Lattice QCD and Muon $g-2$

(USQCD Collaboration)

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After reviewing recent successes of lattice QCD over a broad range of topics, we discuss how lattice QCD calculations can reduce the hadronic uncertainty in the Standard-Model prediction for the anomalous magnetic moment of the muon, $g-2$.

I. INTRODUCTION

During the past decade, lattice QCD has made substantial progress in several areas that influence particle physics, nuclear physics, and astrophysics. Once enough computing and algorithmic power became available to treat virtual quark-antiquark pairs (the "sea" quarks) realistically, the results of lattice-QCD calculations rapidly reproduced a wide variety of hadron properties [1]. The same techniques then enabled genuine predictions of D meson semileptonic form factors, D- and D_s-meson leptonic-decay constants, and the mass of the B_c meson [2]. Lattice QCD now plays an important role in quark flavor physics, yielding indispensible results for neutral meson mixing and leptonic decay rates, and important results for semileptonic form factors [3, 4]. These results not only constrain the Cabibbo-Kobayashi-Maskawa (CKM) matrix but also enable indirect searches for new particles.

The success of lattice QCD is not confined to flavor physics alone. The nucleon mass, one of the original objectives, has been computed with a precision of about 2% [5]. Nambu's ideas of spontaneous chiral symmetry breaking, once strong beliefs, have been verified via direct calculation from the QCD Lagrangian [6]. Connected to these developments are the only *ab initio* determinations of the light-quark masses [7]: the up-, down-, and strange-quark masses turn out to be small—about four, nine, and 180 times the electron mass, respectively. Lattice QCD meanwhile provides the most accurate determinations of the strong coupling α_s [8] and competitive determinations of the bottom- and charm-quark masses. These results connect the QCD probed in high-energy processes with the QCD description of hadrons.

With matrix elements from flavor physics and the nucleon mass under control, a next step is to compute nucleon matrix elements [9]. These are helpful for interpreting experiments on the neutron electric dipole moment, nucleon β decay, and nucleon structure, as well as planning searches for proton decay. Another recent development is the calculation of virtual hadron properties, which influence electroweak parameters. One example is the evolution of QED's fine structure constant from electronic to Z-pole scales. More prominent for the intensity frontier are related calculations of hadronic contributions to the muon's anomalous magnetic moment [10, 11].

There are many other lattice-QCD calculations that are beyond the scope of this document, which we shall mention only briefly. Together with the CKM matrix, the quark masses and α_s constrain speculation about unification of the forces and other physics beyond the reach of accelerators. Calculations of the strangeness content of the nucleon are needed to understand dark-matter detection experiments [12, 13]. Studies of the QCD phase transition with lattice QCD have shown that the quark masses, though small, are just large enough to make the transition a crossover [14, 15]. Previously, research on the early universe assumed the transition was of first order, with phenomena like bubbles of hadrons; we now know that the universe did not cool this way. Calculations of hadron-hadron interactions help us understand the physics of neutron stars, particularly whether neutrons could dissociate into $K\Lambda$ pairs [16]. Lattice gauge theories varying the number of colors and matter content are shedding light on the dynamics of technicolor models of electroweak symmetry breaking [17].

The rest of this document is organized as follows. Section II reviews in detail how lattice QCD plays a key role in determining the hadronic contributions to muon $g-2$. We address both the hadronic vacuum polarization, which seems well under control, and the hadronic light-by-light contribution, where new ideas may be needed. In Sec. III, we present forecasts of future precision that we expect to obtain in hadronic vacuum polarization. We conclude with some remarks of the broader role of lattice QCD at the intensity frontier in Sec. IV.

For similar information on quark-flavor physics and nucleon properties, please consult documents submitted to the Heavy-quark and Nucleons/nuclei/atoms WGs, respectively.

II. HADRONIC CONTRIBUTIONS TO $g-2$ AND LATTICE QCD

The measured value of the anomalous magnetic moment is in apparent disagreement with theoretical calculations based on the Standard Model (SM). The BNL E821 experiment finds [18]

$$a_\mu(\mathrm{Expt}) = 116\,592\,080(54)(33) \times 10^{-11}, \quad (1)$$

where $a_\mu = (g-2)/2$ and the errors are statistical and systematic, respectively. This can be compared with the SM prediction [19]

$$a_\mu(\mathrm{SM}) = 116\,591\,802(42)(26)(02) \times 10^{-11}, \quad (2)$$

where the errors are from lowest-order—$\mathrm{O}(\alpha^2)$—hadronic contributions, higher-order hadronic contributions, and all others. The discrepancy between the measurement and the SM is 3.6σ. (There are many assessments of the SM contribution to $g-2$; here we take results from one source, Ref. [19], for convenience.)

