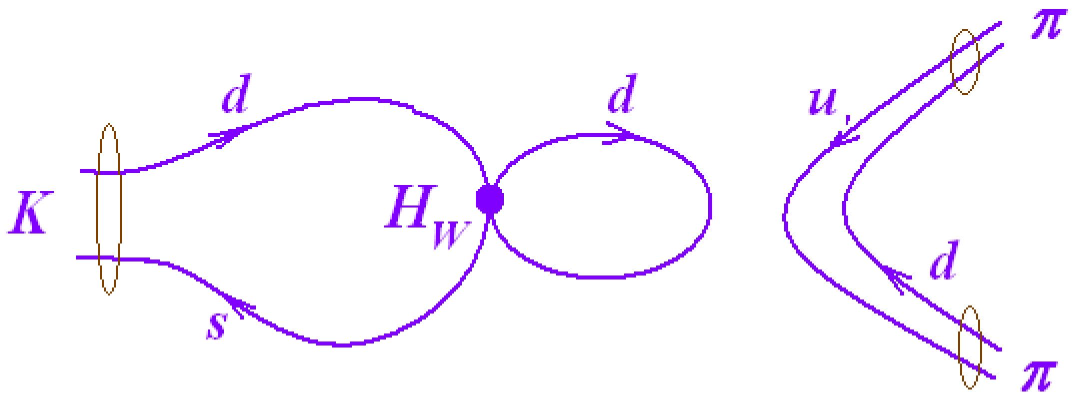
Challenges for lattice methods

- Match lattice and continuum operators
- Eye diagrams contain quadratic divergences

} Ultraviolet

- Difficult $\pi - \pi$ final state
- $\Delta I = 1/2$ amplitudes require disconnected graphs

} Infrared



$K \rightarrow \pi\pi$: an important RBC/UKQCD goal

RBC

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- T. Ishikawa
- T. Izubuchi
- XD Jin
- C. Jung
- M. Lightman
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UKQCD

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- D. Brommel
- J. Flynn
- P. Fritzsch
- N. Garron
- E. Goode
- C. Kelly
- C. Maynard
- C. Sachrajda
- J. Zanotti

Operator Renormalization (NPR)

- Seven $\Delta S = 1$ operators divide into three groups which mix:
  - $O^{(27,1)}$
  - O_7 and O_8
  - O_2, O_3, O_5, O_6
- Accurately handled by RI/MOM (Chris Dawson, Shu Li, Nicolas Garron)
- Mixing with lower dimension operators is a small effect and easily treated.
- Effects of a single gluonic operator not yet included.

