charmonium 'spectroscopy' from lattice QCD

Jo Dudek

Jefferson Lab / Old Dominion University

will cover:

* sub-threshold charmonium
* excited states in charmonium
* radiative charmonium physics

displays most of the 'fundamental' problems
pions show up less

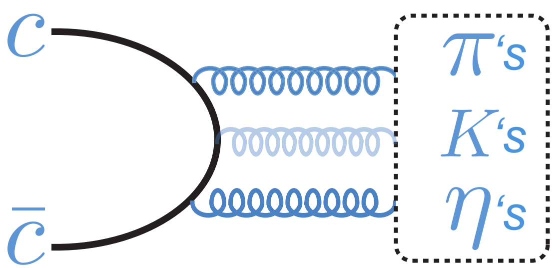
won't cover:

* bottomonium
  please address questions to NRQCD experts (I'm not one)
* heavy-light
  light-quark dominated?
  pions
* baryons

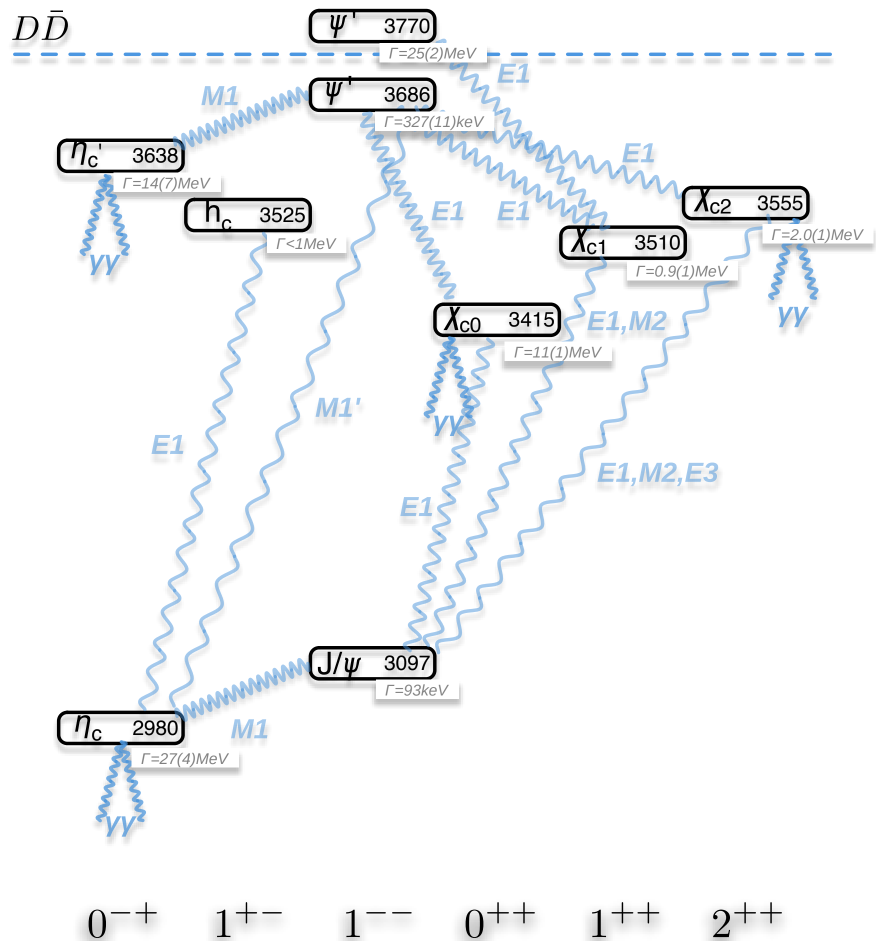
personal opinions - will find out in the next 30mins if these are controversial

spectrum situation

- will focus on charmonium since that's where a lot of the action is currently
- sub-threshold states now mostly established
  - narrow widths from *OZI*



  - radiative transitions
    - e.g. $\chi_{c0} \to J/\psi\ \gamma$
  - $\pi\pi$ transitions
    - e.g. $\psi(3686) \to J/\psi\ \pi\pi$
  - two photon decays
    - e.g. $\chi_{c2} \to \gamma\gamma$



two-point functions

- basic spectral information comes from *hadronic two-point functions*

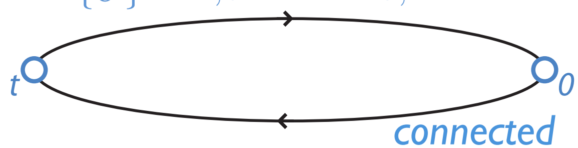
$$
\begin{aligned}
C_{ij}(t) &= \sum_{\vec{x}} \langle 0 | \mathcal{O}_i(\vec{x},t) \mathcal{O}(\vec{0},0) | 0 \rangle \\
&= \sum_N \langle 0 | e^{Ht} \mathcal{O}_i(\vec{0},0) e^{-Ht} | N \rangle \langle N | \mathcal{O}_j(\vec{0},0) | 0 \rangle \\
&= \sum_N \langle 0 | \mathcal{O}_i(\vec{0},0) | N \rangle \langle N | \mathcal{O}_j(\vec{0},0) | 0 \rangle e^{-E_N t}
\end{aligned}
$$

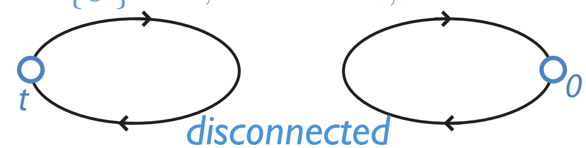
- the states labeled by ***N*** are ***all*** the eigenstates of the QCD Hamiltonian with the external quantum numbers of $\mathcal{O}$
- the interpolating fields (or operators) $\mathcal{O}$ are combinations of quark and gluon fields
  - e.g. a simple (local) interpolating field for a pseudoscalar is $\bar{\psi}(\vec{0},0)\gamma^5\psi(\vec{0},0)$
  - if the quark fields correspond to charm quarks, the sum over ***N*** includes all pseudoscalar states that have a non-zero amplitude to be in a $\bar{c}c$ Fock state with both at the origin

sub-threshold

- sub-threshold states are ideal for lattice computation ('gold-plated')
  - neglect the *OZI* suppressed decays - i.e. don't compute disconnected correlators
    - e.g. using fermion bilinears $C(t) = \langle \bar{q}(t)\Gamma q(t) \cdot \bar{q}(0)\Gamma q(0) \rangle$

$$= \sum_{\{U\}} Q^{-1}_{t,0}[U]\Gamma Q^{-1}_{0,t}[U]\Gamma \quad + \quad \sum_{\{U\}} Q^{-1}_{t,t}[U]\Gamma Q^{-1}_{0,0}[U]\Gamma$$