The hadronic contributions enter at order $O(\alpha^2)$ (vacuum polarization) and $O(\alpha^3)$ (light-by-light scattering) in the fine-structure constant. The former are roughly 17,000 times smaller than the measured value, while the latter are roughly 1 million times smaller. Because the experimental error is so small, about 0.5 ppm, these contributions must be included accurately in order to test the SM or use the discrepancy as a probe of new physics. Moreover, the new Fermilab experiment E989 aims to reduce the uncertainty to 0.14 ppm, providing a challenge to theorists to reduce the hadronic uncertainty in a commensurate way.

The hadronic vacuum polarization (HVP) contribution to a_μ(SM) is computed from the two-point correlation function of the electromagnetic current of the quarks, Fourier transformed to 4-momentum space, and inserted into a one-loop QED integral for the interaction of the muon with an external field, as sketched in Fig. 1. It dominates the total SM uncertainty: QED and electroweak uncertainties are about 100 and 1000 times smaller, respectively, so they will not be relevant even for the new Fermilab experiment. The most accurate value for the HVP contribution [19]

$$a_\mu(\mathrm{HVP}) = 6923(42) \times 10^{-11} \tag{3}$$

is determined from a dispersion relation for the correlation function combined with the cross section for $e^+e^- \to$ hadrons. Using $\tau \to$ hadrons for part of the cross section leads to a somewhat smaller discrepancy, 2.4σ.

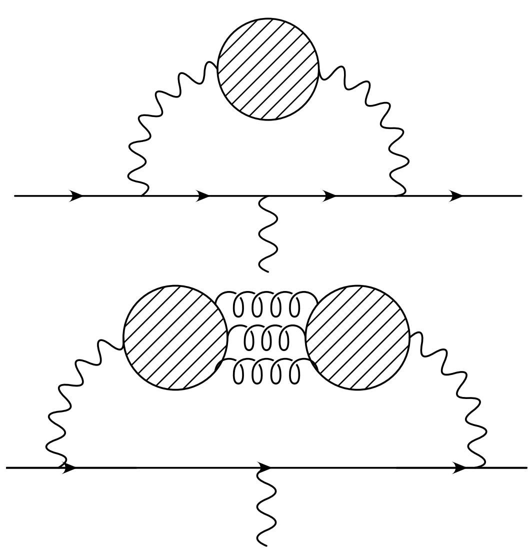

FIG. 1: The hadronic vacuum polarization diagrams contributing to a_μ(SM), the muon anomaly in the standard model. The horizontal lines represent the muon. Upper panel: the blob formed by the quark-antiquark loop represents all possible hadronic intermediate states. Lower panel: disconnected quark-line contribution, in which separate quark loops are connected by gluons.

The e^+e^--scattering and τ-decay determinations of a_μ(HVP) do not agree with each other very well. Some of the e^+e^- data sets do not agree especially well with each other. In order to insert the e^+e^- data into Fig. 1, radiative corrections must be applied. In order to apply τ-decay data, isospin correction must be taken into account, which can entail hadronic uncertainties [20, 21]. These difficulties can be circumvented by computing the HVP directly in QCD.

The HVP resides naturally in the spacelike, or Euclidean, momentum regime, making it a natural quantity to compute with lattice gauge theory [10]. Spurred by both the discrepancy and the prospects for further improvements in the measurement, several groups are calculating this quantity in QCD with $n_f = 2+1$ flavors of sea quarks [22, 23] and with $n_f = 2$ [24, 25]. The most commonly used method computes the vacuum polarization function at several values of spacelike momentum, fits the result to a smooth function and integrates over the loop momentum in Fig. 1. A conservative estimate of the current total accuracy from this technique is 5–10%. A recent calculation using only two flavors of sea quarks and a novel chiral extrapolation quotes a somewhat smaller 3% error [25]. This should be viewed as a proof of principle, and the same group is repeating their calculation with 4 flavors of sea quarks.