Two pion final state

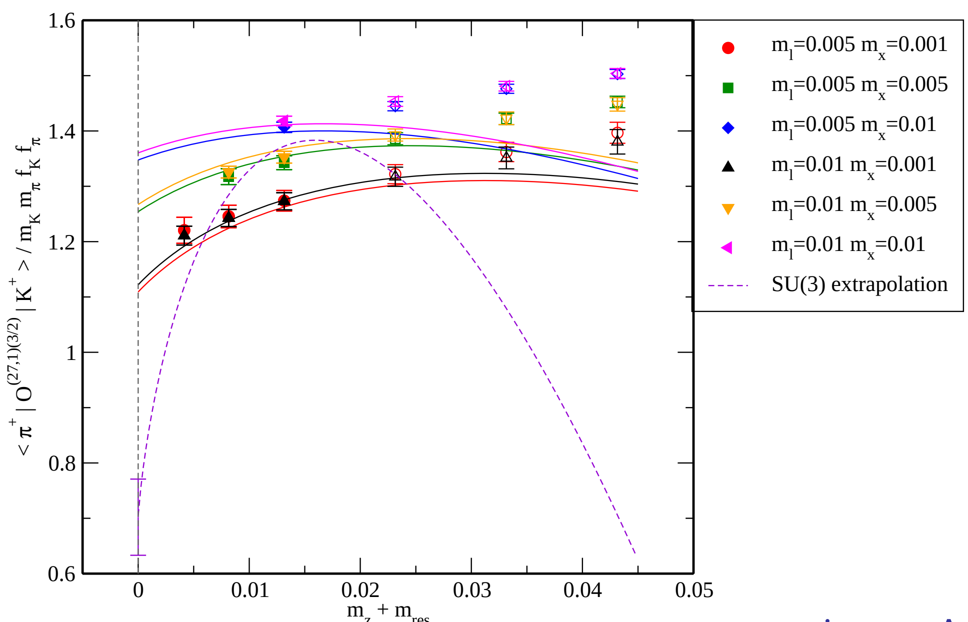
ChPT

SU(3) x SU(3) Chiral Perturbation Theory

- Use "soft-pion" methods to related $K \rightarrow \pi\pi$ to $K \rightarrow \pi$ and $K \rightarrow vac$.
- Earlier RBC 2001 quenched calculations suggested this was promising (but gave $\varepsilon'/\varepsilon = -4.0\pm2.3\ 10^{-4}$).
- However, quenched ChPT highly unphysical (**Golterman and Pallante**).
- Quenched result now replaced by 2+1 flavor, full QCD calculation with lighter quarks.

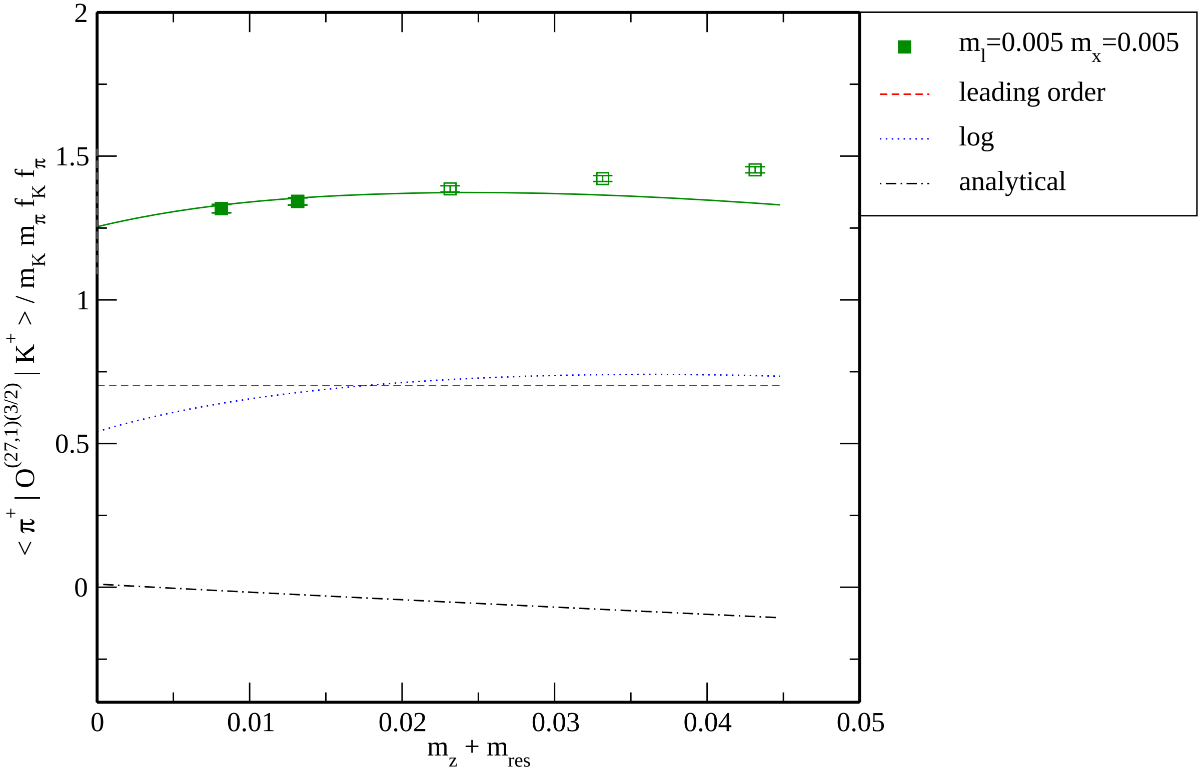
Determination of α_{27}

- Fit to points with $(m_{val} + m_{res})_{avg} \leq 0.013$
- PQChPT describes this data
- Large, ~100% correction!?
- Similar large ChPT corrections as RBC/UQKCD, arXiv:0804.0473
- **Fit does not work without $m_K m_\pi f_K f_\pi$ division.**



