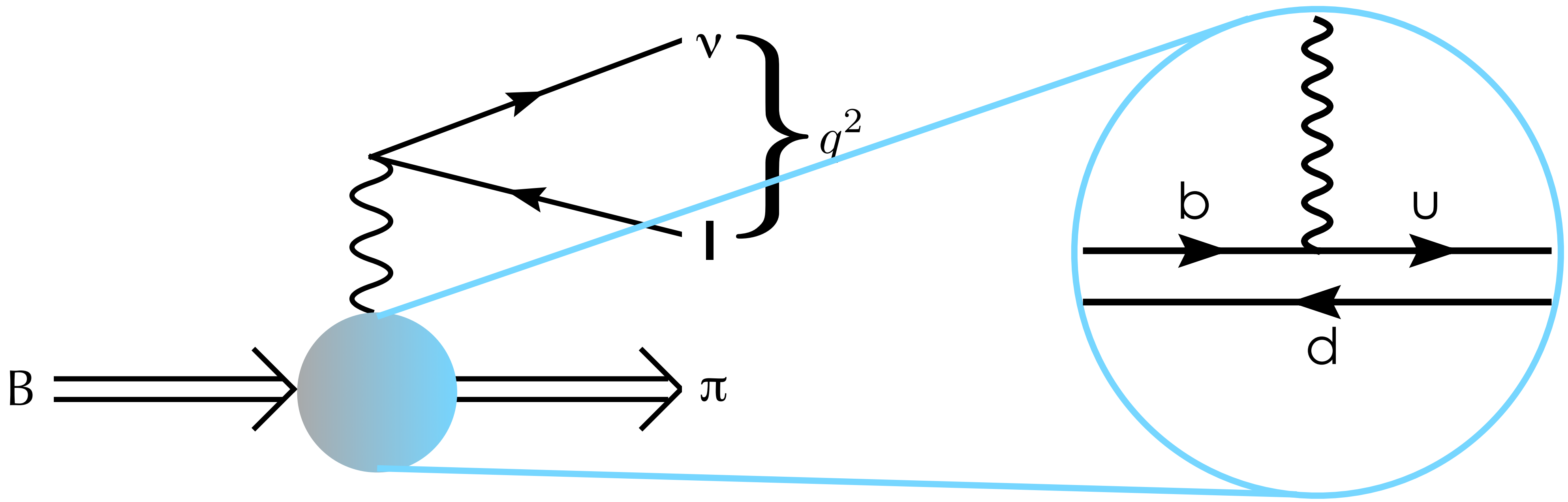
Heavy-to-light semileptonic form factors from lattice QCD

Ruth Van de Water
Fermilab

“Lattice QCD meets experiment”
December 10, 2007

Heavy-to-light semileptonic decays

- This talk will primarily focus on the tree-level decays $D\to\pi\ell\nu$, $D\to K\ell\nu$, and $B\to\pi\ell\nu$:



$$\text{B.R.}(D \to \pi\ell\nu) = |V_{cd}|^2 \int_0^{q^2_{max}} dq^2 f_+^{D\to\pi}(q^2)^2 \times (\text{known factor})$$

$f_+(q^2)$ are the D- and B-meson form factors:

$$\text{B.R.}(D \to K\ell\nu) = |V_{cs}|^2 \int_0^{q^2_{max}} dq^2 f_+^{D\to K}(q^2)^2 \times (\text{known factor})$$

$$\text{B.R.}(B \to \pi\ell\nu) = |V_{ub}|^2 \int_0^{q^2_{max}} dq^2 f_+^{B\to\pi}(q^2)^2 \times (\text{known factor})$$

- In each case experiments measure a **hadronic M.E. times a CKM element**

Why calculate D→π,K and B→π on the lattice?

(1) Can combine experimental measurements of D-meson branching fractions with values of $|V_{cd}|, |V_{cs}|$ from elsewhere to experimentally determine decay constants or form factors, then compare with lattice calculations

- This provides a **test of lattice QCD** methods, *e.g.*:
  - Dynamical (sea) quark effects
  - Light quark formalism
  - Heavy quark formalism
  - Chiral extrapolations

(2) Can combine experimental measurements of branching fractions with lattice calculations of form factors to extract $|V_{cd}|$, $|V_{cs}|$, and $|V_{ub}|$

- Correct lattice QCD results for D-mesons *give confidence in similar lattice calculations with B-mesons*

Status of D→π,K and B→π lattice calculations

CAVEAT: This talk will be restricted to three-flavor unquenched lattice calculations

- Currently two groups calculating heavy-light meson quantities with three dynamical quark flavors: Fermilab/MILC & HPQCD
- Both use the publicly available "2+1 flavor" **MILC configurations** [Phys.Rev.D70:114501,2004] which have three flavors of improved staggered quarks:
  - Two degenerate light quarks and one heavy quark $(\approx m_s)$
  - Light quark mass ranges from $m_s/10 \leq m_l \leq m_s$
- Groups use **different heavy quark discretizations**:
  - Fermilab/MILC uses Fermilab quarks
  - HPQCD uses nonrelativistic (NRQCD) heavy quarks

Systematics in lattice calculations

- Lattice calculations typically quote the following sources of error:

  1. Monte Carlo statistics & fitting
  2. Tuning lattice spacing, a, and quark masses
  3. **Matching lattice gauge theory to continuum QCD**
     - (Sometimes split up into relativistic errors, discretization errors, perturbation theory, ...)
  4. Extrapolation to continuum
  5. **Chiral extrapolation to physical up, down quark masses**

- Errors #3 and #5 are dominant sources of systematic uncertainty in current heavy-light form factor calculations -- will discuss them in turn

Heavy quarks on the lattice

PROBLEM: Generic lattice quark action will have discretization errors $\propto (am_Q)^n$

SOLUTION: Use knowledge of the heavy quark/nonrelativistic quark limits of QCD to systematically eliminate HQ discretization errors order-by-order

FERMILAB METHOD
[Phys.Rev.D55:3933-3957,1997]