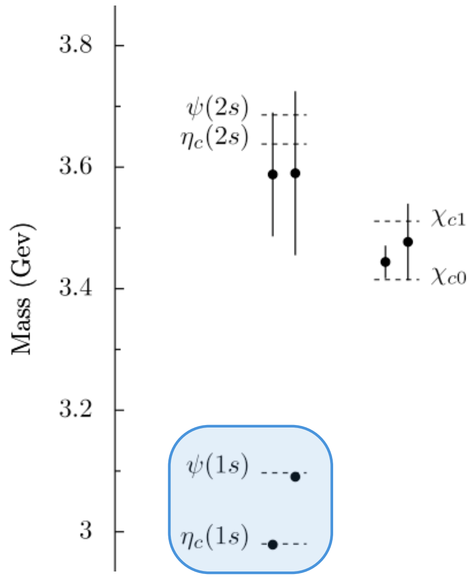
  - most obvious problem in the past is the hyperfine structure, especially the J/ψ-η_c splitting
    - believed to be a short distance effect, so must have short distance physics 'right'
    - lattice is discrete on a scale *a*
    - physics at shorter distance scales controlled by the particular discretisation used - "improved actions"

sub-threshold

- also important that the coupling constant is right at short distances
  - usually set at large distances → runs → need right β function
  - so quenching the light quarks is a bad idea

$$g^2(k) = \frac{g^2}{1 + \frac{g^2}{3(4\pi)^2}(33 - 2N_f)\log\frac{k^2}{\Lambda^2}}$$

- one recent example using an improved action and dynamical light quarks
  - *Phys.Rev.D75:054502,2007 (HPQCD & UKQCD)*
  - *a~0.09*fm , m_π ~ *250* MeV
  - no disconnected correlators



- disconnected diagrams remain the largest challenge (both computationally and theoretically) in this region *(ask me - I have more)*

fine structure

- newly observed spin singlets, $\eta_c(2S)$, h_c, are of interest
- especially their splittings from the $\psi(2S)$, χ_{cJ}
  - *(a good idea to use a basis of operators - technical analysis issue)*
  - this sort of precision measurement requires improved actions, probably light sea quarks and probably inclusion of disconnected diagrams
  - I defer to precision experts here...
    - improving the precision is a very important problem
    - I'd like to focus on the 'other' problem - what further quantities can be computed (worry about the precision later - soon hopefully)

excited states

- lightest state with each J^{PC} is technically easy to extract $C(t \to \infty) \to e^{-m_0 t}$
- excited states are more challenging
  - technical problem / theoretical problem

technical problem:
how do we actually extract reliable estimates of spectral quantities from correlators?

I believe this is 'solved':
variational solution to matrix of correlators

theoretical problem
what happens to the spectrum of states when they can decay?
i.e. "how does one deal with resonances?"

'Realistic' investigation of this has just begun:
Lüscher Method
spectrum in a finite box → S-matrix

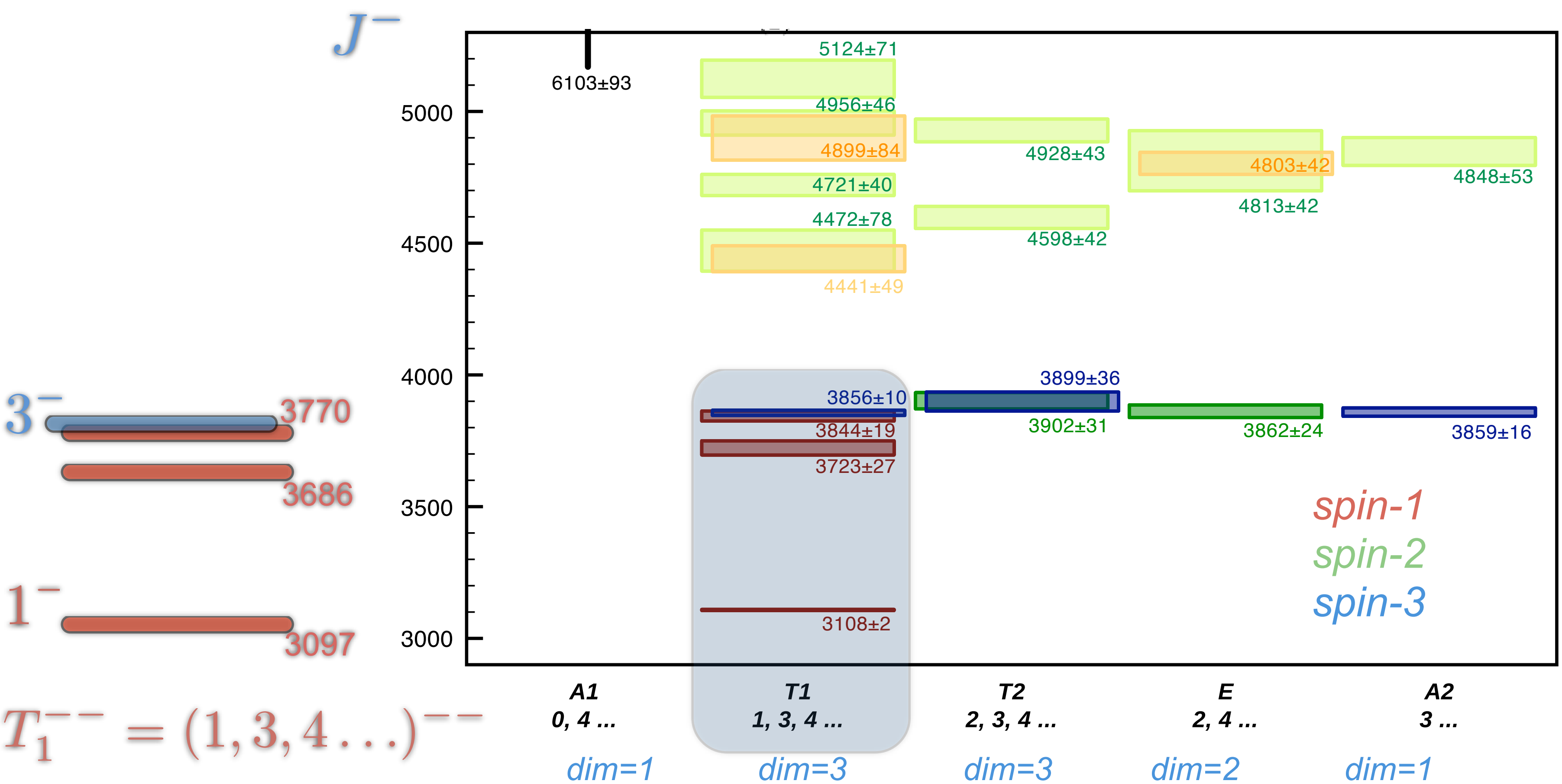
excited states - technical problem

- compute a matrix of correlators $C_{ij}(t) = \langle \mathcal{O}_i(t)\mathcal{O}_j(0)\rangle$
- using a bunch of interpolating fields, e.g. $\bar{\psi}\,\Gamma\psi \quad \bar{\psi}\,\Gamma \overleftrightarrow{D}_k\,\psi \quad \bar{\psi}\,\Gamma \overleftrightarrow{D}_j \overleftrightarrow{D}_k\,\psi$