Relative size of LO and NLO terms

- LO and NLO log terms are the same size.
- Consistent results if we divide by $m_K m_\pi (f_K f_\pi)^2$
- Double the difference between two fits to estimate systematic error.

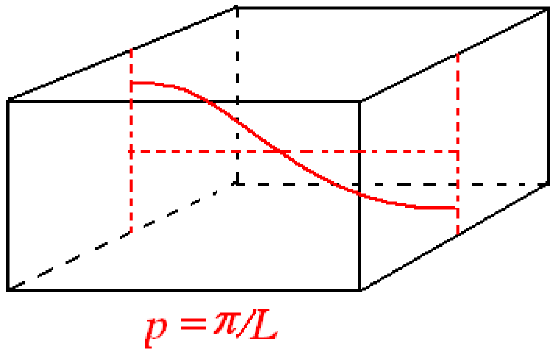


SU(3) x SU(3) ChPT
Critique

- Difficult to extrapolate to chiral limit and extract needed LEC's (240 MeV $\leq m_\pi \leq$ 430 MeV)
- Unrealistic to then use those LEC's to reconstruct physical 495 MeV kaon.
- $\alpha_1^{3/2} = 2.48\ (24)(39)\ 10^{-6}\ (\mathrm{GeV})^4$
- $\alpha_6^{1/2} = -4.1(7)(41)\ 10^{-4}\ (\mathrm{GeV})^4$
- **ChPT methods are too unreliable to be useful.**

Two pion final state
Lellouch-Luscher

Calculate π - π final state directly



- Lellouch-Luscher method:
  - Correct normalization for mixing of different l coming from cubic box.
  - Correctly include π - π interactions
  - No issue with Watson theorem and Euclidean space!
  - Overcome Maiani-Testa theorem by studying 1st or 2nd excited state with physical relative momentum.
- Further refinements:
  - Twisted or G-parity boundary conditions – force π - π to carry physical 205 MeV momentum. (Changhoan Kim)
  - Non-zero cm mass momentum adjusted to make π - π relative momentum physical. (Takeshi Yamazaki)

Δ I = 3/2

$\Delta I = 3/2$ $K \to \pi\pi$

- Usual SU(2) x SU(2) ChPT is not useful: two pions are *hard*
  - New method of Flynn and Sachrajda (arXiv:0809.1229) and Bijnens and Celis, (arXiv:0906.0302)
  - Perhaps ChPT is not needed!
- $I = 2$ final state has no vacuum overlap.
- $I = 2$ quantum number must be carried by four I=1/2 valence quarks.
  - Twist only valence quarks Sachrajda and Villadoro (hep-lat/0411033).
  - Safe to use slightly different valence and sea quark masses.

$\Delta I = 3/2$ $K \to \pi\pi$

(Matthew Lightman and Elaine Goode)