The dominant contribution to a_μ(HVP) comes from very low momenta, $p \sim m_\mu$ [10]. This is a challenge for numerical lattice QCD, because finite computer resources dictate a finite spatial volume, hence, discrete momenta. With typical box sizes $L \approx$ 2–3 fm, a unit of momentum $2\pi/L \approx$ 400–600 MeV. To reach momenta as small as m_μ, typically an extrapolation is carried out. The statistical errors increase as the momenta is decreased, making the extrapolation difficult. Alternatively, one can also use twisted-boundary conditions to enforce arbitrary momenta on the box [23, 24], but this does not improve the statistical errors on small momenta. The biggest difficulty is the sensitivity to the fit function used in the extrapolation to low momenta [22]. Methods to reduce this sensitivity have been proposed [23, 25]. It is also possible to access small time-like momenta on the lattice [26], and calculations of the pion form factor can be used to improve the low-momentum part of the dispersive calculation [27, 28]. Note, however, that there are many applications of lattice QCD that will profit from larger L, so these problems should become less severe even without better methods.

The computation of the hadronic-light-by-light (HLbL) contribution is much more challenging. The size of the HLbL contribution

$$a_\mu(\mathrm{HLbL}) = 105(26) \times 10^{-11}, \tag{4}$$

is not well known: it is based on a consensus on the size of various hadronic contributions estimated in several different models [29]. Its uncertainty, though less than that in a_μ(HVP), seems harder to reduce. Finding a new approach, such as lattice QCD, in which uncertainties are systematically improvable, is crucial for making greatest use of the new Fermilab experiment. With this in mind, E989 recently convened a workshop at the Institute for Nuclear Theory [11]. Workshop participants discussed how models, lattice QCD, and data-driven methods could be exploited to reduce the error on a_μ(HLbL). The outcome of this workshop is that a synthesis of all these methods can be used over the next five years to reduce the error on a_μ(HLbL) to 10% or less.

In the context of lattice QCD, two direct methods for computing the HLbL amplitude are under investigation. One approach, analogous to the HVP calculation, is to compute the correlation function of four electromagnetic currents in QCD and use it to do the two-loop QED integrals of the diagrams in Fig. 2. The second approach is to calculate the entire amplitude of Fig. 2 with lattice gauge theory, including both photons and leptons as lattice fields, even though the QED coupling constant is small.

In the first method, the four-point amplitude is computed for each possible momentum at each of the four electromagnetic vertices. The resulting four-index tensor is fit to a smooth function of the momenta, and plugged into the two-loop QED integrals. The momentum at the external vertex is taken to zero, so overall momentum conservation leads to two independent momenta to integrate over, a costly four-volume-squared computation. These calculations are still in an exploratory phase.

Alternatively, in the second method, the full amplitude shown in Fig. 2, including the muon and photons, is computed nonperturbatively [30]. The calculation proceeds just like a conventional QCD calculation, except the gauge link becomes a combined gluon and photon field [31]. Being a calculation that includes all orders in α, a nonperturbative subtraction of the leading QED contributions is required. By subtracting the product of muon and quark correlation functions from the original combined function, the HLbL contribution is obtained, plus terms of order $O(\alpha^4)$. The leading α^2 pieces in the original correlation function and subtraction are identical, so cancel completely, as do all α^3 contributions except the desired contribution from HLbL [30]. This method is under active research and development, as described below.

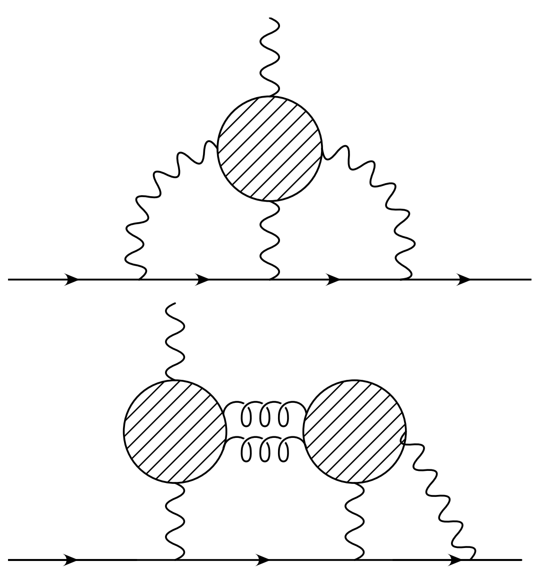

FIG. 2: The hadronic light-by-light scattering diagrams contributing to a_μ(SM), the muon anomaly in the standard model. The horizontal lines represent the muon. Upper panel: the blob formed by the quark loop represents all possible hadronic intermediate states. Lower panel: disconnected quark-line contribution, in which separate quark loops are connected by gluons.