- a variational solution can be found - utilises the orthogonality of state vectors
- very successful at extracting multiple excited states in a given J^{PC}

variational method

- e.g. recent (**quenched**) "charmonium" study (vector channel)



$$T_1^{--} = (1, 3, 4 \ldots)^{--}$$

to my knowledge, the first time this has been seen

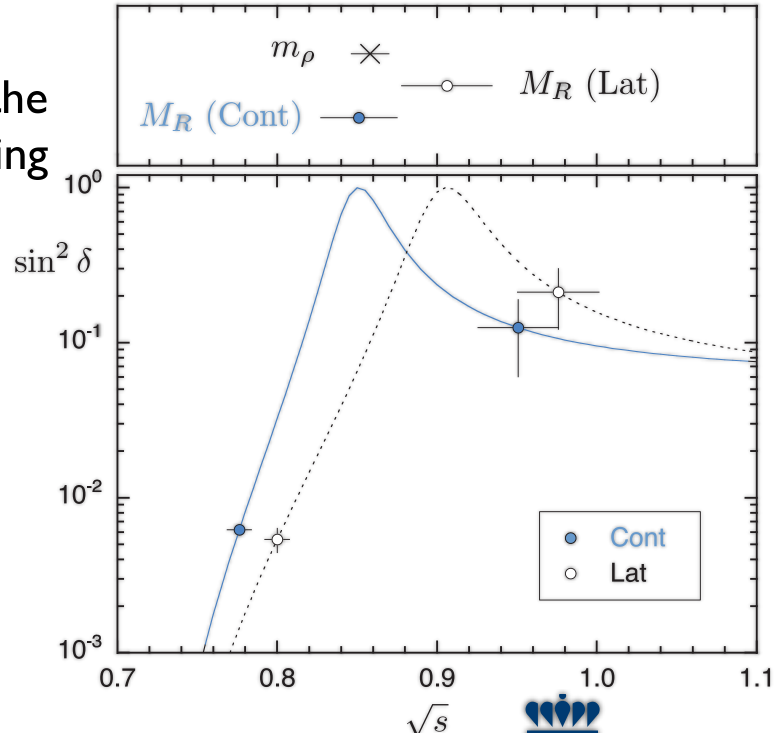
excited states - theoretical problem

- how do we extract the mass and width of a resonance?
  - e.g. consider the ρ meson which decays to $\pi\pi$
  - might try $C(t) = \langle \bar{q}(t)\gamma^i q(t) \cdot \bar{q}(0)\gamma^i q(0)\rangle \to e^{-E_0 t}$ as $t \to \infty$
  - will give us the lightest 1^{--} eigenstate of QCD
    - this is two pions in a *P*-wave - rest energy = $2m_\pi$
    - in an infinite box there are a continuum of such states
    - looks hopeless

- Lüscher (and others) have shown that in the **finite box** we work with in lattice QCD, there is a mapping between the energy levels extracted and the elastic scattering matrix (i.e. the phase shift)
  - in a periodic finite box, all energy levels are discrete so perhaps we can extract a small number and infer something about resonances?
  - one recent 'successful' application

ρ meson as a $\pi\pi$ resonance

- CP-PACS recently used the Gottlieb-Rummukainen extension of the Lüscher method to study the ρ meson as a resonance in $\pi\pi$
- at a fixed lattice volume they extracted two energy levels using a correlation matrix constructed from
  - a "ρ-like" $\bar{q}q$ (wavefunction at the origin) operator
  - a "$\pi\pi$-like" separated $\bar{q}q - \bar{q}q$ operator
- by comparing the extracted energy levels to the expected discrete levels for two non-interacting pions they inferred the $\pi\pi$ phase-shift
  - simulation not at physical point
    - $m_\rho/m_\pi \sim 2.4$
    - $a \sim 0.2$ fm
    - $Lm_\pi \sim 4$



charmonium application?

- the first excited χ'_{c0} state would be a good place to try this out in charmonium
- observed $\chi'_{c2}(3930)$ suggests $\chi'_{c0}(\sim 3900)$ for which only $\bar{D}D$ is open

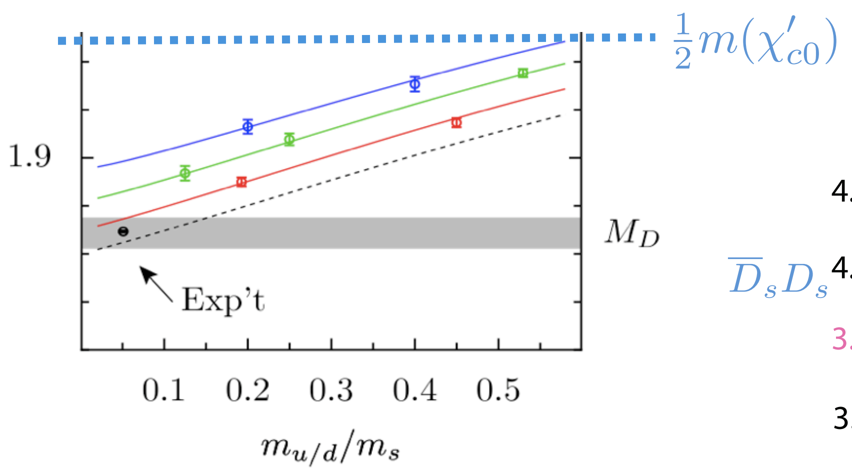
  - is this practical on current lattices?
  - depends upon the *D*-meson mass at available quark masses
  - and the available volumes
  - and the energy resolution