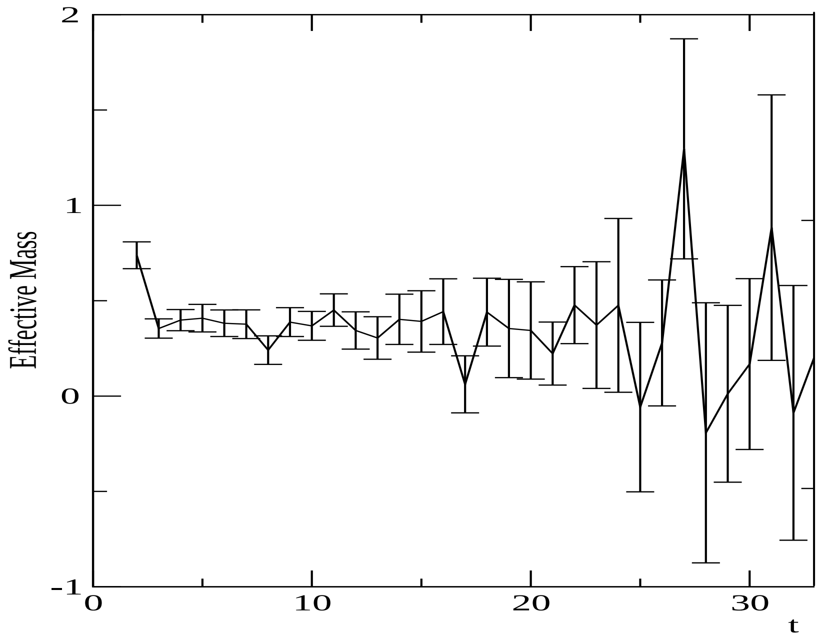
- Use new coarse 4.5 fm DSDR DWF ensembles.
  - m_π = 250 and 180 MeV
  - $1/a$ = 1.4 GeV
  - Finite a errors ≤ 8%.
- Use physical valence light quark mass.
  - Sea quark mass dependence of I=2, $K \to \pi\pi$ exected to be very small
  - m_{sea}= 0.008→0.004, < 3% (Lightman, arXiv:0906.1847 [hep-lat])
- Use anti-periodic boundary condition in two space directions (*30 configurations – highly preliminary!*)
  - $\boldsymbol{m_\pi}$ = 145.8(7) MeV
  - $\boldsymbol{m_K}$ = 518(2)
  - $\boldsymbol{E_{\pi\pi}}$ = 515(8) MeV
- A physical, on-shell, energy conserving K decay with 145 MeV pions and chiral fermions now possible!

$\Delta I = 3/2 \;\; K \rightarrow \pi\pi$

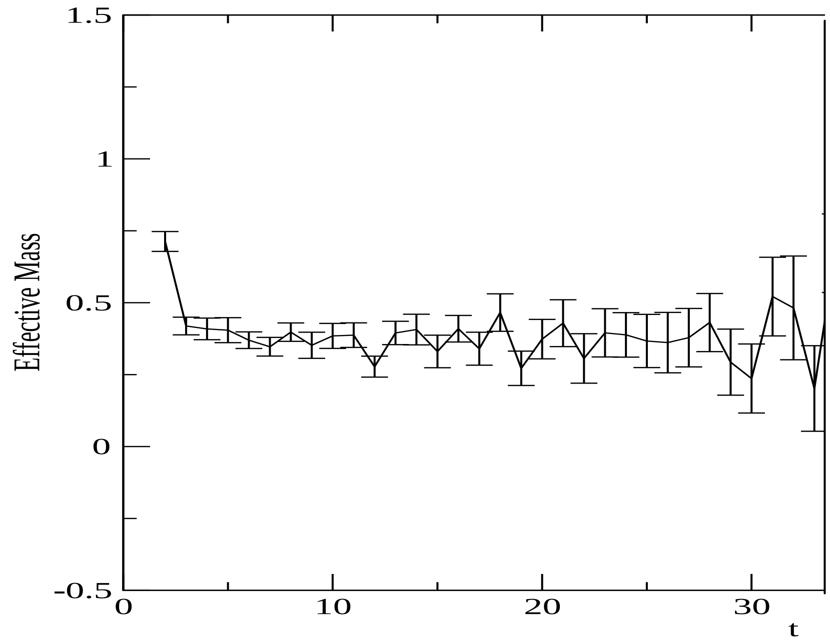
(Matthew Lightman and Elaine Goode)

$\pi\pi$ and K effective mass: $m_{\text{eff}}(t) = \ln\big(C(t) \,/\, C(t{+}1) \big)$

