# Analysis of $B^0 - \bar{B}^0$ mixing parameters on the lattice

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Lattice QCD Meets Experiment Workshop

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# 1. Introduction: $B_0 - \bar{B}_0$ mixing parameters

# Experimental measurements:

$$\boxed{\Delta M_s|_{exp.} = 17.77 \pm 0.10(stat) \pm 0.07(syst)\, ps^{-1}}$$ **CDF**

$$\boxed{\Delta M_d|_{exp.} = 0.507 \pm 0.005\, ps^{-1}}$$ **PDG07 average**

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# Possible new particles show up in the loops.

**New physics can significantly affect** $M_{12}^s \propto \Delta M_s$

* $\Gamma_{12}$ dominated by `CKM-favoured` $b \to c\bar{c}s$ tree-level decays.

- theoretically: In the Standard Model

$$\Delta M_q|_{theor.} = \frac{G_F^2 M_W^2}{6\pi^2} |V_{tq}^* V_{tb}|^2 \eta_2^B S_0(x_t) M_{B_q} f_{B_q}^2 \hat{B}_{B_q}$$

where $x_t = m_t^2/M_W^2$, $\eta_2^B$ is a perturbative QCD correction factor and $S_0(x_t)$ is the `Inami-Lim` function.

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# Non-perturbative input

$\frac{8}{3} f_{B_s}^2 B_{B_s}(\mu) M_{B_s}^2 = \langle \bar{B}_s^0 | Q_1 | B_s^0 \rangle(\mu)$ with $O_1 \equiv [\overline{b^i}\, s^i]_{V-A} [\overline{b^j}\, s^j]_{V-A}$

# For $\Delta\Gamma_q$ one needs either $\langle \bar{B}_q^0 | Q_2 | B_q^0 \rangle(\mu)$ and $\langle \bar{B}_q^0 | Q_1 | B_q^0 \rangle(\mu)$ or $\langle \bar{B}_q^0 | Q_3 | B_q^0 \rangle(\mu)$ and $\langle \bar{B}_q^0 | Q_1 | B_q^0 \rangle(\mu)$

$$O_2 \equiv [\overline{b^i}\, s^i]_{S-P} [\overline{b^j}\, s^j]_{S-P}$$

$$O_3 \equiv [\overline{b^i}\, s^j]_{S-P} [\overline{b^j}\, s^i]_{S-P}$$

## Precise determination of CKM matrix elements

$$\left|\frac{V_{td}}{V_{ts}}\right| = \frac{f_{B_s}\sqrt{B_{B_s}}}{f_{B_d}\sqrt{B_{B_d}}} \qquad \underbrace{\sqrt{\frac{\Delta M_d M_{B_s}}{\Delta M_s M_{B_d}}}}_{\substack{\text{known experiment.} \\ \text{better than } 1\%}}$$

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**Calculating** $\xi = \frac{f_{B_s}\sqrt{B_{B_s}}}{f_{B_d}\sqrt{B_{B_d}}}$ **with a** `few percent` **error**

# 2. Unquenched lattice determinations of $B_0$ mixing parameters

**Quenched approximation**: neglect vacuum polarization effects
→ uncontrolled and irreducible errors ✗

# Unquenched determinations with 2+1 flavours of sea quarks

- HPQCD: E. Dalgic, A. Gray, E. G., C.T.H. Davies,G.P. Lepage, J. Shigemitsu, H. Trottier, M. Wingate
- Fermilab lattice/MILC: R.T. Evans, E.G., A.X. El-Khadra, M. di Pierro
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# 2.1. Fermion formulations and matching

**MILC** $N_f^{sea} = 2 + 1$ configurations

| | HPQCD | Fermilab/MILC |
|---|---|---|
| Light fermions | Asqtad | Asqtad |
| Heavy fermions | NRQCD | Fermilab |
| Matching | Perturbative: one-loop | Perturbative: one-loop |

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  ( El-Khadra, Kronfeld, Mackenzie )

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- **Improved gluon action**

| | $a$ | $\#m^{sea}_{light}/m^{sea}_s$ | $\#m^{valence}$ |
|---|---|---|---|
| HPQCD | 0.12 fm | 4 | full QCD |
| | 0.09 fm | 2 | |
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$\Longrightarrow$ Light ($m_u^{sea} = m_d^{sea}$) sea and valence quark masses as low as $\simeq m^s_{phys.}/8 \rightarrow$ chiral regime