Continuum QCD
↓ (using HQET)
Lattice gauge theory

LATTICE NRQCD
[Phys.Rev.D46:4052-4067,1992]

Continuum QCD
↓
Nonrelativistic QCD
↓
Lattice gauge theory

- Both methods require tuning parameters of lattice action
- For heavy-light decays, must also match lattice *currents* to continuum
- Typically calculate matching coefficients in lattice perturbation theory [Phys.Rev.D48:2250-2264,1993]

Matching errors

- In principle, can remove errors of any order in heavy quark mass, but, in practice, becomes increasingly difficult at each higher order
- $\Longrightarrow$ Must estimate size of errors due to inexact matching

FERMILAB METHOD

QCD

↓ "heavy quark discretization effects"

LGT

- Combine all errors associated with discretizing action
- Estimate errors using knowledge of short-distance coefficients and power-counting

LATTICE NRQCD

QCD

↓ "relativistic errors", e.g. $O(\alpha_S \Lambda_{QCD}/m_Q)$ & $O(\Lambda_{QCD}^2/m_Q^2)$

NRQCD

↓ " perturbation theory errors", e.g. $O(\alpha_S^2)$

LGT

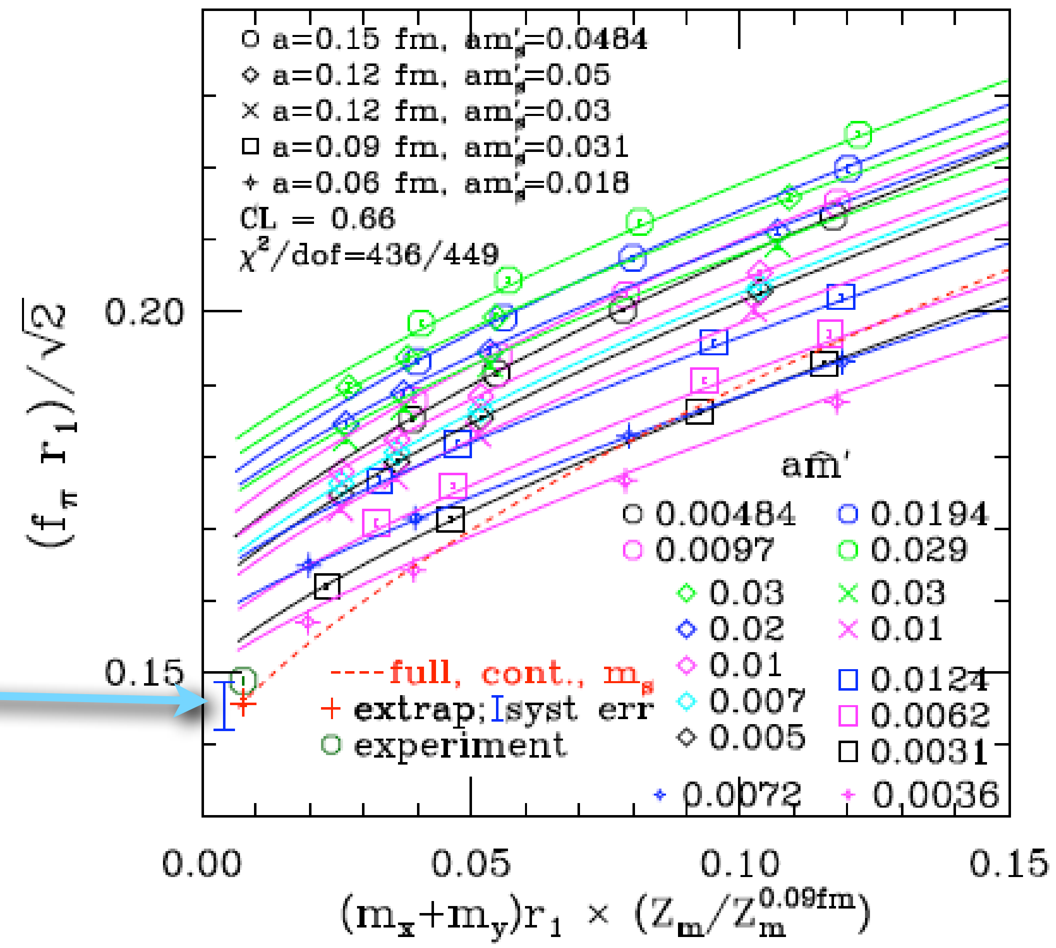
- Estimate errors using power-counting

Light quarks on the lattice

PROBLEM: Simulating light quarks at the physical up,down, and strange quark masses is prohibitively computationally expensive

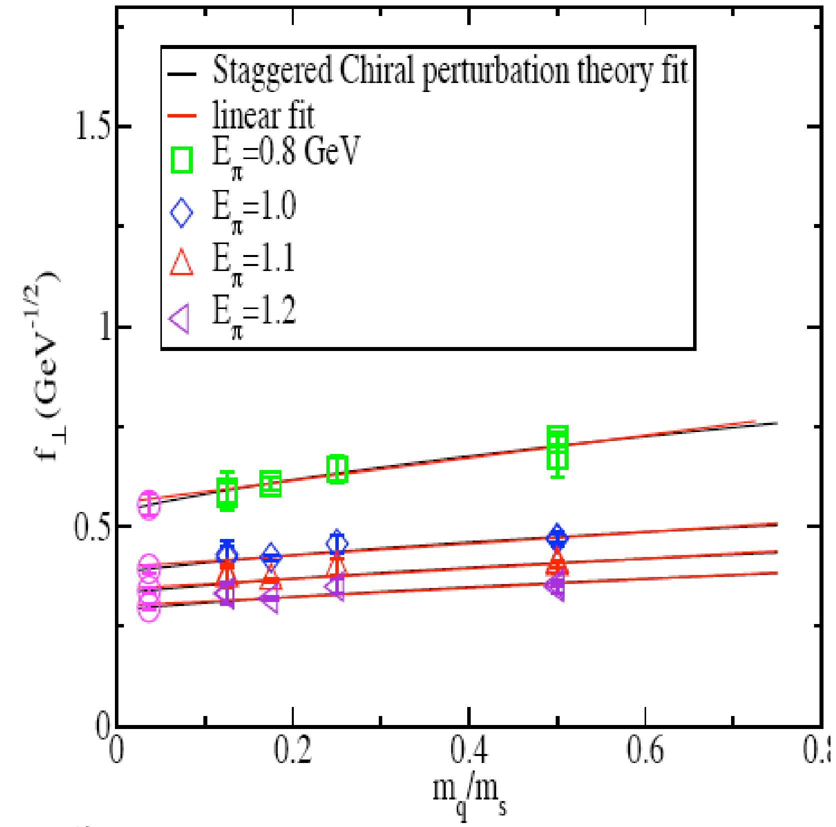
SOLUTION: Use expressions derived in chiral perturbation theory to extrapolate to the physical quark masses in a controlled way