$\pi\pi$



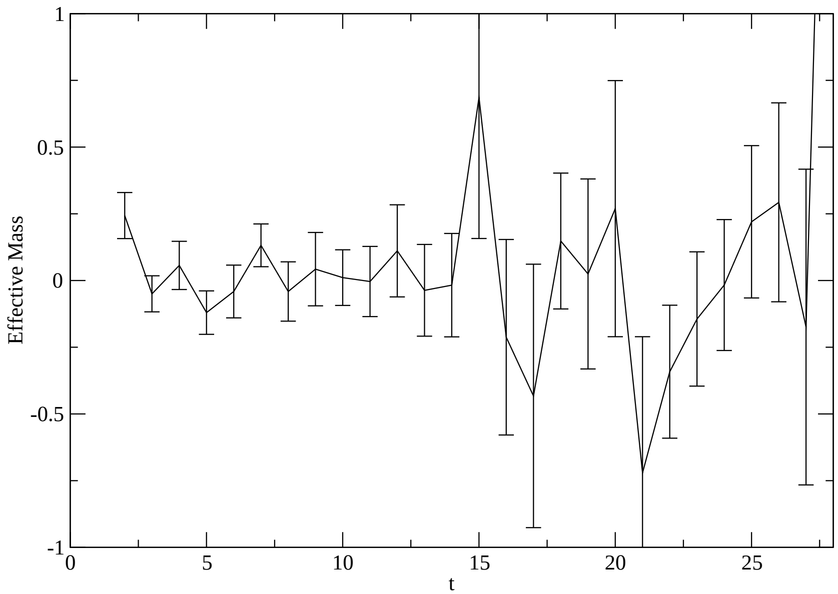
K



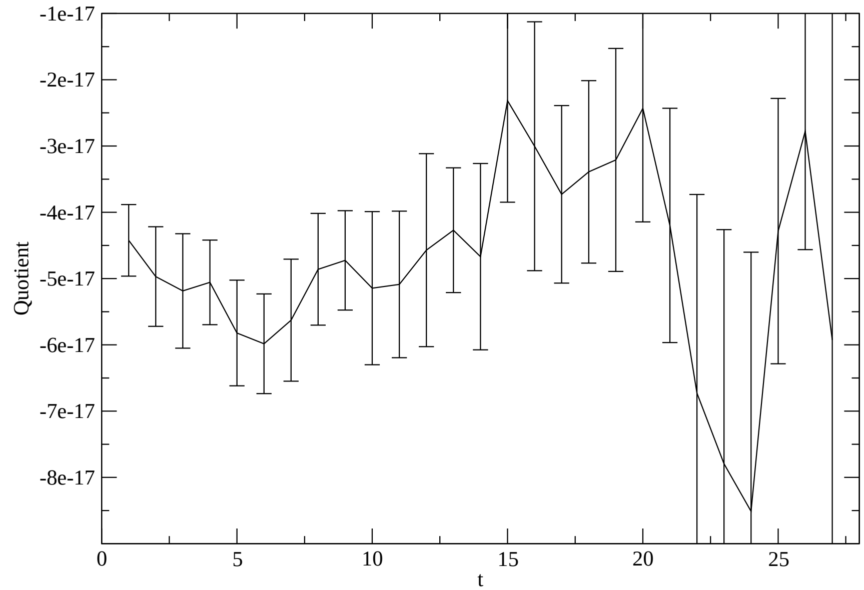
$< \pi\, \pi \,|\, O^{(27,1)} \,|\, K >$ from 29 configurations

(Matthew Lightman and Elaine Goode)

$O^{(27,1)}$ Effective mass



$O^{(27,1)}$ quotient



$O^{(27,1)}$	0.000926(59)
$O^{(8,8)}$	0.0187(11)
$O^{(8,8)m}$	0.0625(38)