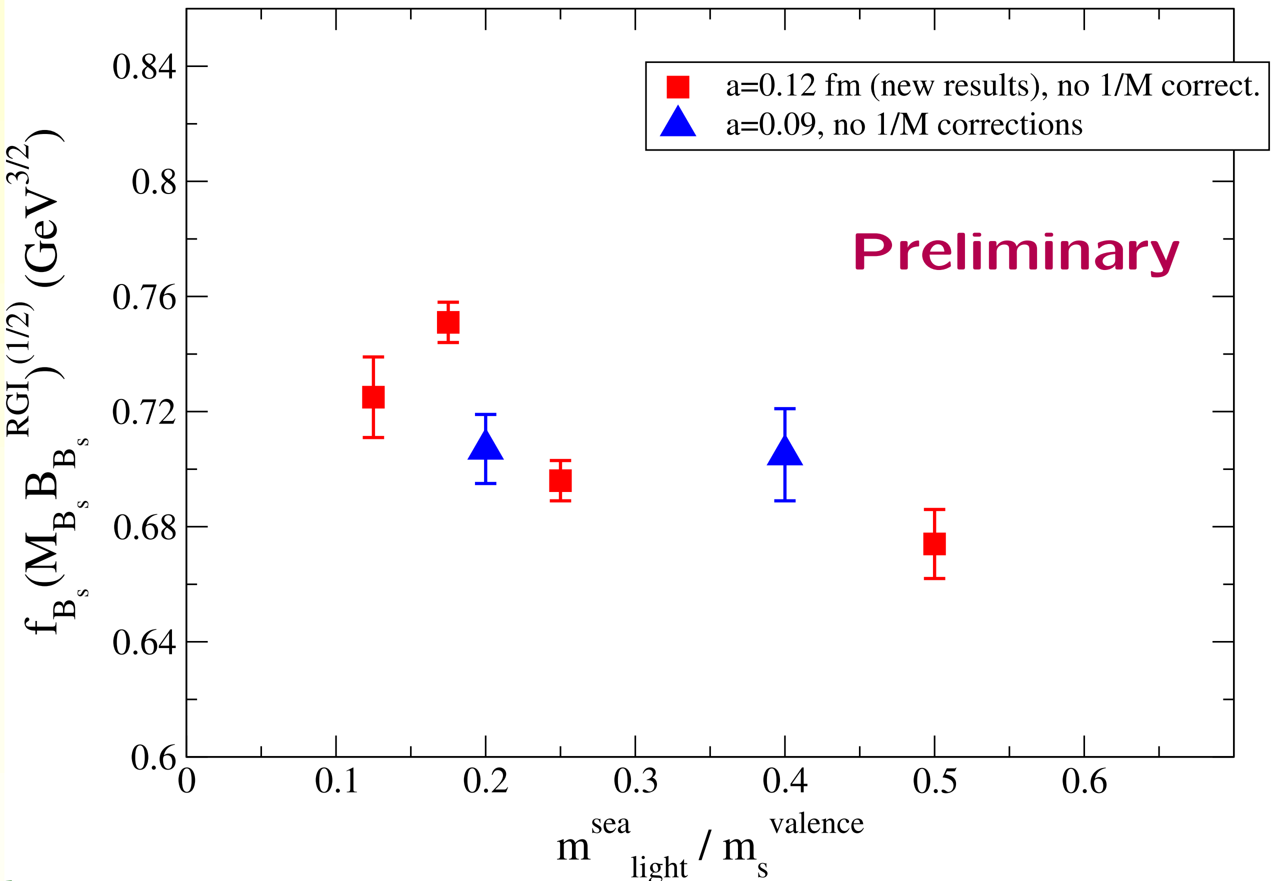
$*$ Lightest pions $m_\pi \sim 230\ MeV$.

$\Longrightarrow$ Valence $m_b$ fixed to its physical value. Sea and valence $m_s$ close to its physical value.

# 4. Preliminary results for $f_{B_q}\sqrt{B_{B_q}}$

$f_{B_s}\sqrt{M_{B_s}\hat{B}_{B_s}}(GeV^{3/2})$

HPQCD




with $m_s^{valence}$ fixed to its physical value and $m_s^{sea}$ very close to it.