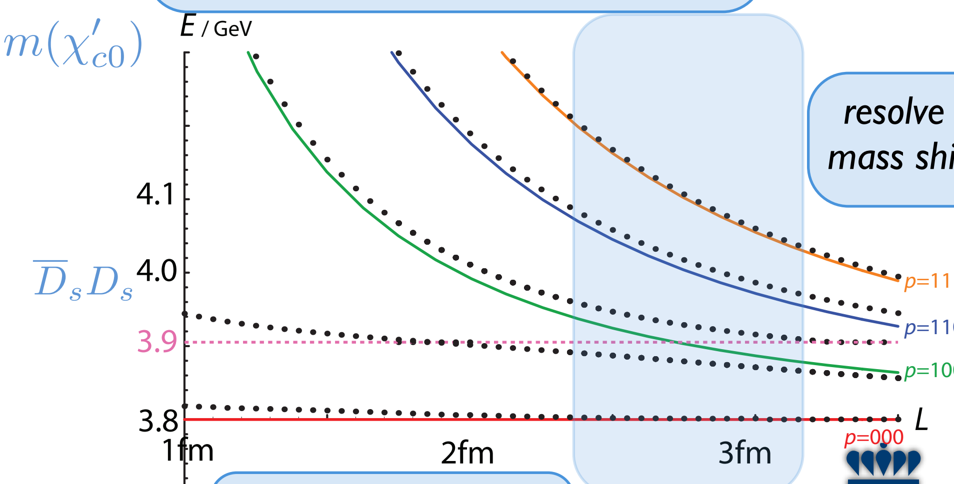
assuming $m_D \sim 1.9$GeV,

and $m(\chi'_{c0}) \sim 3.9$GeV, $\Gamma(\chi'_{c0}) \sim 50$MeV

would get something like

HISQ on MILC



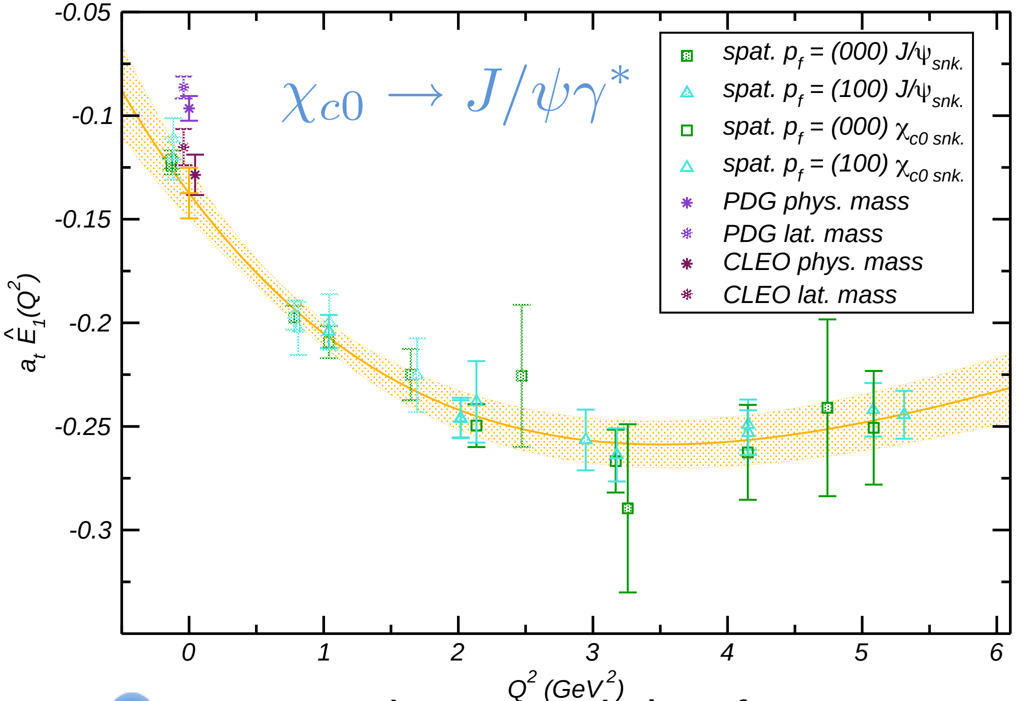


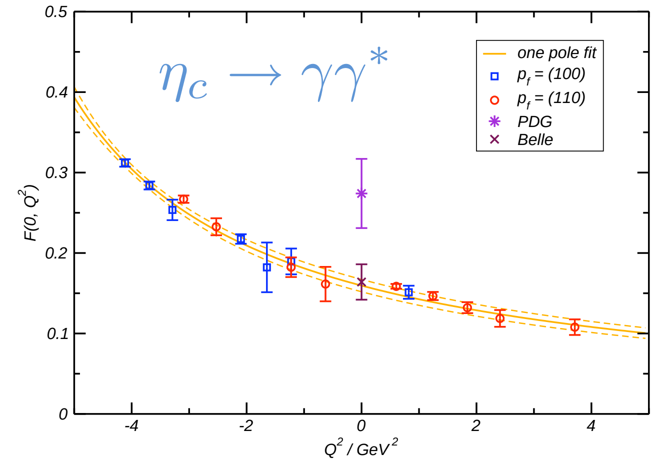
resolve 30 MeV mass shifts?

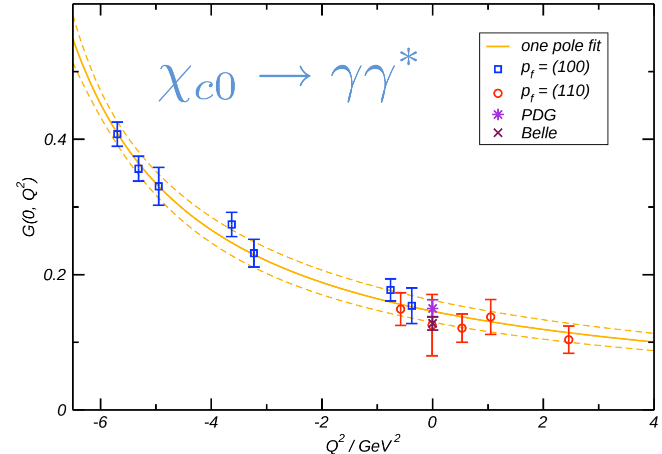
χ'_{c2} would be similar

other 'spectroscopic' quantities

- radiative transitions and two-photon couplings can be obtained from three-point correlators
  - *N.B.* two-photon fusion is the production mechanism for the new $\chi'_{c2}(3930)$
- so far only calculations are on ***quenched*** lattices







- no excited states or *J*>*1* so far
  - JLab group working on it - *much* harder

X(3872) - an interesting case

- *X(3872)*
  - unreasonably close to thresholds for $D^+D^{*-},\ D^0\overline{D}^{*0}$ (1 MeV away?)
  - so close that isospin violation comes into play
- **no hope** of tuning everything to the precision required for direct lattice study
  - if 'binding' energy really ±1 MeV, potentially long distance tail to wavefunction
- but still interesting to observe behaviour as quark mass & lattice volume change
- perhaps as input to effective field theory models?