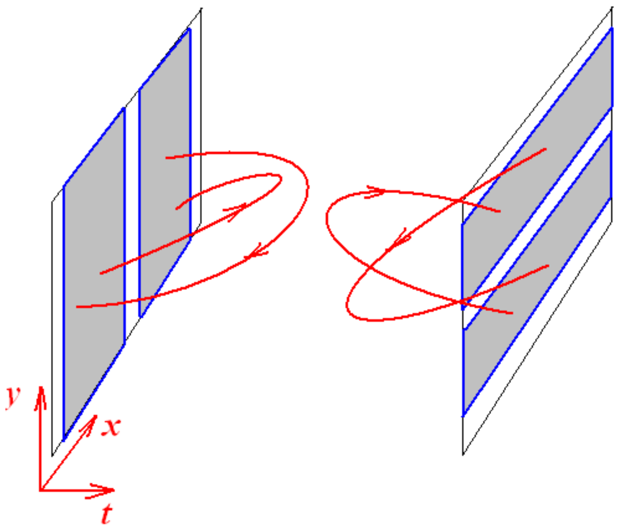
Δ I = 1/2

$\Delta I = 1/2 \ \ K \rightarrow \pi\pi$
(Qi Liu)

- $I = 0$ final state overlaps with vacuum.
- Disconnected diagrams require statistical cancellation to realize $e^{-2m_\pi t}$ decrease.
- Begin $16^3 \times 32$, $1/a = 1.73$ GeV, $m_\pi = 420$ MeV high-statistics experiments
  - Calculate 32 propagators for each time slice
  - $I = 0$, $\pi - \pi$ scattering
  - $\eta - \eta'$ masses and mixing
  - $K \rightarrow \pi\pi$

$I = 0$, $p = 0$, $\pi - \pi$ scattering
(Qi Liu)

- 120 configurations (wall source)
  - $E_{\pi\pi} = 0.451(33)$
- 30 configurations (split source)
  - $E_{\pi\pi} = 0.455(15)$
  - $2\ m_{\pi} = 0.4866(24)$
  - Attraction too strong to use Luscher's formulae



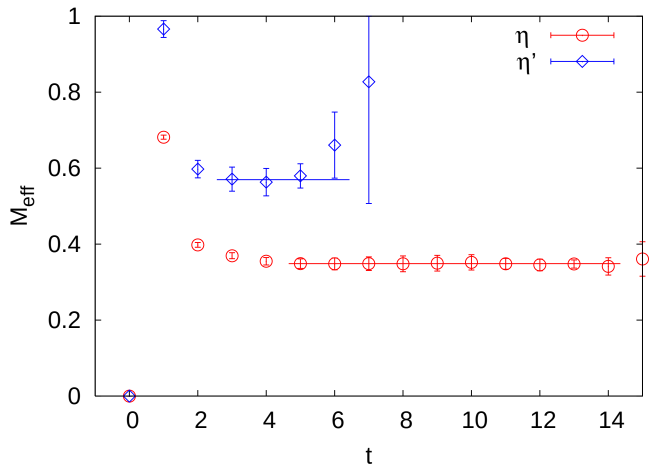
- 4x needed inversions
- 4x overall statistical gain
- Care needed to avoid unwanted momentum

$\eta - \eta'$ masses and mixing

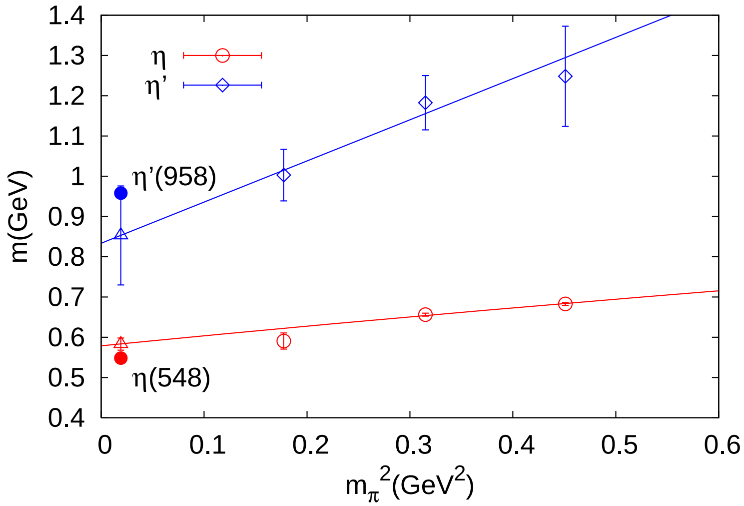
(Qi Liu)