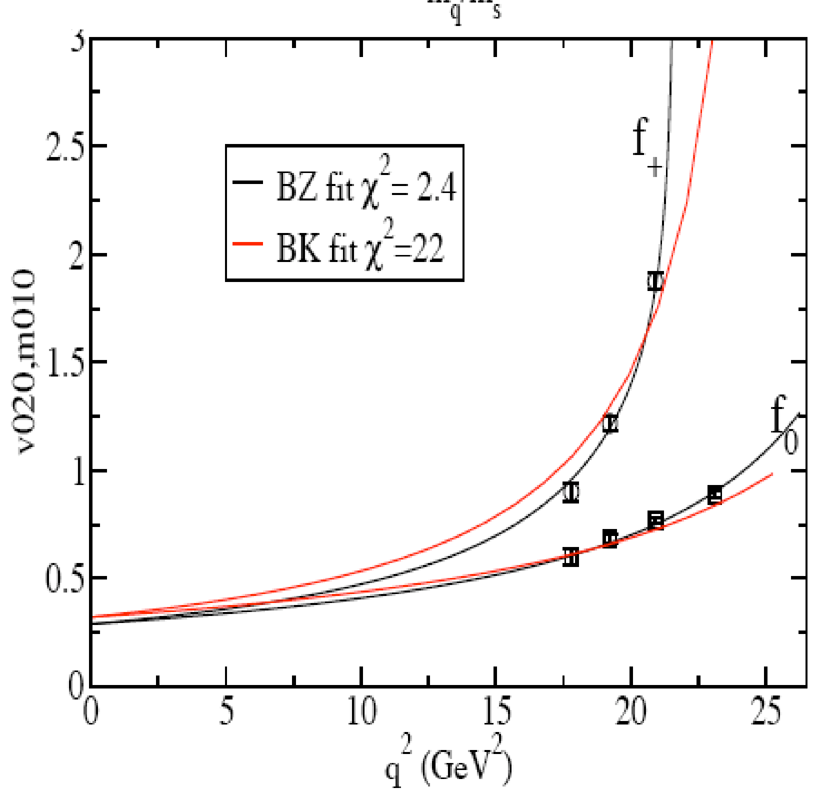
- ✦ For MILC 2+1 flavor lattices, must use **staggered chiral perturbation theory** [Lee & Sharpe, Aubin & Bernard, Sharpe & RV]
  - ❖ Accounts for next-to-leading order light quark mass dependence
  - ❖ Also accounts for light quark discretization effects through $O(\alpha_S^2 a^2 \Lambda_{QCD}^2)$
  - ❖ Extremely successful for light-light meson quantities such as f_π (MILC Lat'07 arXiv:0710.1118 [hep-lat])



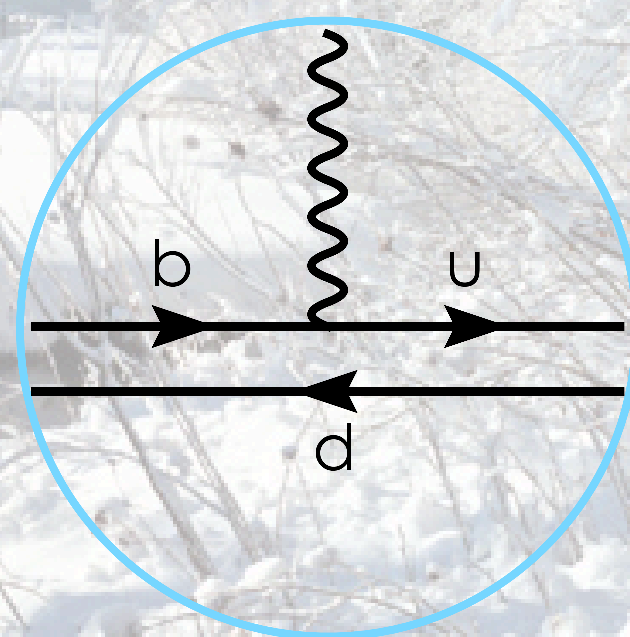
Chiral extrapolation and q^2 interpolation errors

- In the case of heavy-light form factors it is convenient to extrapolate in the light quark mass at fixed values of the pion recoil energy
- Present calculations first interpolate in q^2 to fiducial values of E_π, then extrapolate in m_q using an ansatz for the form factor shape -- introduces systematic error due to choice of model
- Note: Fermilab uses the Becirevic-Kaidalov (BK) parameterization and HPQCD uses the Ball-Zwicky (BZ) parameterization for their central values
- This can be avoided by performing a simultaneous fit in m_q and E_π using chiral perturbation theory -- *in progress by RV*
- Estimate remaining chiral extrapolation error by varying parameters and higher-order terms