In addition to these direct approaches, there is ongoing work on lattice-QCD calculations that check or supplement the model calculations. For example, it is well-known that the pion pole (namely, $\gamma\gamma^* \to \pi \to \gamma^*\gamma^*$) provides the largest contribution to the QCD blob in Fig. 2. Just as experiments are being mounted to examine this physics (*e.g.*, PrimEx at JLab and KLOE at LNF), several groups [26, 32, 33] are using lattice QCD to compute the amplitudes for $\pi \to \gamma\gamma^*$ and $\pi \to \gamma^*\gamma^*$ (with one or two virtual photons).

III. FORECASTS

In order to make forecasts of the precision attainable in future lattice-QCD calculations, one must begin with assumptions about the computing landscape. The discussion here is given from the perspective of the USQCD Collaboration, an umbrella organization that coordinates computing and software for the US lattice-gauge-theory community. Once assumptions about the cost and availability have been spelled out, we can proceed to forecasting the precision that can be attained for the most important flavor-physics matrix elements.

A. Assumptions

The DOE HEP and NP program offices fund dedicated computing resources for lattice gauge theory through the LQCD Infrastructure Project on behalf of the USQCD Collaboration. The Collaboration then allocates these computing resources to its members. This Project has a decade's experience constructing computing clusters based on Intel or AMD chips and has, thus, been able to measure the cost per delivered floating-point operation per second (flop/s). The measurements of flop/s delivered are based on real computations of two of the most popular quark propagators, and the performance relies on a low-latency network; in practice, recent clusters have used Infiniband. Over several years's experience, including our most recent clusters, `10q` and `Ds`, we remain on a trend line, for which costs halve in 1.6 years.

The LQCD Project received \$9.2M for four years, FY2006–2009. The Project is now being extended for another four years with approximately \$4M per annum. For our forecasts below, we assume that the budget will remain flat and that the costs will fall close to the historical trend line.

In addition to its dedicated high-capacity hardware, which is used mostly for physics analysis, USQCD has received significant resources from the DOE's leadership-class computing facilities at Argonne and Oak Ridge National Labs. USQCD has mostly used these high-capability resources to generate the ensembles of lattice gauge fields that underlie all physics projects. Groups within USQCD also receive significant resources on supercomputers at NERSC and at the centers supported through the NSF's XSEDE Program. Here we assume that USQCD and its members will receive a similar fraction of the planned upgrades to these facilities.

A further assumption we must make is for software support. The Moore's Law seen in our clusters has, lately, depended on new CPUs with many cores. In the last two years, a further hardware development has been use of graphics cards (GPUs), which have hundreds of cores. USQCD has been able to respond to these changes thanks to software funding from the DOE's SciDAC program. We assume that this, or equivalent, support will continue. Indeed, if GPUs live up to their early promise, the cost of computing may fall faster than our observed trend; we do not, however, assume such decreases here.

Until recently, lattice-QCD calculations of HVP and, especially, HLbL have consumed a small fraction of the computing resources available for lattice QCD. With more and more groups being motivated by the new experiment to address these problems, it seems natural that more computing resources will be devoted to these problems. However, even though we expect several teams to be working on $g-2$, here we do *not* reduce error forecasts by imagining an average over several results. We take the perspective that lattice-QCD calculations are still in an era where cross-checks are affordable and desirable. Methods for the HLbL contribution, in particular, are still developing.

Finally, we assume that funding to support junior researchers—graduate students, postdocs, and junior faculty—does not decrease.

B. Forecasts

To match the expected uncertainty of the new Fermilab experiment, the theoretical errors on both a_μ(HVP) and a_μ(HLbL) will have to be reduced to the level of 10×10^{-11}. This corresponds to reducing the error on the HVP contribution by a factor of 4, and the error on the HLbL contribution by a factor of 2.5.

The sources of uncertainty on the HVP are typical in lattice calculations: statistical, finite volume, nonzero lattice spacing, and (chiral) extrapolation to the physical quark masses. We briefly discuss each.

For the HVP there is no obstacle in attaining sub-percent statistical precision (except for the so-called disconnected diagram shown in the lower panel of Fig. 1, which we discuss below). We assume that all future calculations will achieve this.

Nonzero lattice spacing errors are small with modern lattice actions, and several (three or more) spacings can be used to essentially eliminate these by extrapolation.

Next, systematic uncertainty stems from the chiral extrapolation to the physical quark masses. The quark mass dependence of a_μ(HVP) is very large [22–24], owing to the ρ pole in the timelike region. A recent 2-flavor calculation [25] has shown that adjusting the kernel in the QED one-loop integral dramatically reduces the dependence of a_μ(HVP) on the quark mass [25]. Furthermore, lattice-QCD calculations at physical quark mass, which are now starting [34–36], avoid these issues.

