LATTICE GAUGE THEORIES AT THE ENERGY FRONTIER

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I. EXECUTIVE SUMMARY

This report on *Lattice Gauge Theory at the Energy Frontier* is a companion document to three other reports: *Lattice QCD at the Intensity Frontier*, *Lattice QCD for Cold Nuclear Physics*, and *Computational Challenges in QCD Thermodynamics*. Together they address prospects in the next five years to advance lattice field theory calculations to support the high-energy physics and nuclear physics programs.

Lattice based studies of strongly coupled gauge theories proposed for the description of physics beyond the standard model (BSM) have accelerated in recent years. New numerical and theoretical tools have enabled physicists to address a broad range of theories that could, for example, lie at the heart of the dynamics of the Higgs effective theory of electroweak symmetry breaking. With the dramatic discovery at the LHC of a 126 GeV Higgs-like particle, lattice BSM research is moving to the study of mechanisms for the origin of a Higgs scalar field that are consistent with this discovery. Rather than accepting an elementary Higgs boson with no new physics at accessible scales, lattice BSM research explores its possible origin from TeV-scale physics involving compositeness and new symmetries. Studying these theories requires non-perturbative lattice methods, which build directly on substantial USQCD lattice-BSM accomplishments of recent years:

- ◦ Investigations of strongly coupled BSM gauge theories identified conformal or near conformal behavior, demonstrating that the anomalous mass dimensions and chiral condensates are enhanced near conformality, with interesting implications for model building.
- ◦ Electroweak precision experimental constraints were compared with numerical estimates of the S-parameter, W-W scattering, and the composite spectra. In particular in contrast with naive estimates, these studies demonstrate that the S-paramenter in near-conformal theories may be reduced in better agreement with experimental constraints.
- ◦ Investigations of $\mathcal{N} = 1$ supersymmetric Yang Mills theory (gauge bosons and gauginos) produced estimates of the gluino condensate and string tension in these theories.

Building on these significant and computationally demanding accomplishments, the USQCD BSM program has developed three major directions for future lattice BSM research with well-defined calculational goals:

- To determine whether a composite dilaton-like particle or light Higgs can emerge in near-conformal quantum field theories.
- To investigate strongly coupled theories with a composite Higgs as a pseudo-Goldstone boson.
- To investigate the nature of $\mathcal{N} = 1$ SUSY breaking with matter multiplets and $\mathcal{N} = 4$ conformal SUSY as a test bed for AdS/CFT theoretical conjectures.

For each, we describe challenges and prospects with an emphasis on the broad range of phenomenological investigations underway and the development of new lattice-field-theory methods targeted at BSM research. We identify the computational resources needed to make major contributions to our understanding of gauge theories and to apply the theoretical framework to illuminate experiments at the energy frontier.

II. INTRODUCTION

From its inception in the 1970s, lattice gauge theory has been employed with great success to study QCD, the one strongly coupled gauge theory known to describe the real world. The remarkable progress of recent years includes increasingly precise computations of hadron structure, finite-temperature behavior, and the matrix elements crucial for precision tests of the standard model (SM).

During this same period, the many questions posed by the standard model have led theorists to propose and study a vast array of other strongly coupled gauge theories. They have included QCD-like theories, supersymmetric (SUSY) gauge theories, conformal field theories, and others. These beyond-standard-model (BSM) theories could play a role in extending the standard model to ever higher energies, and perhaps lead to an understanding of the many parameters of the standard model. Theorists have envisioned these theories to describe a range of phenomena including electroweak symmetry breaking, SUSY breaking, and the generation of the flavor hierarchies of the standard model. While not yet making definitive contact with experimental measurement, this work has given birth to many ideas for exploring SUSY and non-SUSY gauge field theories. Approaches such as degree-of-freedom based constraints on renormalization-group flow, duality and AdS/CFT constraints on SUSY gauge theories, and symmetry-based constraints on conformal field theories have provided important new insights.

In recent years, lattice simulations have taken a prominent place among these tools. They have been brought to bear on a variety of strongly coupled gauge field theories, exploring QCD-like theories extensively and laying the groundwork for similar studies of SUSY gauge theories. For certain QCD-like theories, dramatic quantitative changes have been observed as the number of light fermions is increased, or their color representation is changed, showing a much smaller relative scale for vacuum chiral symmetry breaking. In fact, these same studies raise the strong possibility of the emergence of conformal symmetry as the fermion content is increased. Rather than confining and spontaneously breaking chiral symmetries as in QCD, these theories may exhibit an exact or approximate infrared conformal symmetry. The nature of this transition, including the spectrum of particle states and other properties, is the focus of intensive, ongoing research. If these theories play a role in electroweak breaking, then these transitional features could have immediate measurable consequences. Lattice simulation of SUSY gauge theories will be especially exciting, making contact with other powerful approaches to these theories.