Y(4260) (1^{--})

- model suggestions that it might be a non-exotic hybrid

- on a lattice this guy will be very tough!
  - above threshold for decay to $\bar{D}D,\ \bar{D}D^*,\ \bar{D}^*D^*,\ \bar{D}_sD_s,\ \bar{D}_sD_s^*$
  - and above the J/ψ, $\psi(2S)$ and probably three resonances, $\psi(3770)$, $\psi(4040)$, $\psi(4150)$

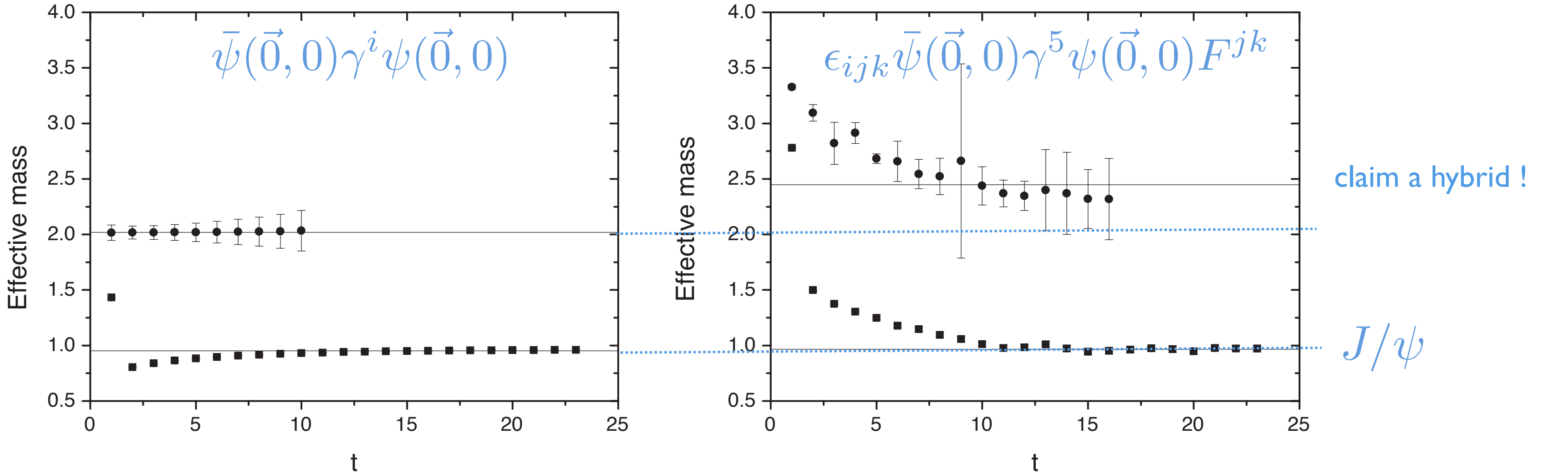
- need some major theoretical advances to consider this state
  - but far from understood phenomenologically!

modelling job

  - *decay into $\pi\pi\, J/\psi$ seen, but hadronic width is such it must be decaying elsewhere too*
  - *but doesn't show up as a peak in the exclusive*
  - *or (visibly) in the new exclusive data (CLEO, Belle, BaBar)*
  - *needs a (unitary!) coupled-channels fit including interferences properly*

interpolating fields & state interpretation

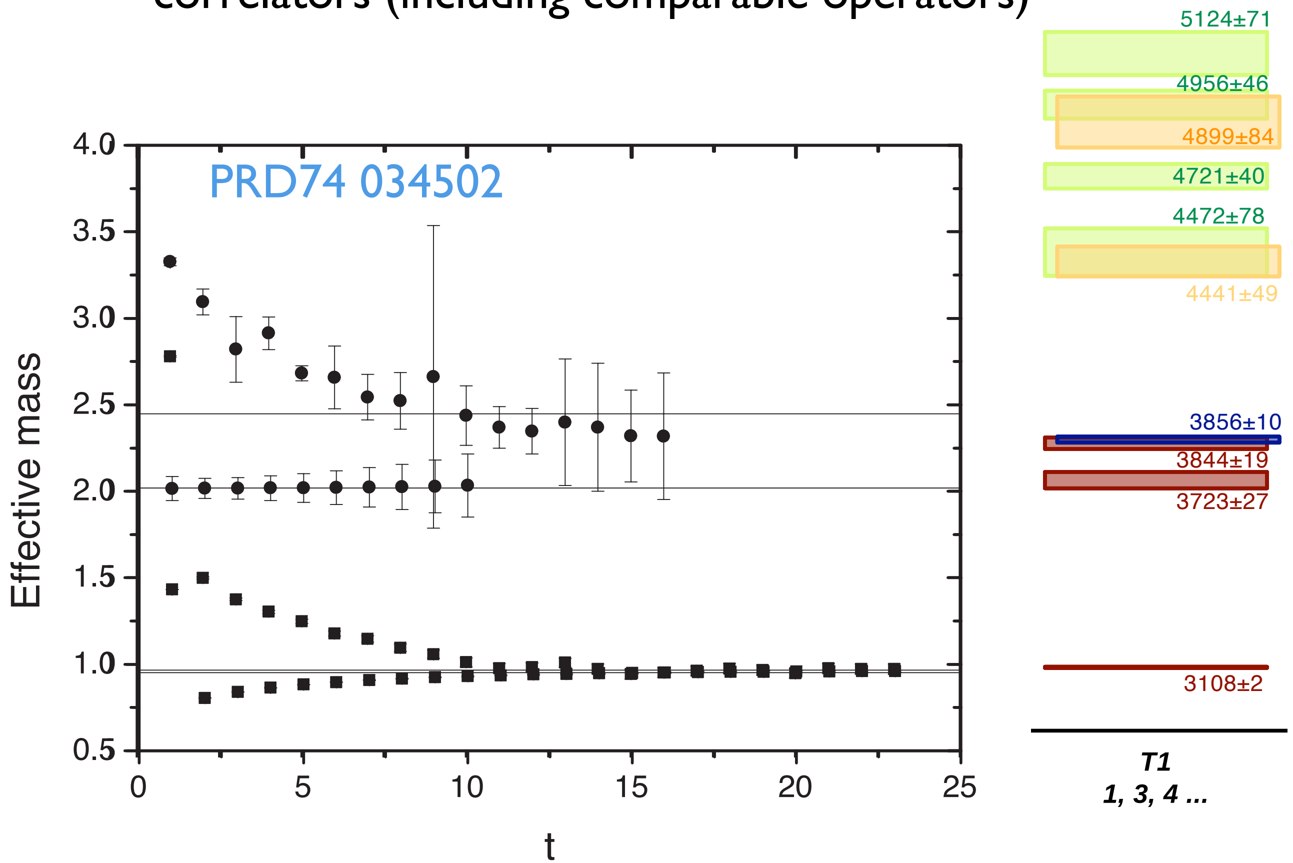
- it is a common mistake (even one made by some lattice theorists) to think that the “internal” structure of the interpolating field directly tells you the nature of states
  - e.g. any state produced using $\bar{\psi}(\vec{0},0)\gamma^i\psi(\vec{0},0)$ must be a “conventional” $\bar{c}c\ 1^{--}$
  - any state produced using, say, $\epsilon_{ijk}\bar{\psi}(\vec{0},0)\gamma^5\psi(\vec{0},0)F^{jk}$, must be a “hybrid” because of the gluonic factor
  - see e.g. *“Gluonic excitation of non-exotic hybrid charmonium from lattice QCD”* X-Q. Luo & Y. Liu (PRD74 034502) *quenched*



if a “hybrid” is anything overlapping the second operator, then do we have to rethink the J/ψ ?