**statistics+fitting errors** $\sim 1-3\%$

\# $1/M$ corrections not included yet.

Same for $f_{B_d}\sqrt{M_{B_d}B_{B_d}}$

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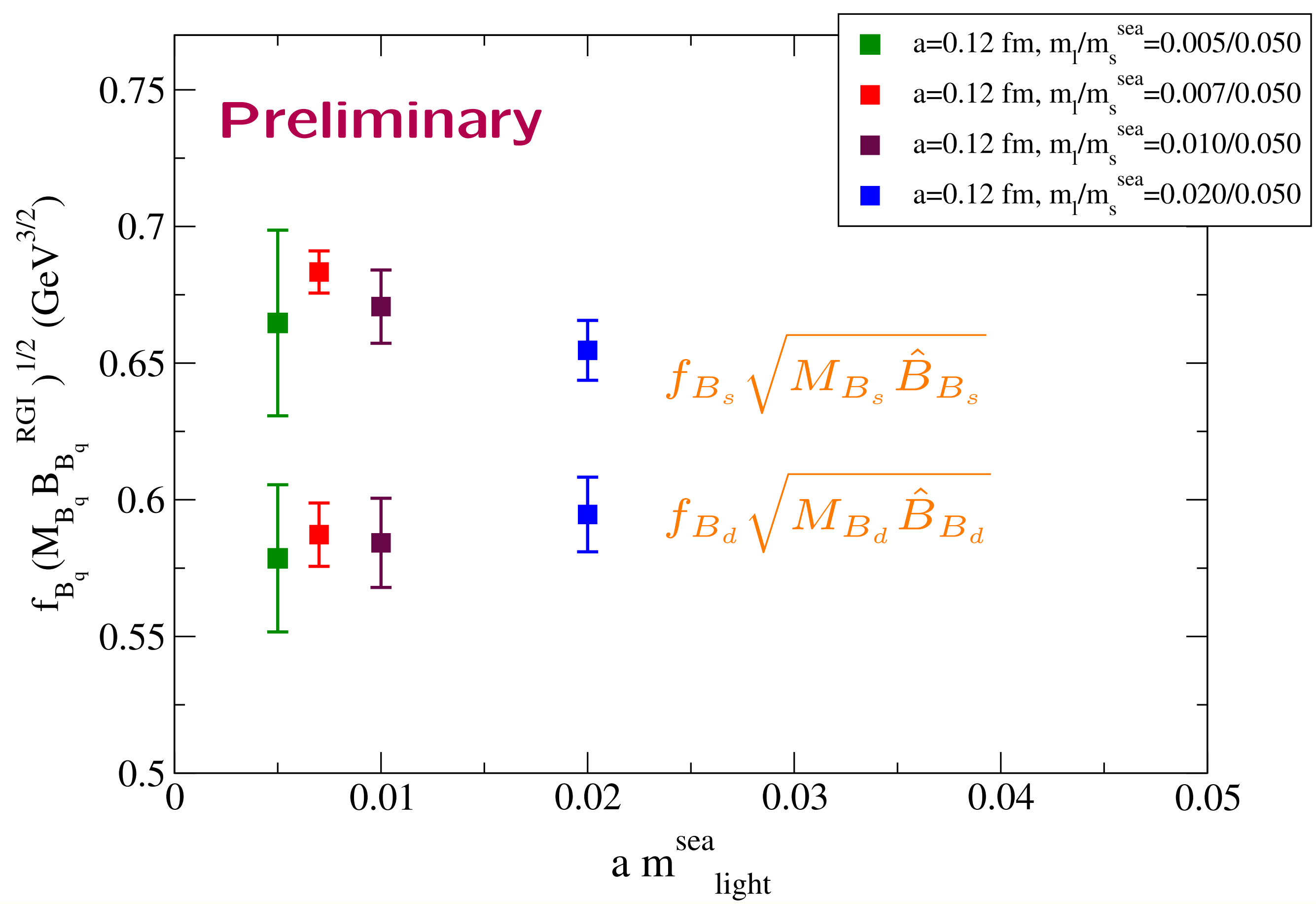