Lattice-based work on BSM theories is already being powerfully shaped by the dramatic discovery at the Large Hadron Collider (LHC) of a 126 GeV Higgs-like particle. An immediate question is whether a strongly-coupled gauge theory can produce a composite Higgs boson, which is light relative to all the heavier resonances on the TeV scale. This parametric light scalar maybe an approximate Nambu-Goldstone boson associated either with the breaking of an approximate conformal symmetry or with the spontaneous breaking of a global symmetry. Both of these possibilities are receiving much recent attention.

As lattice-based studies of BSM theories have blossomed in recent years, a whole set of simulation and analysis tools have been developed, sharpened, and shared. They are widely applicable, and will no doubt continue to evolve and enable BSM studies as new possibilities and directions are identified. All of this progress and promise for the future depends critically on the availability of large-scale computational resources. It is an investment that is beginning to pay off, promising to deepen our understanding of gauge field theories and substantially increasing our ability to interpret experiments at the energy frontier.

III. LATTICE BSM RESEARCH

The recent discovery of the Higgs-like resonance at 126 GeV by the CMS [1] and ATLAS [2] experiments at the Large Hadron Collider (LHC) provides the first insight into the origin of electroweak symmetry breaking (EWSB) in the standard model. The minimal realization of EWSB is implemented by introducing an SU(2) doublet scalar Higgs field whose vacuum expectation value, v = 246 GeV, sets the electroweak scale, or VEV. This simple "Mexican-hat" solution should be regarded as a parametrization rather than a dynamical explanation of EWSB. In particular, the mass-squared parameter of the light Higgs has to be finely tuned, leading to the well-known hierarchy problem. Searching for a deeper dynamical explanation, and resolving the shortcomings of the minimal standard model with its elementary Higgs doublet, the USQCD BSM program has developed three research directions. One employs strongly coupled gauge theories near conformality [3–33], another envisions the new particle as a light pseudo-Goldstone boson (PNGB) in the spirit of little Higgs scenarios [34–41], and the third begins to explore SUSY gauge theories. In each of these three research directions, new degrees of freedom are expected at the TeV scale.

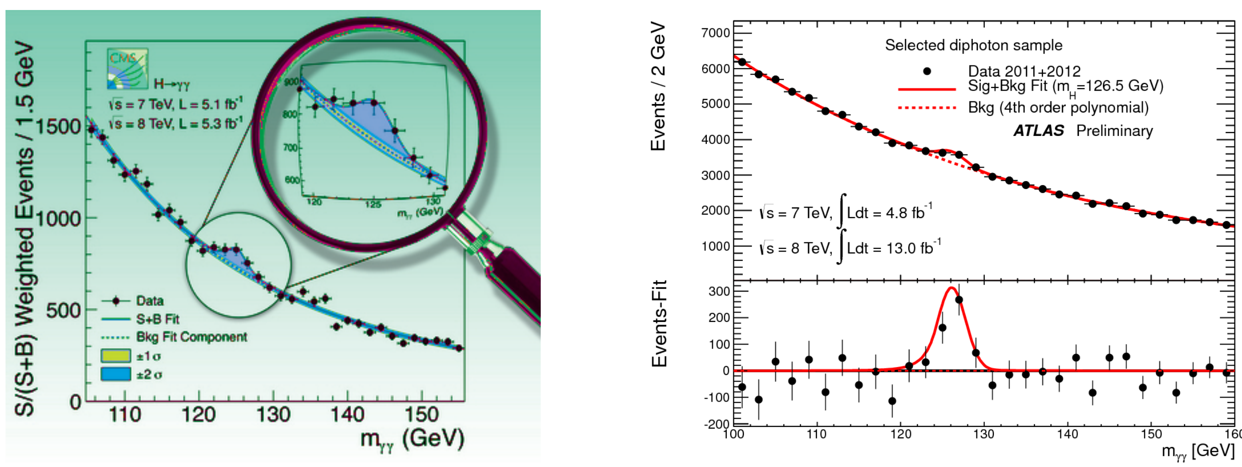

FIG. 1. The plots illustrate the Higgs discovery of the CMS and ATLAS collaborations in the prominent two-photon decay channel. On the right, ATLAS data is shown for the invariant mass distribution of diphoton candidates for the combined $\sqrt{s} = 7$ TeV and $\sqrt{s} = 8$ TeV data samples. On the left, the two-photon signal of the CMS collaboration is shown.