Finally, there are so-called disconnected diagrams—lower panel, Fig. 1—that have not yet been included in lattice-QCD calculations of the HVP. (In lattice jargon, quark lines or loops connected only by gluons, are called disconnected, because the avergage over gauge fields includes all gluons as a matter of course.) Their contribution cancels in the SU(3) flavor limit and is Zweig-supressed. One group found them to be zero within statistical errors [25], but in chiral perturbation theory their value is known to be a significant fraction of the connected contribution of the pions [37], so more work is needed to nail down their contribution which could be as large as 1–2%. Low eigenmode methods, which are becoming standard, should help significantly here.

In summary, the systematic errors are well understood, and with concerted effort and sufficient computational resources, as outlined above, they will be brought under full control. Taking all of the above into account, we see no obstacle to reduce the lattice-QCD uncertainty on a_μ(HVP), currently $\sim$ 5%-level, down to the 1–2% level within the next 3–5 years. This stage is already interesting, because it could help resolve tension between the timelike methods for determining a_μ(HVP). With increasing experience and computer power, it should be possible to compete with the e^+e^- determination of a_μ(HVP) by the end of the decade.

The state of the HLbL calculation is less well understood, and the calculations are still at a very early stage. The LbL amplitude in pure QED has been calculated using the nonperturbative method [30] described in Sec. II. The result is consistent with well-established perturbative results, albeit with large finite volume corrections since the test was done on a small lattice. Subsequent tests on a larger volume show the finite-volume errors are manageable. Using the same method, pilot calculations for the HLbL amplitude appear promising: statistically significant results are obtained. Done at unphysical quark and muon masses, and at non-zero momentum transfer, these are not yet directly comparable to model calculations, but on-going calculations are moving in this direction. As with the HVP, the quark-line disconnected diagram shown in Fig. 2 has not yet been calculated. Except for being Zweig-supressed, there is no reason to expect its contribution to be small compared to the connected one.

For the conventional method described in Sec. II, all-to-all propagator techniques are being developed to tackle the four-point vector-current correlation function. Exploratory calculations are needed to begin to understand the systematics and required level of statistics. It is also cumbersome in a mundane way: the four-point function has over 100 Lorentz structures, of which 32 contribute to $g-2$. With no real benchmark calculations as a starting point, it is hard to forecast the level of accuracy on the HLbL amplitude. That said, this challenging problem is attracting considerable attention: reducing the error on a_μ(HLbL) to 10% within five years (with lattice QCD alone) is neither guaranteed nor out of the question.

Short of a full lattice-QCD calculation of a_μ(HLbL), there are intermediate steps that are less expensive and will be taken along the way. First, one may simply compute only a few, well-chosen values of the momenta and use these to check model calculations. With enough values, a smooth function may be fit to them to fill in the rest, assuming a well-motivated functional form can be found. Second, the more straightfoward computations of the pion to off-shell photons, described in Sec. II, will provide data for the models, in a way complementary to the two-photon experimental data.

We close by reiterating that lattice QCD can point to success over a wide range of problems, and the challenge that $g-2$ presents is generating many ideas. It may well be that the idea that solves the problem is not yet available.

IV. OUTLOOK

The intensity frontier complements high-p_T physics in at least two ways. Observations discrepant with the Standard Model are discoveries in their own right. More generally, precise measurements offer constraints on the identity of high-mass particles, such as those that may be observed at the LHC.

The interpretation of precise experiments at the intensity frontier requires comparable precision in the corresponding theoretical calculations. In many experiments at the intensity frontier, hadrons are involved in an essential way, leading inevitably to the need to calculate hadronic properties, in particular matrix elements of operators that arise when integrating out short-distance SM or BSM particles. Even in some leptonic observables, the precision is such that virtual hadrons make a significant contribution. Lattice gauge theory provides a set of numerical methods for computing these hadronic properties, within a framework where uncertainties can be systematically reduced.

In the past several years, the right combination of algorithms, computing power and infrastructure, and collaboration structure has come together, leading to a plethora of results. Some of these results are quantitatively impressive and bode well for future experiments at the intensity frontier. Others are qualitatively interesting and connect to the energy and cosmic frontiers. We see special opportunities in quark flavor physics, nucleon matrix elements, and muon $g-2$. With continued support, we look forward to the coming decade's interplay between experiment, theory, and lattice QCD.

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