Full QCD

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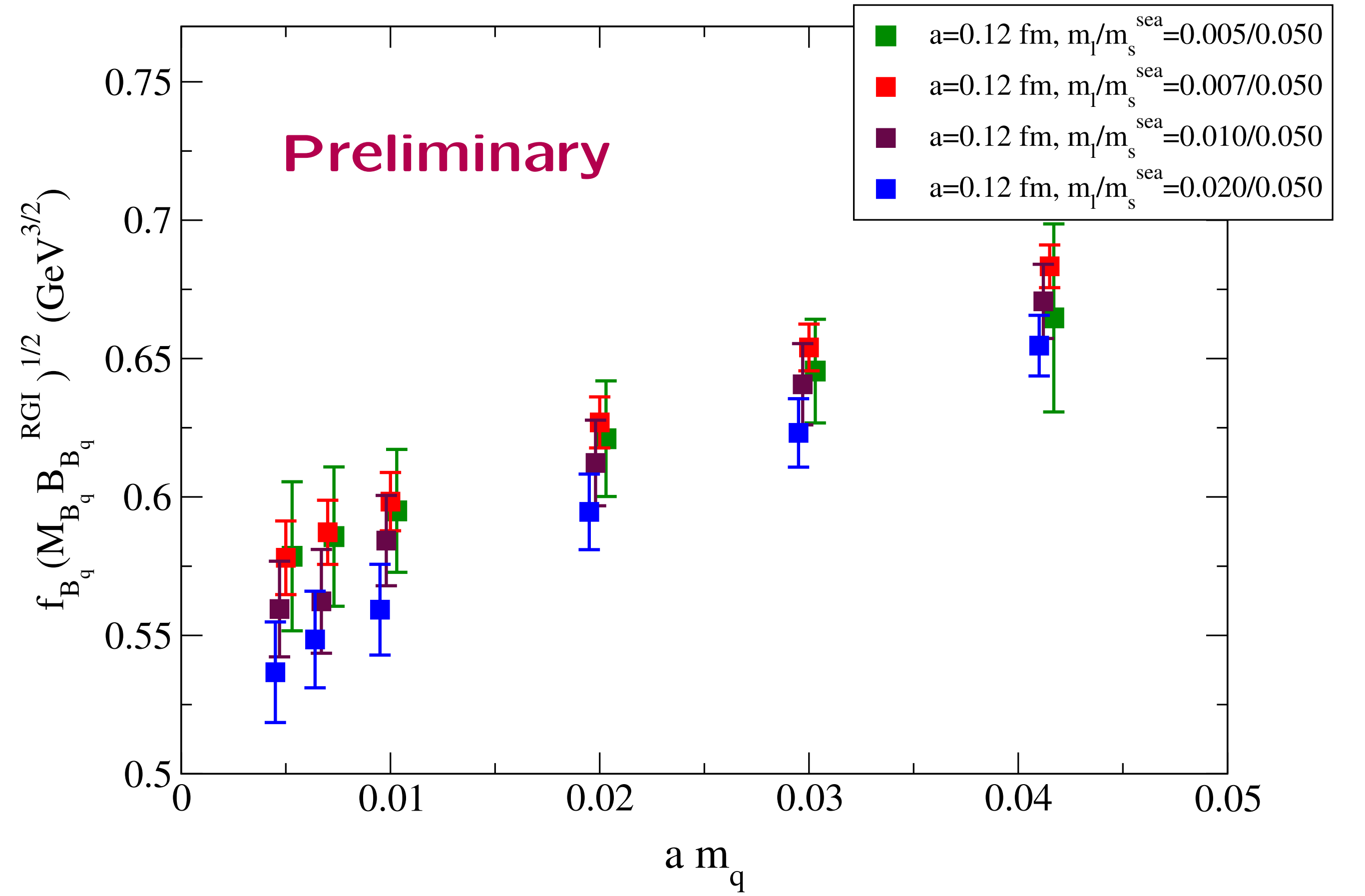