The dramatic LHC discovery presents a challenge for four-dimensional BSM gauge theories where the electroweak symmetry is broken via strong dynamics. Can such theories lead to a naturally light and narrow Higgs-like state in the scalar spectrum with 0^{++} quantum numbers? An approach to the viability of this class of models, the main focus of the first two of the three current BSM directions, envisions strong dynamics that could produce a Higgs-like light and narrow scalar composite state with 0^{++} quantum numbers. One possibility is that an underlying strongly coupled gauge theory could produce this state as a dilaton, the PNGB of conformal symmetry breaking. The minimal standard model itself, with a light Higgs particle, has this approximate symmetry at the classical level. The landscape of strongly interacting gauge theories also provides intriguing chiral symmetry-breaking patterns in which the light state can exist as a PNGB of an approximate expanded global symmetry, similar in spirit to the little Higgs scenarios. In addition, the landscape of SUSY gauge theories includes many attractive features, among them natural flat directions which can make plausible the dilaton interpretation of the 126 GeV particle.

A. Highlights of Recent Results and Future Goals

Rather than accepting an elementary Higgs boson with no new physics at accessible scales, lattice BSM research explores its possible origin from TeV-scale physics involving compositeness and new symmetries. Studying these theories requires non-perturbative lattice methods, which build directly on substantial lattice-BSM accomplishments of recent years:

- Investigations of strongly coupled BSM gauge theories identified conformal or near conformal behavior, demonstrating that the anomalous mass dimensions and chiral condensates are enhanced near conformality, with interesting implications for model building.
- Electroweak precision experimental constraints were compared with numerical estimates of the S-parameter, W-W scattering, and the composite spectra. In particular in contrast with naive estimates, these studies demonstrate that the S-paramenter in near-conformal theories may be reduced in better agreement with experimental constraints.
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Building on these significant and computationally demanding accomplishments, the USQCD BSM program has developed three major directions for future lattice BSM research with well-defined calculational goals:

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For each we describe challenges and prospects with an emphasis on the broad range of phenomenological investigations underway and the development of new lattice-field-theory methods targeted at BSM research. We identify the computational resources needed to make major contributions to our understanding of gauge theories and to prepare the theoretical framework to illuminate experiments at the energy frontier.

Each of our three BSM research directions is supported by an expanding USQCD BSM lattice tool set, including techniques and phenomenological approaches (such as chiral perturbation theory) which are useful for all three directions. In this white paper, we first present in sections IV-VI the physics motivation and current status of each of the major research directions: Higgs as a dilaton, Higgs as a PNGB, and supersymmetric theories. In section VII we give an overview of phenomenological applications, generally using methods developed in QCD simulations. Section VIII presents new methods under development specifically for the study of BSM theories on the lattice, while section IX identifies computational resources needed during the next five years for important representative projects of the three research directions.

IV. THE LIGHT HIGGS AND THE DILATON NEAR CONFORMALITY

It is a truth universally acknowledged that at high energies particle masses become unimportant. In the absence of electroweak symmetry breaking, the interactions of standard-model gauge bosons and fermions show approximate conformal symmetry down to the QCD scale. In this sense, electroweak symmetry breaking in the standard model appears together with the dynamical breaking of scale invariance. This opens up the possibility that the Higgs mode and the dilaton mode, the pseudo-Goldstone boson of spontaneously broken scale invariance, are perhaps intimately related. The important properties of the standard-model Higgs boson are basically determined by the approximate conformal invariance in the limit when the Higgs potential is turned off. In this case the Higgs VEV is arbitrary (it is a flat direction), and its value at v = 246 GeV will spontaneously break the approximate conformal symmetry and the electroweak symmetry. In this limit the Higgs particle can be identified with the massless dilaton of conformal symmetry breaking [27] at the scale $f_\sigma = v$. Higgs properties in this weak coupling scenario are associated with approximate conformal invariance.

The possibility of approximate conformal symmetry and its breaking in strong gauge dynamics motivates intriguing BSM possibilities close to the conformal window. The onset of the chiral condensate with electroweak symmetry breaking and the spontaneously broken conformal invariance with its dilaton might create a scenario consistent with the standard-model Higgs mechanism and the light Higgs impostor, in a natural setting and with minimal tuning [28, 31, 33] .