large basis of operators

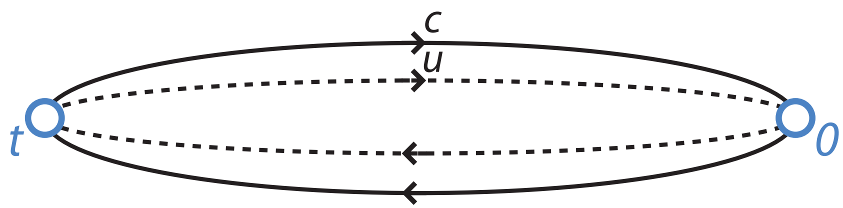
- similar (quenched study) using a matrix of correlators (including comparable operators)



- appears to be far more mundane explanations for these correlators

more (quenched) Y(4260) claims

- Chiu & Hsieh *"Y(4260) on the lattice"* (PRD73 094510)
- compute correlators with multiquark operators, e.g. $(\bar{q}\gamma_i c)(\bar{c}q)$
  - all quarks at same space point
- connected diagrams only



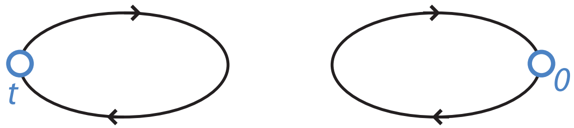
  - technically this means *isospin=1*, but *isospin=0* might be degenerate?

- find ground state masses in the region of *4300-4500* MeV (after a crude extrapolation in m_q)

$Z^+(4430)$??

- 'seen' as a resonance in $\pi^+\psi(2S)$ by Belle via B-decays
- isospin one with large affinity for charmonium - might be interesting !?
- potentially lots of decay channels open so similar problem to $Y(4260)$

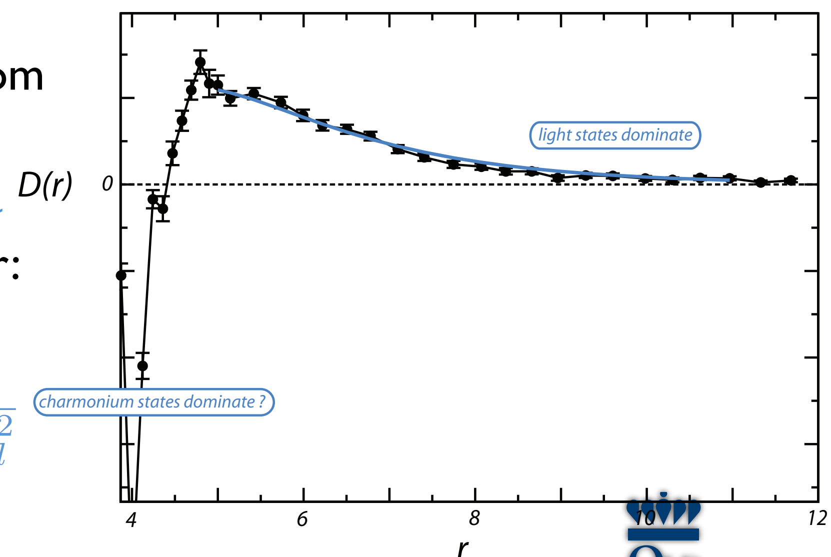
disconnected contributions



- technically challenging (noisy signals)
- theoretically challenging - what are we measuring?
  - e.g. pseudoscalar "charmonium" interpolating field $\bar{c}\gamma^5 c$
  - at large Euclidean times (separations) should get the lightest QCD eigenstate that is psuedoscalar and isosinglet (and has some amplitude to be $\bar{c}c$)
  - this is the η-meson (via its hidden charm component) !

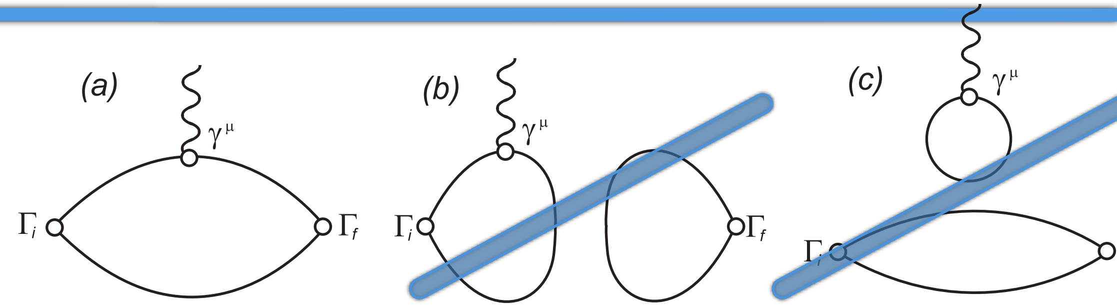
- separating properties of charmonium from the light-quark stuff is tricky
  - see recent work by DeTar & Levkova
  - non-trivial modeling of the correlator:

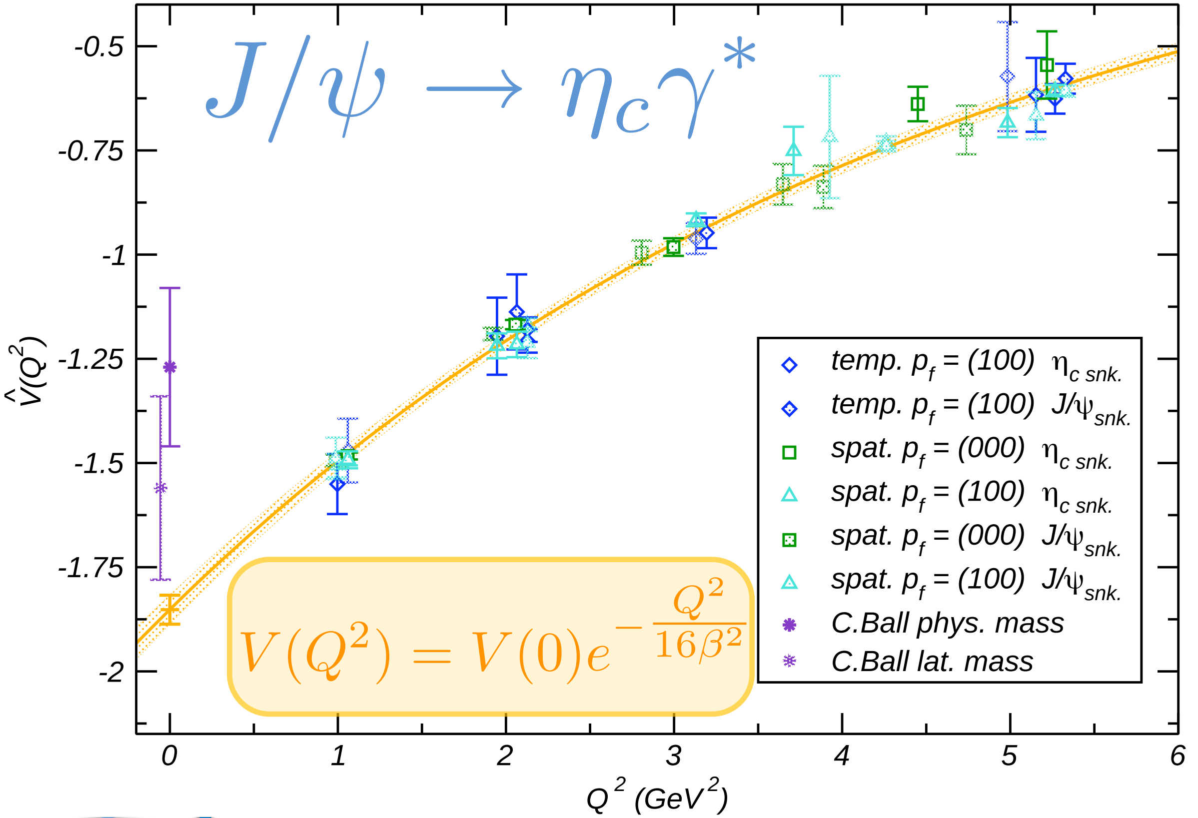
$$D(p^2) = \left[\frac{f(p^2)}{p^2 + m_{\bar{c}c}^2} + \frac{g(p^2)}{p^2 + m_{\bar{c}c^*}^2} \right]^2 \frac{1}{p^2 + m_l^2}$$