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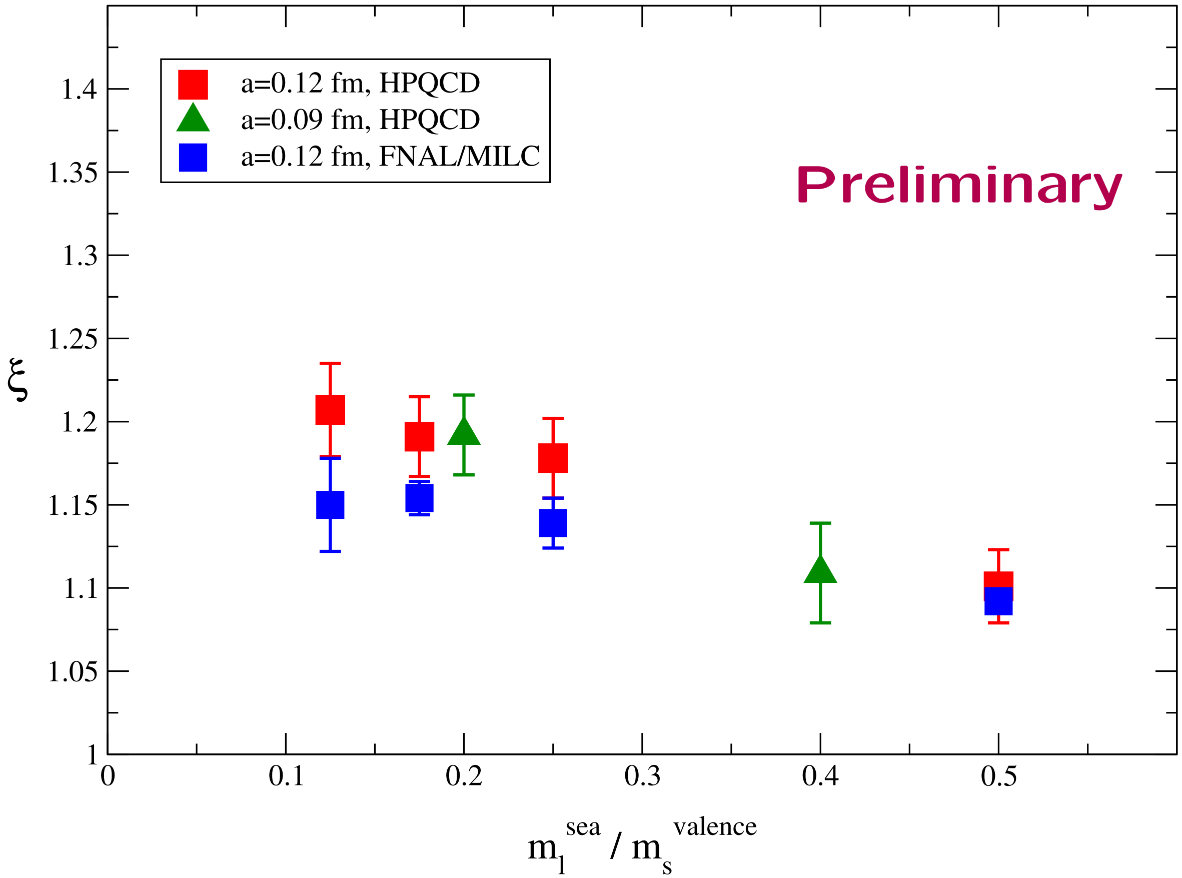
\# One-loop renormalization coefficients need to be checked (`not included`).

# Preliminary results for $\xi$: Full QCD

$$\xi = (f_{B_s}\sqrt{B_{B_s}})/(f_{B_d}\sqrt{B_{B_d}})$$




**statistics+fitting errors $\sim 1-2\%$**

# Discussion of errors (2 lattice spacings)

(ranges cover both HPQCD and FNAL/MILC calculations)

| | $f_{B_q}\sqrt{B_{B_q}}$ | $\xi$ |
|---|---|---|
| statistics+fitting | $1-3\%$ | $\sim 1-2\%$ |
| inputs ($a$, $m_b$ ...) | $2.5\%$ | $< 0.1$ |
| Higher order matching | $\sim 3.5\%$ | cancel to a large extent |
| Heavy quark action | $1.5-2\%$ | $< 0.2\%$ |
| Light quark discret. + $\chi$PT fits | $2-4\%^*$ | $< 2\%^*$ |

# Higher order matching errors naively estimated $\mathcal{O}(1 \times \alpha_s^2)$

# Difference between `tree level` and `one-loop` results $< 0.5\%$ in $\xi$

(to be compared with a $5-7\%$ shift in $f_B\sqrt{B_B}$).