The knowledge of the phase diagram of nearly conformal gauge theories is important as the number of colors N_c, number of fermion flavors N_f , and the fermion representation R of the color gauge group are varied in theory space. For fixed N_c and R the theory is in the chirally broken phase for low N_f and asymptotic freedom is maintained with a negative β-function. On the other hand, if N_f is large enough, the β-function is positive for all couplings, and the theory is trivial. The conformal window is defined by the range of N_f for which the β-function has an infrared fixed point, where the theory is in fact conformal.

Close to the conformal window, BSM gauge theories with strong dynamics have a broad appeal. If the strongly coupled BSM gauge model is very close to the conformal window with a small but nonvanishing β-function, a necessary condition is satisfied for spontaneous breaking of scale invariance and generating the light PNGB dilaton state. Since these models exhibit chiral symmetry breaking (χSB) a Goldstone pion spectrum is generated and when coupled to the electroweak sector, the onset of electroweak symmetry breaking with the Higgs mechanism is realized. The very small beta function (walking) and χSB are not sufficient to guarantee a light dilaton state if scale symmetry breaking and χSB are entangled in a complicated way.

It is far from clear that the dilaton mechanism is naturally realized in strong dynamics. However, a light Higgs-like scalar might still be expected to emerge near the conformal window as a composite scalar state with 0^{++} quantum numbers, not necessarily with dilaton interpretation. This scalar state has to be light and would not be required to exhibit exactly the observed 126 GeV mass. The dynamical Higgs mass $\mathrm{M}_{\mathrm{H}}^{0^{++}}$ from composite strong dynamics is expected to be shifted by electroweak loop corrections, most likely dominated by the large negative mass shift from the top quark loop [42]. The Higgs mass emerging from strong gauge dynamics cannot be very heavy, but an $\mathrm{M}_{\mathrm{H}}^{0^{++}}$ in the range of several hundred GeV, before electroweak corrections shift it downward, should not be dismissed for viable model building.

A high priority goal for non-perturbative BSM lattice studies of the USQCD collaboration is to determine the scalar spectrum of strong gauge dynamics with Higgs quantum numbers, close to the conformal window. This presents an enormous challenge in BSM lattice simulations, and the first pilot studies are just beginning to emerge, outlining a roadmap toward this goal. Once the scalar spectrum is established, the possibility of an intriguing connection with the dilaton mechanism can be further investigated with the potential of important phenomenological consequences.

A. The 0^{++} Scalar Spectrum in BSM Lattice Models

The generic features and the required resources for the scalar spectrum to include a state as light as the experimentally observed Higgs particle are illustrated with a pilot study of a frequently discussed BSM gauge theory with twelve fermion flavors in the fundamental representation of the SU(3) color gauge group. With $\mathrm{N_f} = 12$ being close to the critical flavor number of the conformal edge, the model has attracted a great deal of attention in the lattice community [43–56], and off-lattice as well. Although the precise position of the model with respect to the conformal window remains unresolved, the required lattice technology to capture the scalar spectrum in BSM lattice models is quite robust in a model independent way. In Section 8 the resource estimate will be presented for another frequently discussed strongly interacting gauge theory with a fermion flavor doublet in the two-index symmetric (sextet) representation of the SU(3) color gauge group [57–59]. Close to the conformal edge, the scalar spectrum of this model is being explored for the minimal realization of the composite Higgs mechanism.

In BSM models, close to the conformal window, a low mass 0^{++} glueball is expected to mix with the composite scalar state of the fermion-antifermion pair. With the ultimate goal of reaching to the chiral limit of massless fermions, the mixing in the scalar spectrum has to include Goldstone pairs with vacuum quantum numbers and exotic states made of two fermions and two antifermions with 0^{++} quantum numbers. Realistic studies require a 3-channel solution, even if exotica are excluded from the analysis. The pilot study presented here for future planning is restricted to the single channel problem using scalar correlators which are built from connected and disconnected loops of fermion propagators [60].