radiative transitions

- first computation (quenched)
- no disconnected diagrams

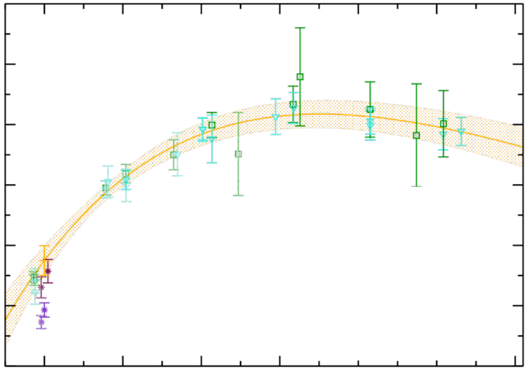




1P→1S transitions

- fit form inspired by potential models with spin-dependent corrections

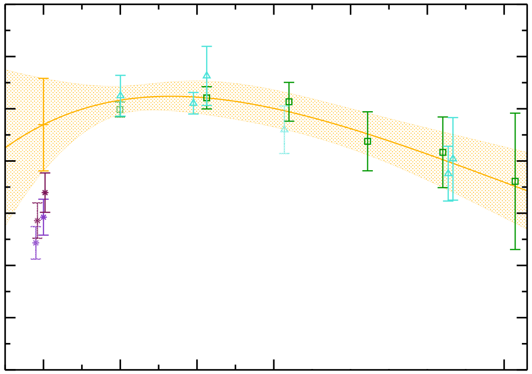
$$E_1(Q^2) = E_1(0)\left(1 + \frac{Q^2}{\rho^2}\right) e^{-\frac{Q^2}{16\beta^2}}$$