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# Heavy quark discretization and relativistic effects estimated by power counting for the fine lattice ($a = 0.09 fm$).

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| **Total** (estimate) | $5-7\%$ | $2-3\%$ |

# Staggered $\chi$PT can be used to remove the leading light quark discretization effects.

* Estimate based on previous $f_B$ studies.

# Discussion of errors: what can be expected from lattice in 2 years?

* Better statistics: More configurations, improved techniques for correlation fits (`smearing, random wall sources`)

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* Smaller values of lattice spacing: $a = 0.09\ fm$ (`fine`) $\rightarrow$
$a = 0.06\ fm$ (`hyperfine`)

Discretization (Fermilab action): $\sim 1.5$
Discretization (NRQCD): $\sim 2$
Discretization (light quarks): $\sim 2$
Matching: $3.5\% \rightarrow 2.6\%$

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* Better determination of inputs: $a$, $m_b$, ...

* Two-loop or non-perturbative renormalization

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statistics+fitting: $1.5-2$

**Reduction of errors by a factor of $1.5-2$**

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\# Effects of heavy new particles seen in the form of effective operators built with **SM** degrees of freedom

\# The most general `Effective Hamiltonian` describing $\Delta F = 2$ processes is

$$\mathcal{H}_{eff}^{\Delta F=2} = \sum_{i=1}^{5} C_i Q_i + \sum_{i=1}^{3} \widetilde{C}_i \widetilde{Q}_i \qquad \text{with}$$

$$Q_1^q = \left(\bar{\psi}_f^i \gamma^\nu (\mathrm{I} - \gamma_5) \psi_q^i\right) \left(\bar{\psi}_f^j \gamma^\nu (\mathrm{I} - \gamma_5) \psi_q^j\right) \qquad \textbf{SM}$$

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$$Q_4^q = \left(\bar{\psi}_f^i (\mathrm{I} - \gamma_5) \psi_q^i\right) \left(\bar{\psi}_f^j (\mathrm{I} + \gamma_5) \psi_q^j\right) \qquad Q_5^q = \left(\bar{\psi}_f^i (\mathrm{I} - \gamma_5) \psi_q^j\right) \left(\bar{\psi}_f^j (\mathrm{I} + \gamma_5) \psi_q^i\right)$$

$$\tilde{Q}_{1,2,3}^q = Q_{1,2,3}^q \text{ with the replacement } (\mathrm{I} \pm \gamma_5) \rightarrow (\mathrm{I} \mp \gamma_5)$$

where $\psi_q$ is a heavy fermion field ($b$ or $c$) and $\psi_f$ a light fermion field.

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- $C_i, \widetilde{C}_i$ Wilson coeff. calculated for a particular BSM theory
- $\langle \bar{F^0} | Q_i | F^0 \rangle$ calculated on the `lattice`

# Comparison of contributions from these extra operators, together with the SM prediction, with experiment can constraint some BSM parameters and help to understand BSM physics. Studies done by:

- **F. Gabbiani et al**, Nucl.Phys.B477 (1996) general SUSY extensions
- **D. Bećirević et al**, Nucl.Phys.B634 (2002) general SUSY models
- **P. Ball and R. Fleischer**, Eur.Phys.J. C48(2006); extra Z' boson, SUSY
- **U. Nierste**, talk at CTP Symposium on Supersymmetry at LHC; SUSY
- **J.K. Parry and H.H. Zhang**, hep-ph/07105443, SUSY

* Quenched lattice calculation of matrix elements still the only ones available for these studies

- **Bećirević et al**, JHEP 0204 (2002), Wilson fermions and static limit

**Need an unquenched determination of the BSM matrix elements**

## $\langle \bar{B}^0 | Q_i | B^0 \rangle$ calculated on the lattice

# Strong interactions conserve parity → $\langle \widetilde{Q}_{i=1,2,3} \rangle = \langle Q_{i=1,2,3} \rangle$.

5 different matrix elements, $\langle \bar{B}^0{}_{d(s)} | Q_{i=1-5} | B^0_{d(s)} \rangle$.