- Use three $16^3 \times 32$ dyamical configurations.
- m_π = 421, 561 and 672 MeV
- $\bar{u}u+\bar{d}d$ and $\bar{s}s$ are ***NOT*** eigenstates!
- arXiv:1002.2999 [hep-lat]

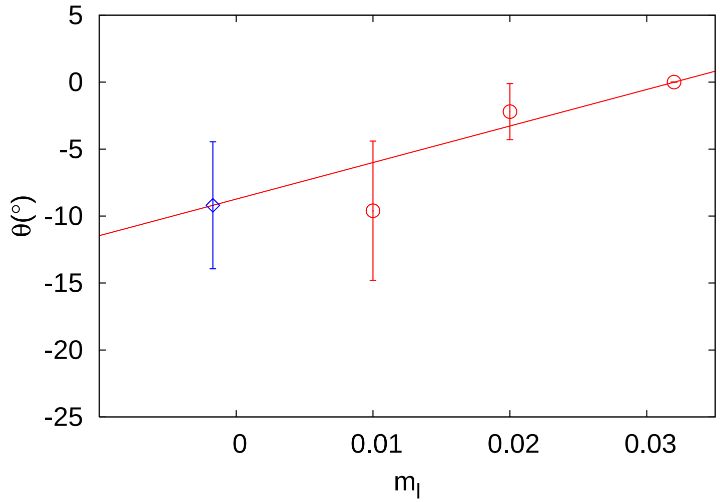
η - η' effective masses



m_η and $m_{\eta'}$

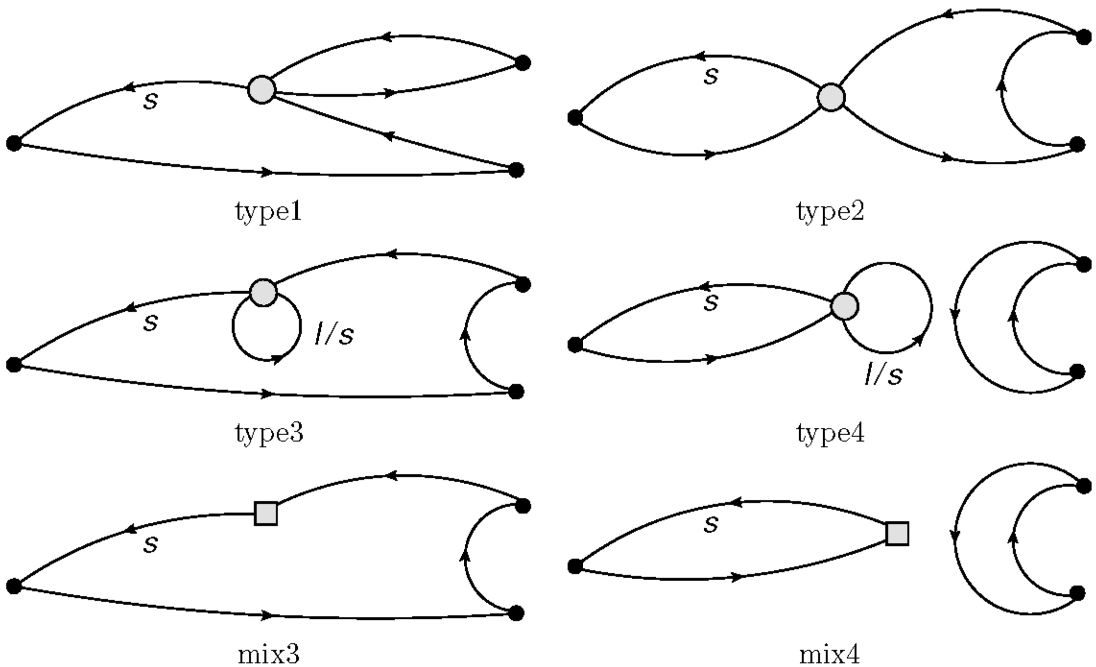


η - η' mixing



$\Delta I = 1/2$ $K \rightarrow \pi\pi$
(Qi Liu)