Lattice results for D→π,K and B→π

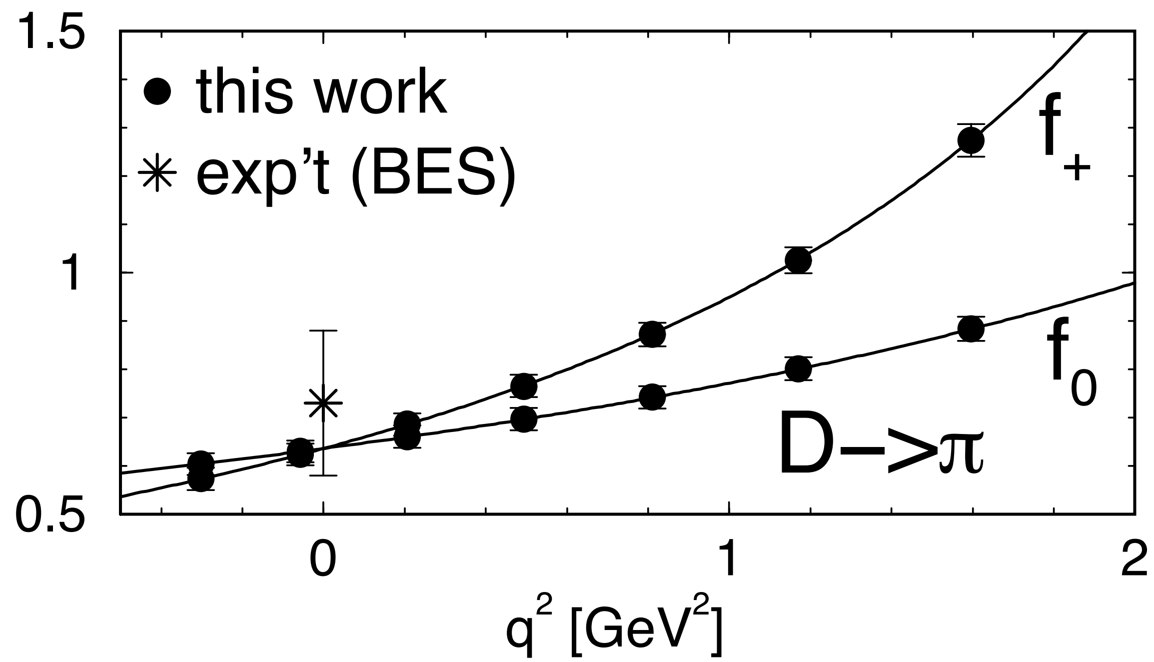


Fermilab/MILC calculation of $D \to \pi \ell \nu$

stat. sys.

$$\boxed{\begin{aligned} f_+^{D\to\pi}(0) &= 0.64(3)(6) \\ |V_{cd}| &= 0.239(10)(24)(20) \end{aligned}}$$

stat. sys. exp.



(Statistical errors only)

[Phys.Rev.Lett.94:011601,2005]

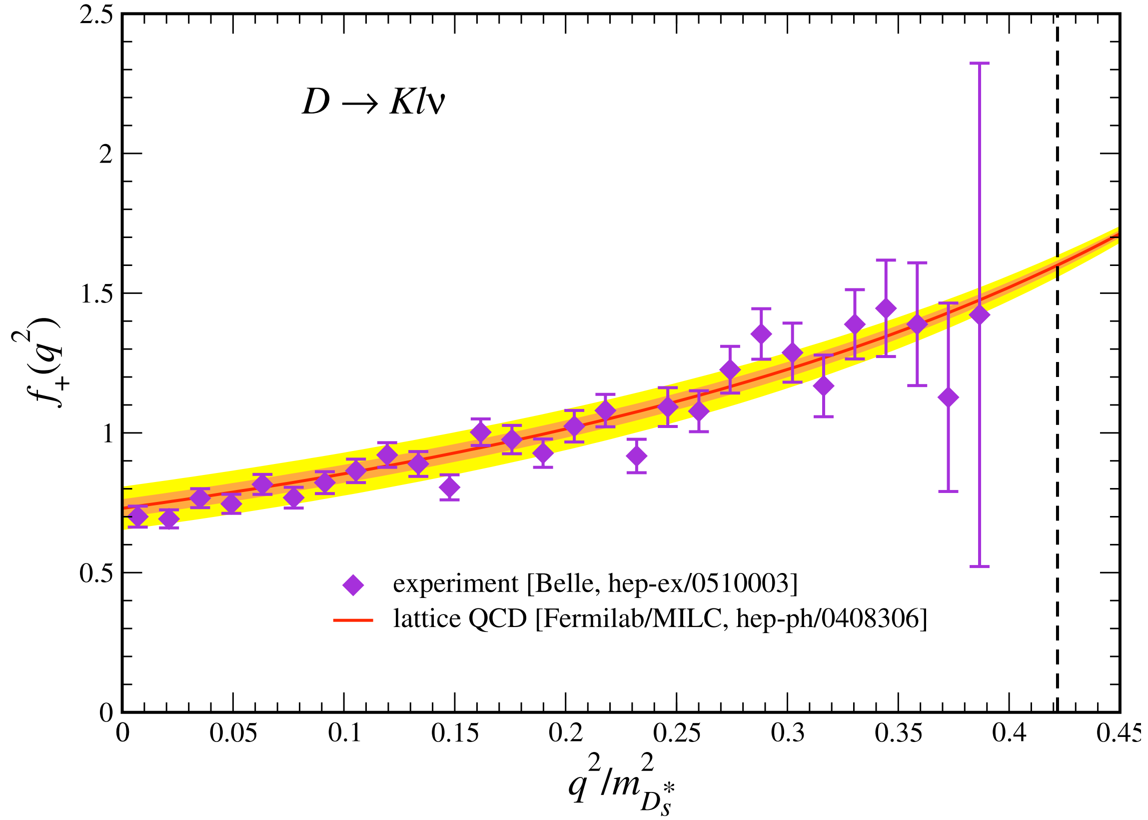
- Given $|V_{cd}|$, result for f(0) consistent with experiment
- Conversely, 14% measurement of $|V_{cd}|$
- Note: 8% experimental error based on 2004 PDG → 3% using 2007 PDG
- Dominant systematic uncertainty **discretization effects (9%)**
  - ⟹ Reducible by adding a finer lattice spacing -- *in progress by Jon Bailey and RV*

Fermilab/MILC calculation of $D \to K\ell\nu$

$$f_+^{D\to K}(0) = 0.73\underbrace{(3)}_{\text{stat.}}\underbrace{(7)}_{\text{sys.}}$$

$$|V_{cs}| = 0.969\underbrace{(39)}_{\text{stat.}}\underbrace{(94)}_{\text{sys.}}\underbrace{(24)}_{\text{exp.}}$$

[Phys.Rev.Lett.94:011601,2005]



- ✦ Form factor shape and normalization **consistent with experiment**
- ✦ Calculations preceded Focus, Belle, BaBar measurements ⇒ lattice *prediction*
- ✦ Error mostly discretization effects (9%)
- ✦ Correct determinations of the $D\to\pi\ell\nu$ and $D\to K\ell\nu$ form factors give **confidence in lattice calculations** of $B\to\pi\ell\nu$ and the resulting exclusive determination of $|V_{ub}|$ . . .