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# Same programme can be applied

- Chiral perturbation theory more involving (extra free parameters):

$$\langle \overline{B^0_{d(s)}} | Q_{i=1-5} | B^0_{d(s)} \rangle \rightarrow_{chiral} \Gamma_i (1 + L) + \underbrace{\Gamma'_i L'}_{i \neq 1} + \text{analytic terms}$$

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- Chiral extrapolations under control for **Fermilab Lattice-MILC** and **HPQCD** studies
  → errors not expected to be much larger than for the SM matrix element

# On-going calculation: **HPQCD** col., E. G. et al.

$2+1$ unquenched analysis
`NRQCD` heavy + (`staggered`) `Asqtad` light

- First step: Calculation of matching coefficients `lattice`-$\overline{MS}$
  - Some continuum renormalization coefficients for BSM operators not available in the literature.

# Complete analysis of $\Delta B = 2$ matrix elements expected from both **Fermilab lattice-MILC** and **HPQCD** collaborations in 2 years with `errors`$< 10\%$.

# 6. $D_0$ mixing: $\Delta\Gamma_D$ and $\Delta m_D$ calculations

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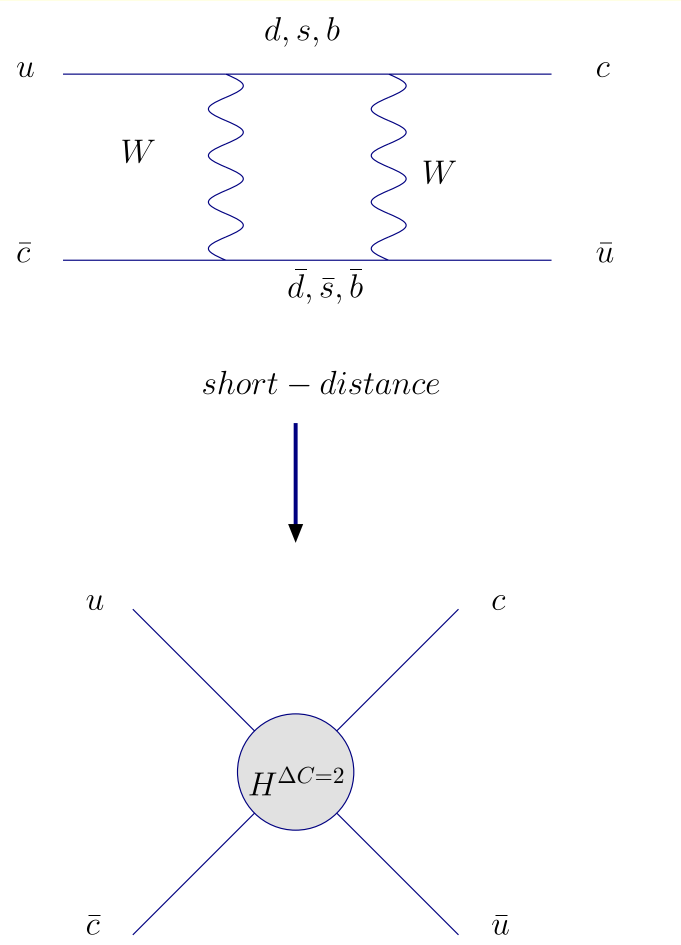


* Contribution from $b$ negligible $(V_{cd}V_{ub}^*)$
* Contribution from $s$ is very much suppressed by powers of $m_s^2/m_c^2$

($B_0$ mixing is dominated by short-distance contributions with an internal top)