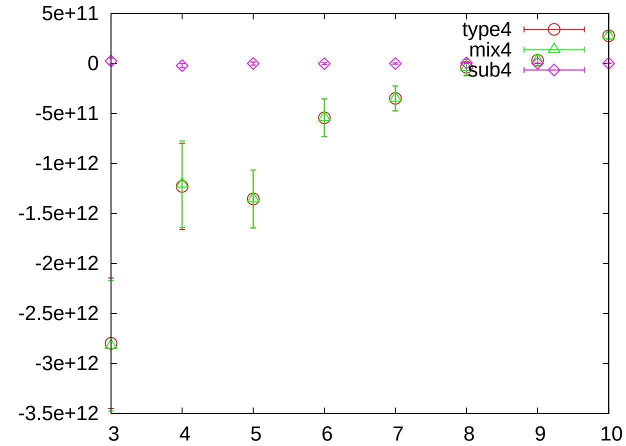
- Code 50 different contractions
- Use Ran Zhou's deflation code
- For each of 100 (→ 400) configurations invert with source at each of 32 times.



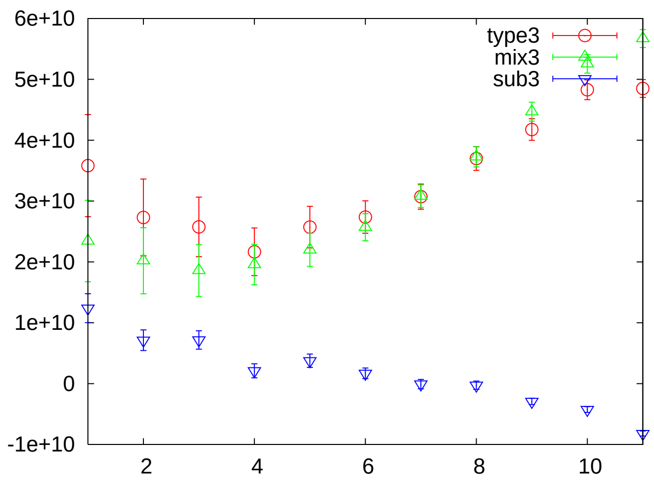
Divergent O_6 matrix elements
(Qi Liu)

- Strong "penguin" matrix elements: divergent $\bar{s}\gamma^5 d$ term
- Vanishes on-shell
- Explicit subtraction needed

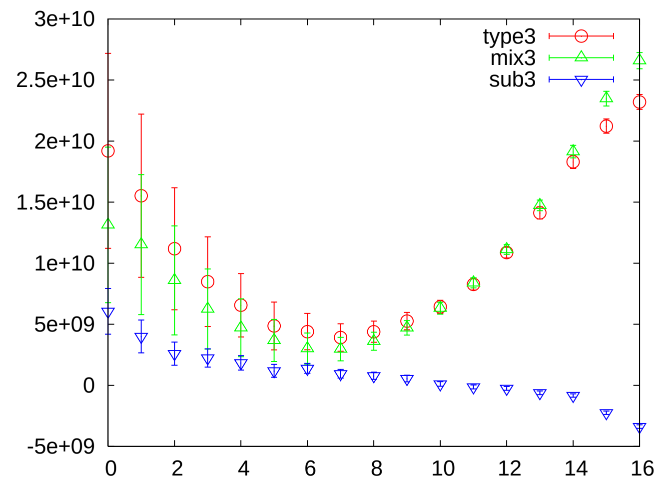
O_6 , sep. 12 (type 4)



O_6 , sep. 12 (type 3)



O_6 , sep. 16 (type 3)



Conclusion

- Calculation of re A_2 and im A_2 to ~10% a realistic 1 - 2 year goal
- re A_0 and im A_0 more difficult
  - Theoretical issues are resolved.
  - Disconnected diagrams easiest in this $\pi - \pi$ case.
  - Faster computer hardware needed for definitive results: Next generation IBM BG/Q machine should be sufficient!
- Expect 20% result for ΔI = ½ rule and ε'/ε in 2 - 3 years!