Fermilab/MILC calculation of B→πℓν

- Primary differences in Fermilab/MILC error budgets for D- and B- decays:
  - Discretization error decreases
  - Systematic from q^2 interpolation using BK ansatz increases

decay	$D \to \pi(K) l \nu$	$B \to \pi l \nu$
CKM matrix element	$\lvert V_{cd(s)} \rvert$	$\lvert V_{ub} \rvert$
discretization effect	9%	~~9%~~ → 7%
fitting 3- and 2-point functions	3%	3%
chiral extrapolation	3%(2%)	4%
q^2 dependence (BK parameterization)	2%	4%
current renormalization	0%	1%
a uncertainty	1%	1%
total systematic	10%	~~11%~~ → 10%

- Result from M. Okamoto's Lattice 2005 proceedings (arXiv:hep-lat/0510113) with an improved estimate of discretization errors
- Uses HFAG branching fractions from EPS '05
- Dominant uncertainties are statistics (8%) and discretization effects (7%)

$$\boxed{|V_{ub}| \times 10^3 = 3.78\underbrace{(30)}_{\text{stat.}}\underbrace{(34)}_{\text{sys.}}\underbrace{(25)}_{\text{exp.}}}$$

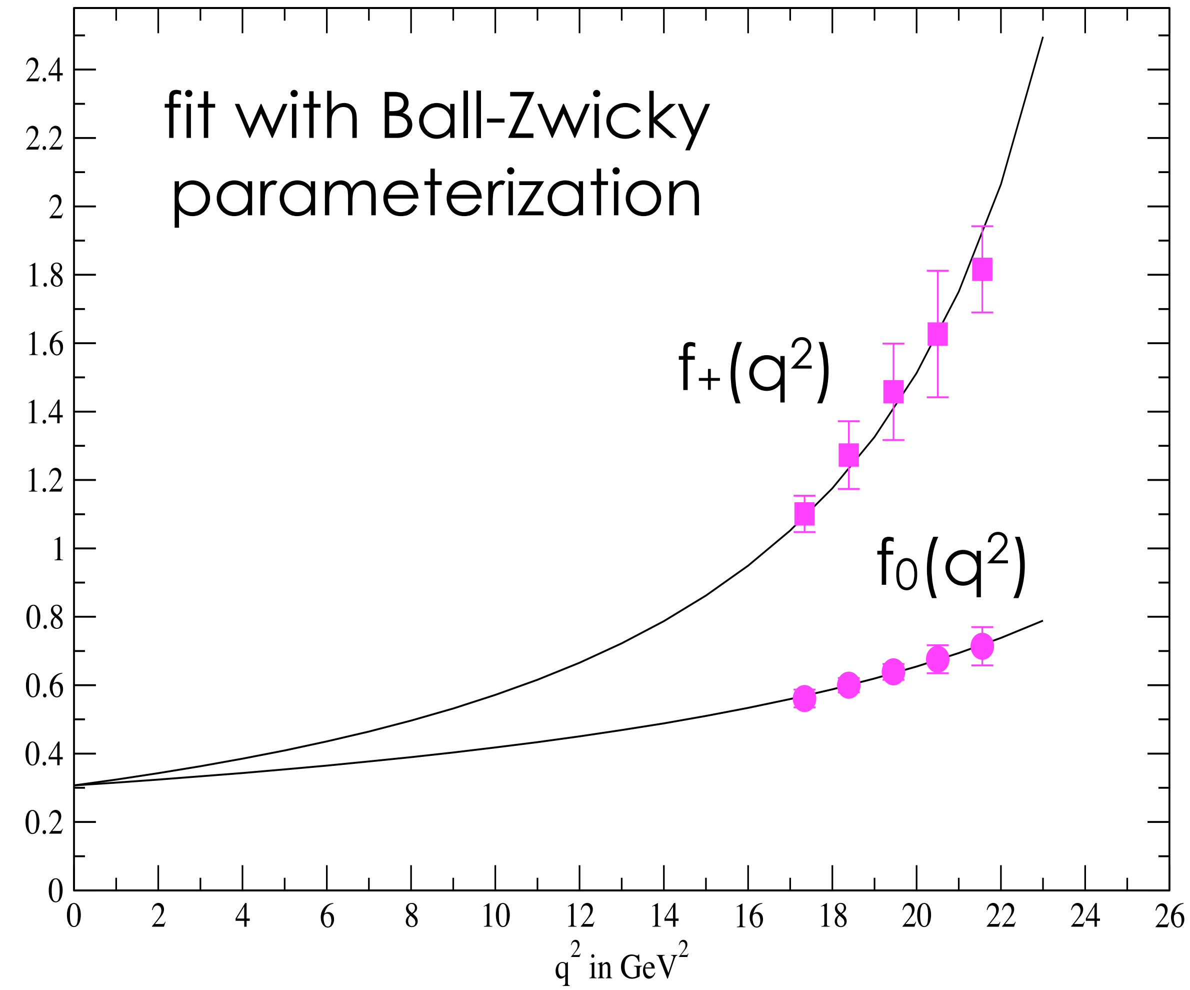
HPQCD calculation of $B \to \pi \ell \nu$

$$|V_{ub}| \times 10^3 = 3.55(25)(50)$$

exp. theory

[Phys.Rev.D73:074502,2006, Erratum-ibid.D75:119906,2007]

source of error	size of error (%)
statistics + chiral extrapolations	10
two-loop matching	9
discretization	3
relativistic	1
Total	14