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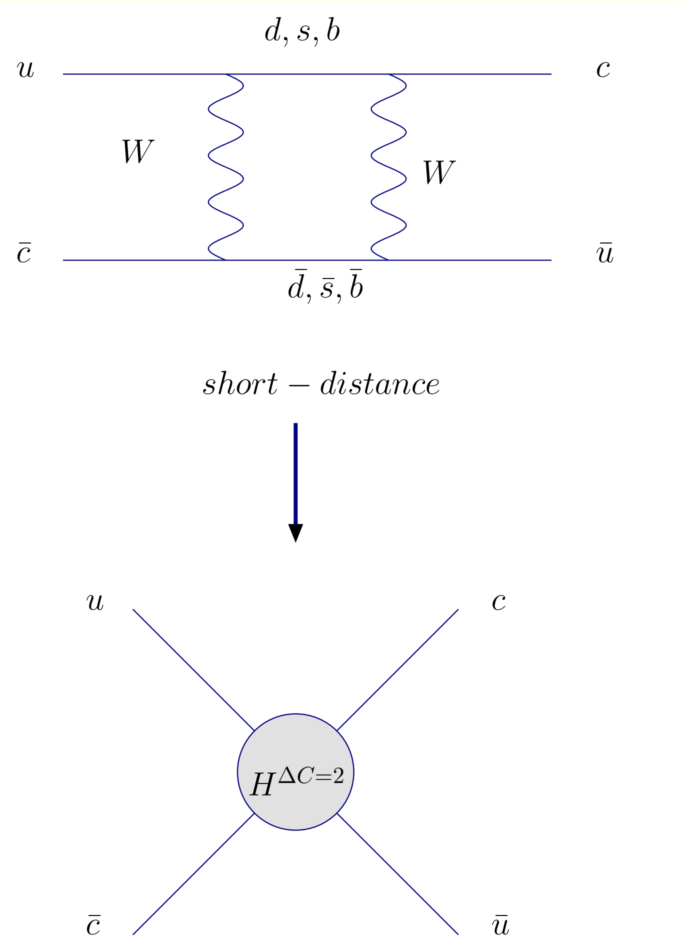


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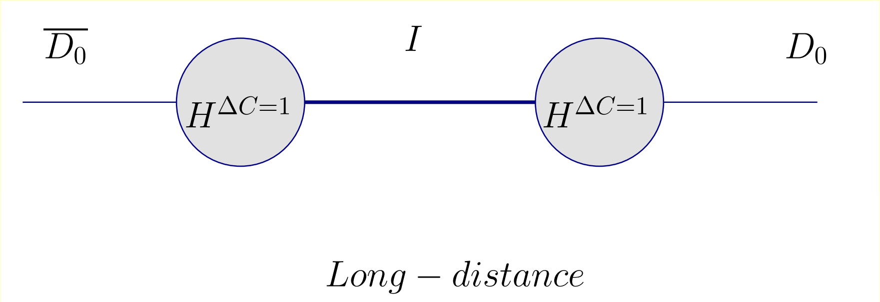
**SM** short-distance $<<$ experiment

$(x_D \sim y_D)$

## * **SM** Long-distance :




$*$ Under some model-dependent assumptions:

**A.F. Falk et al**, Phys.Rev.D69 (2004)

| |
|---|
| **SM** long-distance can account for experimental result |

$$(x_D \sim y_D)$$

$*$ $D^0$ is not light enough for its decays to be dominated by just by two-body states $\rightarrow$ very large uncertainties.

| |
|---|
| SM contribution of the order of experiment<br>and dominated by long-distance effects |

# What can lattice calculate?

# Long-distance:

Current lattice techniques are inefficient
for calculating non-local operators

* Straightforward approach requires a unreasonable increase of computing time to account for non-locality.

$\Downarrow$

* Need to develop new techniques to have accurate ($\sim 10\%$ errors) results.

## What can lattice calculate?

# Short-distance: We can calculate the matrix involved in the the SM and general BSM analysis on the lattice.

* Same techniques and effective hamiltonian as for $B^0$ mixing.

* This kind of studies can exclude large regions of parameters in many models, constraining BSM building.

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* **FNAL/MILC** col. plans to calculate these matrix elements in the next 2 years with at least a $10\%$ precission.

# 7. Summary and future work

# Results for the $B_s^0$ and $B_d^0$ mixing parameters ($\Delta M$ and $\Delta\Gamma$) in the **SM** from both the **Fermilab lattice-MILC** and **HPQCD** are coming soon with a $5-7\%$ error for $f_B\sqrt{B_B}$ and $2-3\%$ error for $\xi$.

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# $D^0$ mixing : We can not (efficiently) calculate the long-distance contributions that seems to dominate the SM predictions with current techniques.

→ Need to develop more intelligent techniques

* We can calculate short-distance contributions from general BSM extensions:

→ **FNAL/MILC** work planned for next year ($\leq 10\%$ accuracy).