$\chi_{c0} \to J/\psi \gamma_{E1}$

$\beta = 542(35)\ \mathrm{MeV}$

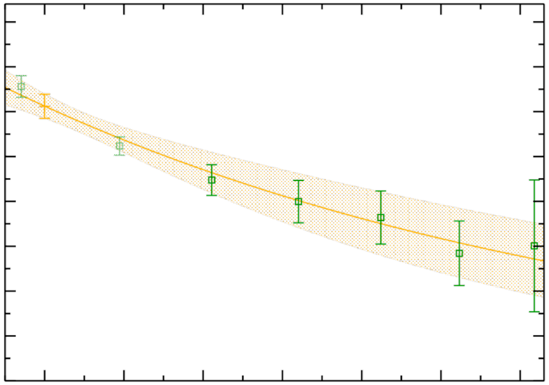
$\rho = 1.08(13)\ \mathrm{GeV}$



$\chi_{c1} \to J/\psi \gamma_{E1}$

$\beta = 555(113)\ \mathrm{MeV}$

$\rho = 1.65(59)\ \mathrm{GeV}$



$h_c \to \eta_c \gamma_{E1}$

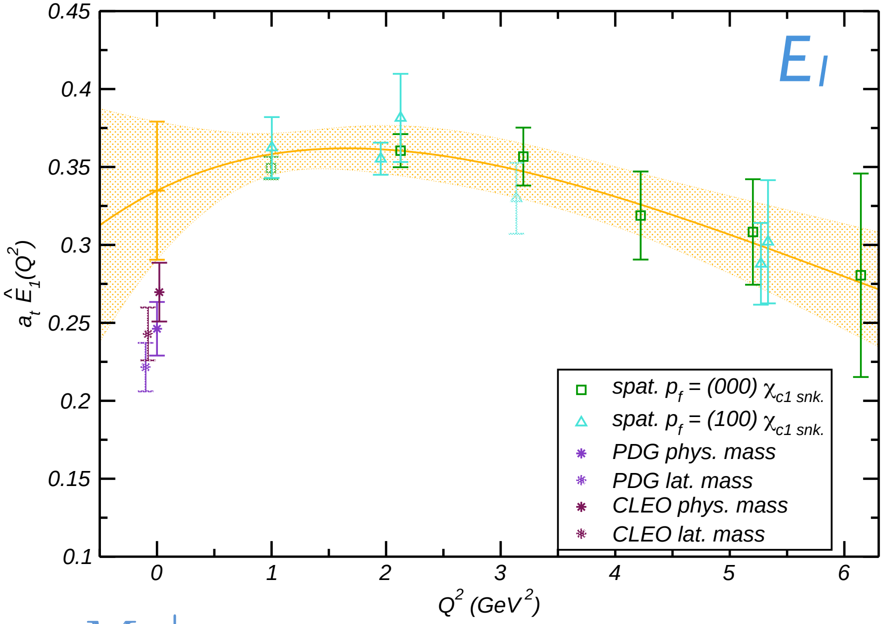
$\beta = 689(133)\ \mathrm{MeV}$

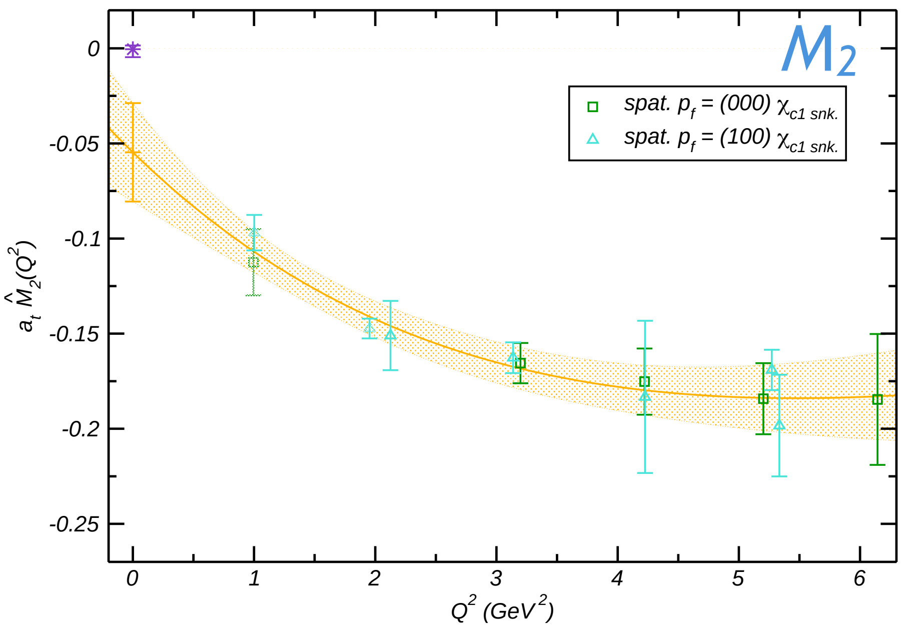
$\rho \to \infty$

simplest quark model has all β equal and $\rho(\chi_{c0}) = 2\beta$, $\rho(\chi_{c1}) = \sqrt{2}\cdot\rho(\chi_{c0})$, $\rho(h_c) \to \infty$

$\chi_{c1} \rightarrow J/\psi\,\gamma$ transition

- automatically access both the electric dipole and the magnetic quadrupole transitions





$$\left.\frac{M_2}{E_1}\right|_{\substack{\text{quen.}\\\text{lat}}} = -0.2(1)$$

less theoretical motivation for the fit-form here

$$\left.\frac{M_2}{E_1}\right|_{\text{expt}} = -0.002\left({}^{+8}_{-17}\right)$$

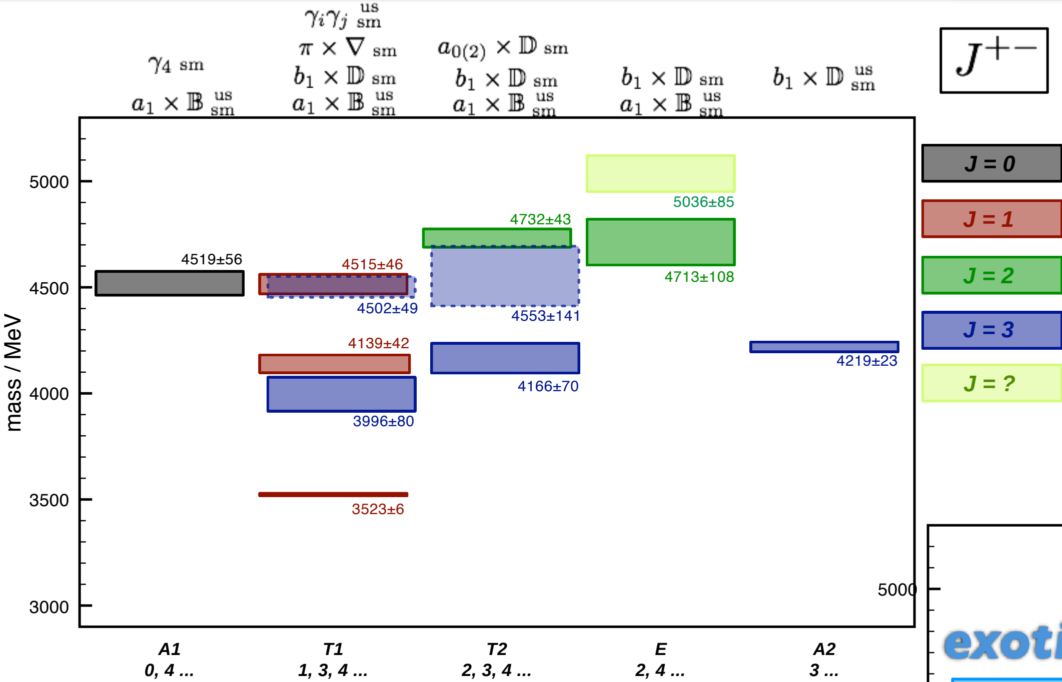
exotic quantum numbers ?

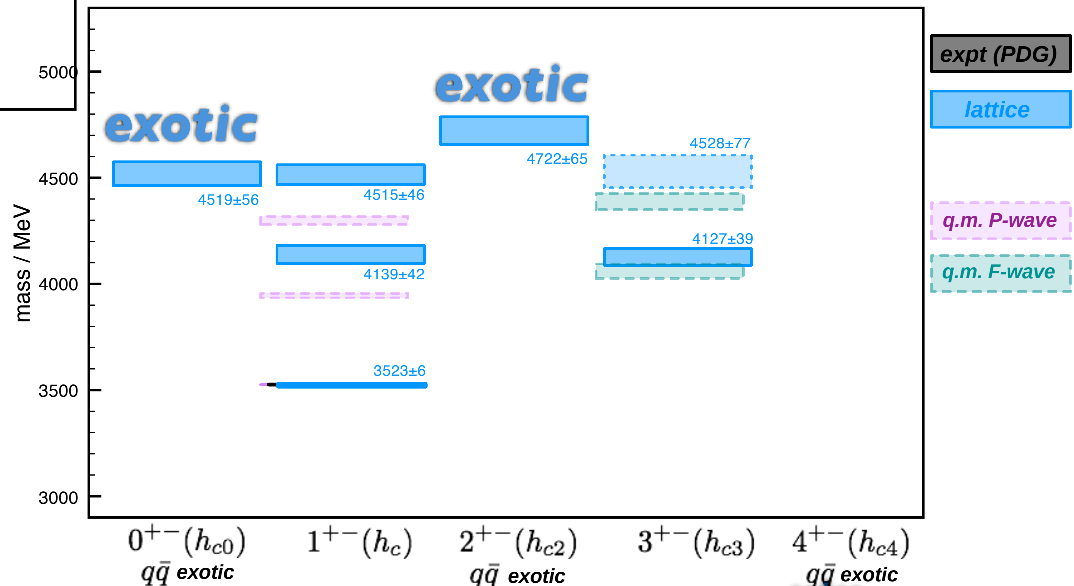
- $0^{--}, 0^{+-}, 1^{-+}, 2^{+-}$ cannot be constructed from a Fock state
- hence 'exotic' - no experimental charmonium candidates (to my knowledge)

- just build an operator with these quantum numbers!
- actually not quite a simple as it appears - lattice symmetry is not continuum rotations, discrete cubic rotations
  - *A_1 - 0, 4...*
  - *T_1 - 1, 3, 4...*
  - *T_2 - 2, 3, 4...*
  - *E - 2, 4...*
  - *A_2 - 3...*

- so the $0^{+-}, 2^{+-}$ are probably straightforward
- but 1^{-+} could be confused with a non-exotic 4^{-+}

exotic quantum number?





exotic quantum number?