- Dominant uncertainties are statistics and two-loop matching (9%)
- Consistent with Fermilab/MILC result with comparable errors

Comment on "experimental error" in $|V_{ub}|$

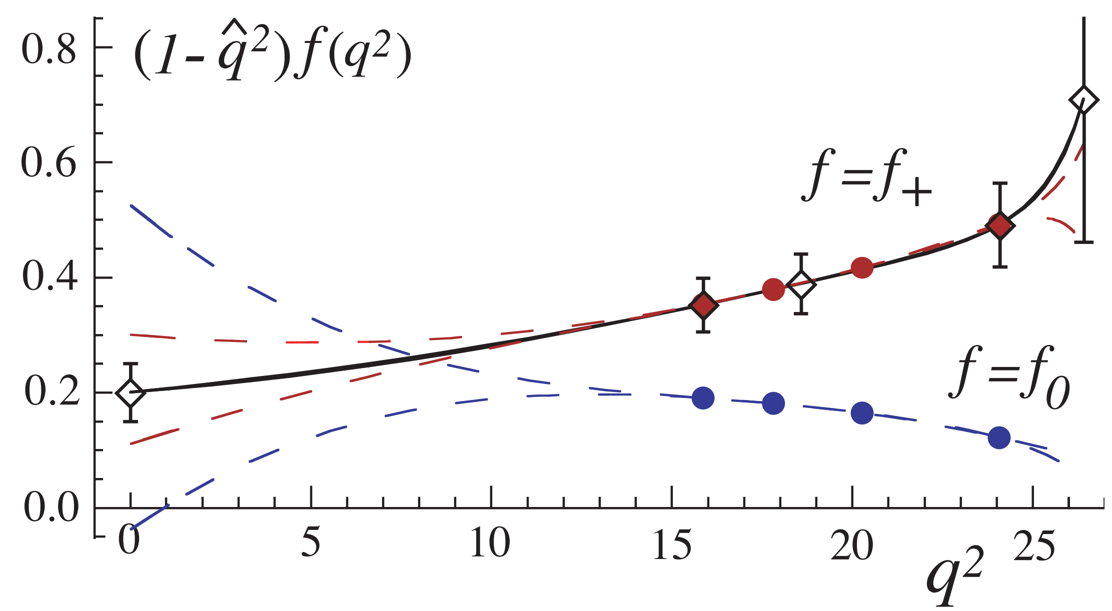
- 7% "experimental error" really from combining lattice form factor results with experimental branching fractions to determine $|V_{ub}|$

- Large because of the poor overlap in q^2 between lattice and experiment $\Longrightarrow$ not just the burden of experimentalists

- Two promising methods for reducing experimental error:

  - **Moving NRQCD:** generate lattice data directly at low q^2 while keeping statistical errors under control [Foley & Lepage; Davies, Lepage, & Wong]

  - **"z-fit":** combine lattice and experimental data over full q^2 region using model-independent expression based on analyticity and unitarity [Arnesen *et al.*; Becher & Hill; P. Ball; Mackenzie & RV]



[Arnesen *et al.*, Phys.Rev.Lett.95:071802,2005]

Improvements-in-progress

Both:

- Improved B-meson source will reduce statistical errors
- Finer a=0.09 fm lattice spacing will reduce the dominant systematics for both HPQCD (two-loop matching) and Fermilab/MILC (discretization effect)

HPQCD:

- Random-wall source for pion will reduce statistical errors (K.Wong Lat'07, arXiv: 0710.0741 [hep-lat])
- May also allow direct simulation of lower q^2 points

Fermilab/MILC:

- Simpler correlation function fits will reduce statistical/fit errors (Mackenzie & RV Lat'07)
- Simultaneous chiral extrapolation in m_q and pion energy
- Model-independent method for combining lattice results and experimental data using z-expansion should minimize the resulting error in $|V_{ub}|$ [Mackenzie & RV Lat'06, PoS LAT2006, 097 (2006)]

Longer-term improvements in methods

For HPQCD:

- 2-loop perturbative (or nonperturbative) matching
- Highly-improved staggered quarks (HISQ) for charm quarks to calculate D-meson form factors (*already used for f_D* -- arXiv:0706.1726 [hep-lat])
- **Moving NRQCD** to generate lattice data at lower values of q^2 [Foley & Lepage; Davies, Lepage, & Wong]

For Fermilab/MILC:

- 2-loop matching of heavy-light current ρ-factor
- Nonperturbative determination of clover coefficient in heavy-quark action (*e.g.* see Lin & Christ)
- **Improved heavy-quark action** (*in progress* -- Kronfeld & Oktay)

"to-do" For Lattice Community:

- Unquenched (three-flavor) heavy-light calculations with different light quark action, e.g domain-wall (RBC/UKQCD) or overlap fermions (JLQCD)

Planned improvements in lattice parameters

- Based on 2007 USQCD Collaboration white paper "Fundamental parameters from future lattice calculations" (http://www.usqcd.org/documents/fundamental.pdf)
- Not guaranteed, but a good approximate timeline ...

year	a(fm)	$m_{u,d}/m_s$
2007	0.06	0.10
2008	0.045	0.20
2009	0.06	0.05
	0.045	0.10
2010	0.06	1/27
	0.045	0.05
2011	0.045	1/27

- Note: this table only applies to *generation* of the MILC Asqtad configurations
  - Will take longer to reach "ultra-fine" lattice spacings and physical quark masses for other light quark formalisms
  - Will take a few years to produce first results with these configurations

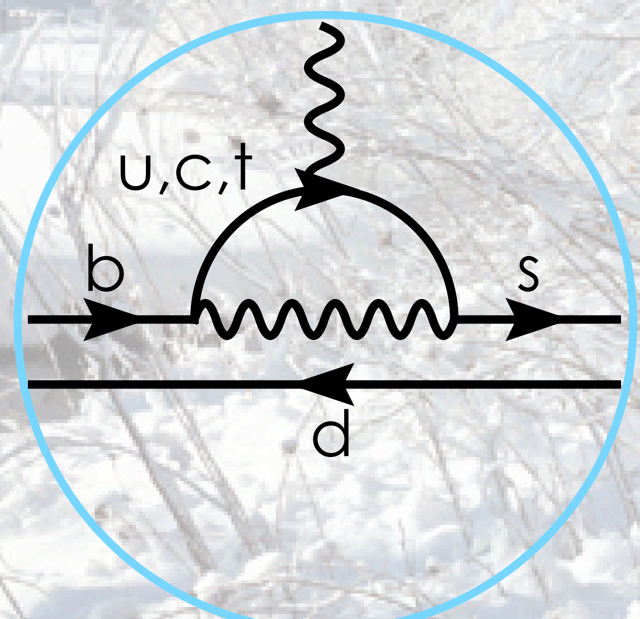
Predicted errors in CKM elements

- Estimates based on Fermilab/MILC systematic error budget and assume fixed experimental error: 2007 PDG for $D\to\pi\ell\nu$ and $D\to K\ell\nu$, EPS '05 HFAG for $B\to\pi\ell\nu$
- **1 year** estimate assumes:
  - 2x statistics, improved correlator fits, no q^2 interpolation, a=0.09 fm lattice data (reduces dominant discretization error from 7-9% → ~3%)
- **5+ years** estimate assumes everything mentioned above plus:
  - 10x statistics, a=0.045 fm lattice data, physical up/down quark masses
  - Also cuts $B\to\pi\ell\nu$ "experimental error" by factor of 2 -- *just a guess* for improvement from z-fit, moving NRQCD, or something else . . .

CKM Element	Now	1 year	5+ years
$\lvert V_{cd}\rvert$	11%	6%	4%
$\lvert V_{cs}\rvert$	11%	5%	2%
$\lvert V_{ub}\rvert$	14%	10%	4%

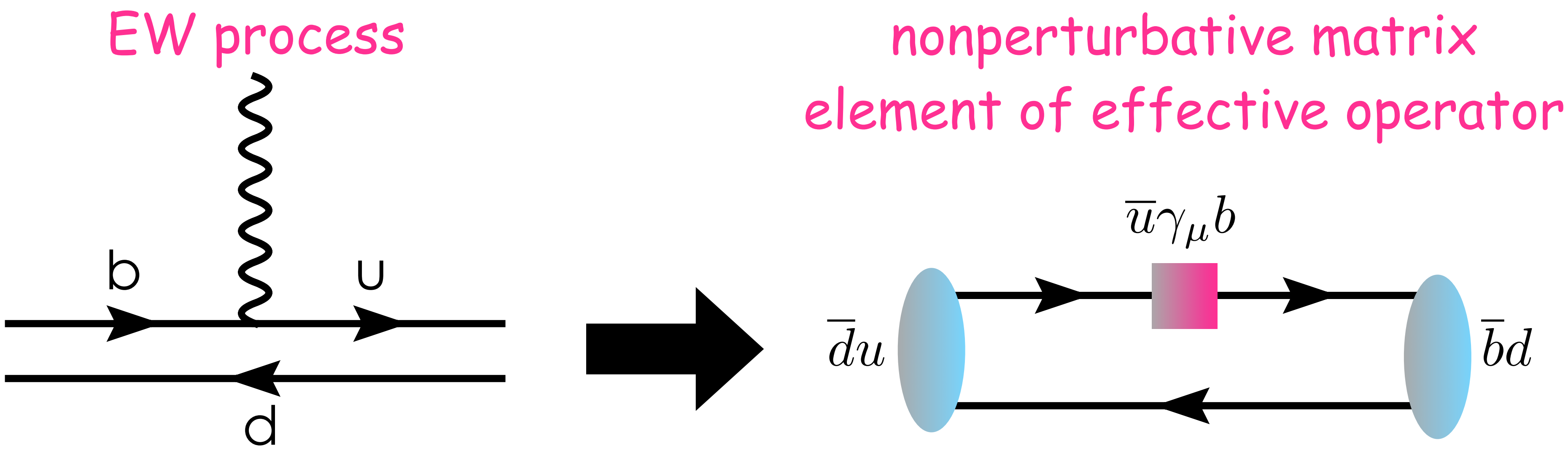
- 5+ year $\lvert V_{cd}\rvert$ estimate limited by experiment -- should actually be better after new CLEO-c measurements later this year
- $\lvert V_{ub}\rvert$ also limited by experiment -- *strong motivation for super-B factory!*

Future lattice calculations of heavy-to-light semileptonic decays



Semileptonic decays on the lattice

- To calculate form factors, lattice simulations compute matrix elements of effective operators:



- In principle can calculate any heavy-to-light meson matrix element on existing lattice configurations with light quarks down to ~$m_{strange}/10$

... So why not calculate all semileptonic form factors with lattice QCD now?

- In order to calculate form factors **accurately** must extrapolate to physical quark masses and continuum while **controlling all sources of systematic error**

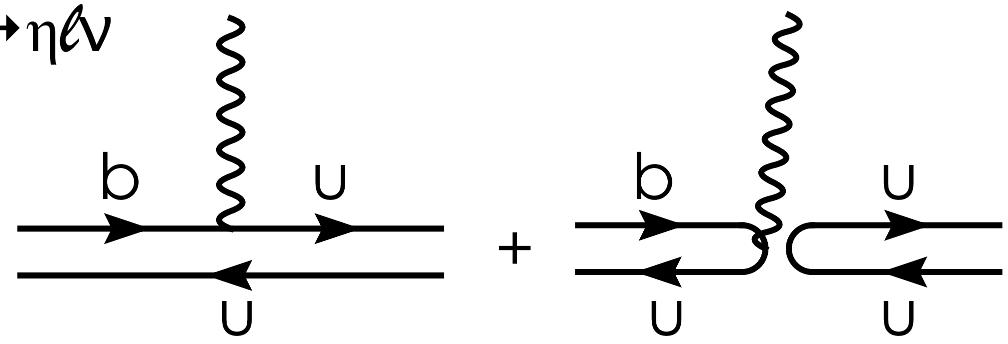
- This is currently possible for $D\to\pi\ell\nu$, $D\to K\ell\nu$, and $B\to\pi\ell\nu$ -- now discuss prospects for lattice calculations of other heavy-light decays . . .

Other b→u decays

✦ Semileptonic form factors with final state mesons other than charged pions and kaons still problematic for lattice QCD for two primary reasons:

(1) Light-light meson is **flavor-neutral**, e.g. $B \to \eta \ell \nu$

❖ Matrix element has contributions from quark-disconnected diagrams:



❖ Still possible to compute on lattice, but **costly** (~10-100x statistics needed)

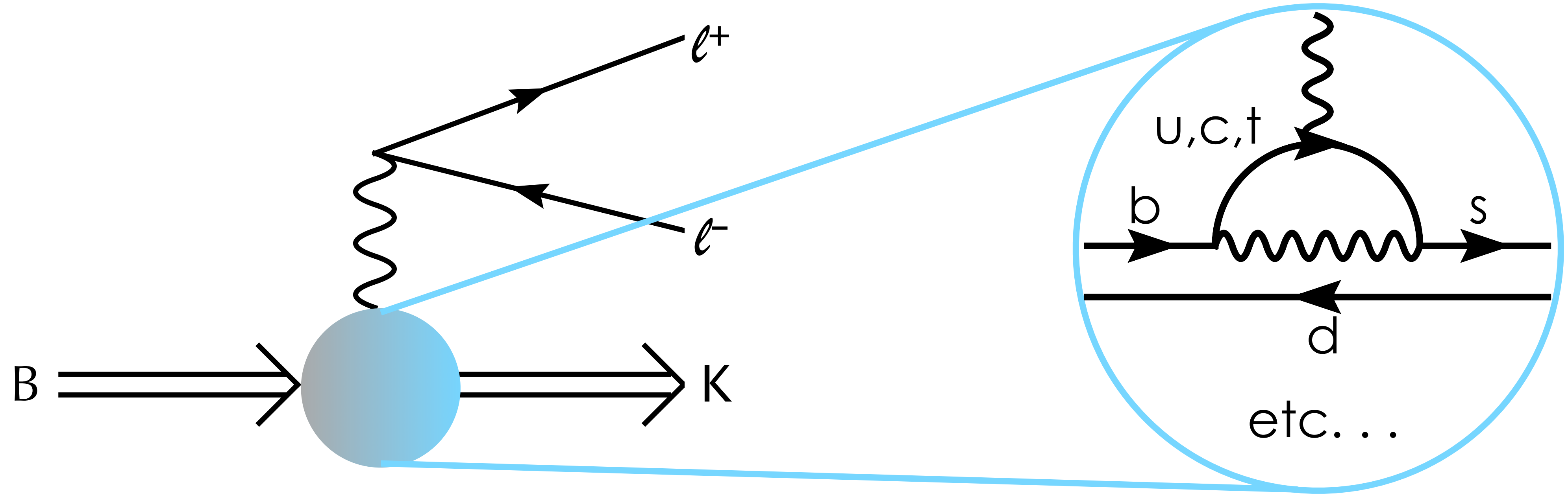
(2) Light-light meson is **unstable**, e.g. $B \to \rho \ell \nu$

❖ Extrapolation in m_q and lattice spacing will be complicated because of cusp at $\pi - \pi$ threshold, but not describable by chiral perturbation theory $\Rightarrow$ **difficult to estimate chiral/continuum extrapolation error**

❖ Cautious approach is to wait for physical quark masses and a=0.045 fm lattices so the correct $\pi - \pi$ threshold is apparent in the data

❖ Must *at least* get ρ mass right before attempting the $B \to \rho$ form factor . . .

b→d & b→s rare decays



- Lowest-order contributions 1-loop and hence **small**, therefore:
  - Beyond the standard model (BSM) contributions could be of same size
  - Potentially **stronger probes of new physics than b→u** decays
- In order to search for new physics:
  - Calculate form factors with lattice QCD
  - Calculate Wilson SM or BSM coefficients in perturbation theory
  - Combine them to make predictions for these processes in SM or BSM theories and compare with experimental measurements

Rough timescale for moving beyond $B \to \pi \ell \nu$

SL decay	flavor neutral	unstable	now	5 years
$B \to \eta \ell \nu$	✓		✓*	
$B \to \eta' \ell \nu$	✓	✓		✓
$B \to \rho \ell \nu$		✓		✓
$B \to \omega \ell \nu$	✓	✓		✓
$B \to K \ell \ell$			✓	
$B \to K^* \ell \ell$		✓		✓
$B \to \varphi \ell \ell$	✓	✓		✓
$B \to K^* \gamma$		✓		✓

*possible, but expensive

physically interesting and computationally affordable -- just need someone to work on it!

Summary and outlook

- Lattice calculations of semileptonic D and B-decays currently allow ~10-15% determinations of CKM matrix elements $|V_{cd}|$, $|V_{cs}|$, $|V_{ub}|$
- D-meson decays also allow **important test of lattice QCD methods**
- Current round of lattice calculations will likely reduce errors in $|V_{cd}|$, $|V_{cs}|$ exclusive to ~5-6% and in $|V_{ub}|$ to ~10%
- In 5 or so years, lattice simulations at a=0.045 fm and physical up/down quarks masses will likely reduce these errors to 2-4%
- Lattice QCD can also currently begin to calculate rare decays ($B\to K\ell\ell$)
- Even more calculations (e.g. $B\to\rho\ell\nu$) will become possible as computing resources increase and once physical up/down quark masses are achieved
- Still not enough people to calculate/measure everything -- our job (both theorists & experimentalists) is to **prioritize** based on new physics discovery potential . . .