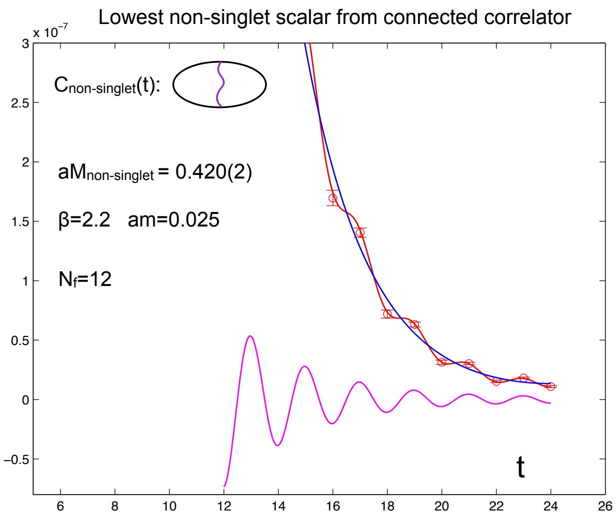

FIG. 2. The fermion loops were evaluated using stochastic methods with full time dilution and 48 noise vectors on each gauge configuration [60]. The correlator $\mathrm{C_{conn}(t)}$ on the left plot and the correlator $\mathrm{C_{singlet} = C_{conn} + C_{disc}(t)}$ on the right plot were assembled from the stochastic fermion propagators. The left side plot shows the mass of the lowest non-singlet scalar (blue exponential fit). The plot also displays the oscillating pseudo-scalar parity partner (magenta) and the full correlator (red) fitting the data. On the right side plot, with larger errors in the limited pilot study, the scalar singlet mass is considerably downshifted (blue exponential) and the presence of a pseudo-scalar parity partner is not detectable. The conventional $\beta = 6/g^2$ lattice gauge coupling, setting the lattice spacing a, is shown in addition to the finite fermion mass am of the simulation.

The staggered lattice fermion formulation is deployed in the pilot study to demonstrate feasibility with control of χSB and serves as a lower bound for the required resources. Domain wall fermions would be 10-20 times more demanding. The Symanzik improved tree level gauge action is used with stout smeared gauge links to minimize lattice cut-off effects in the study. A staggered operator which creates a state that lies in the spin-taste representation $\Gamma_S \otimes \Gamma_T$ also couples to one lying in the $\gamma_4\gamma_5\Gamma_S \otimes \gamma_4\gamma_5\Gamma_T$ representation. Thus a staggered meson correlator has the general form

$$\mathrm{C(t)} = \sum_{\mathrm{n}} \left[\mathrm{A_n e^{-m_n(\Gamma_S\otimes\Gamma_T)t}} + (-1)^{\mathrm{t}}\mathrm{B_n e^{-m_n(\gamma_4\gamma_5\Gamma_S\otimes\gamma_4\gamma_5\Gamma_T)t}}\right]$$

with oscillating contributions from parity partner states. For the scalar meson ($\Gamma_S \otimes \Gamma_T = 1 \otimes 1$), the parity partner is $\gamma_4\gamma_5 \otimes \gamma_4\gamma_5$ which corresponds to one of the pseudoscalars in the analysis. For flavour singlet mesons, the correlator is of the form $\mathrm{C(t) = C_{conn}(t) + C_{disc}(t)}$ where $\mathrm{C_{conn}(t)}$ is the correlator coupled to the non-singlet meson state and $\mathrm{C_{disc}(t)}$ is the contribution of disconnected fermion loops in the annihilation diagram. Figure 2 on the left shows the propagation of the lowest flavor-nonsinglet state together with its oscillating parity partner, as determined by $\mathrm{C_{conn}(t)}$. The singlet scalar mass, the Higgs particle of the strongly coupled gauge model, is determined from the flavor singlet correlator $\mathrm{C(t)}$

that includes the annihilation diagram on the right side of Figure 2. The result shows a significant downward shift of the Higgs mass from the annihilation diagram of the scalar singlet correlator, as expected close to the conformal edge. Very large scale simulations are needed with substantial resources to confirm the emerging picture of this pilot study.

Once the technology of scalar spectra is under control, the USQCD BSM groups plan to investigate the dynamics of strongly coupled BSM gauge theories that could be responsible for producing a light dilaton-like state. This possibility remains consistent with the preliminary values of the observed decay modes of the Higgs-like resonance discovered at the LHC. Using symmetry arguments, the dilaton couplings are generically $\mathrm{v}/\mathrm{f}_\sigma$ suppressed compared to standard model Higgs couplings. Fermion couplings are also modified by the anomalous dimensions of the SM fermions. Couplings to massless gauge bosons are loop induced and are determined by the β-function coefficients of the composites. The coupling to the W and Z can be made realistic and consistent with electroweak precision tests only if the scale of conformal breaking is close to the scale of electroweak symmetry breaking with $\mathrm{v}/\mathrm{f}_\sigma \approx 1$.

B. The PCDC Dilaton Relation on the Lattice

The USQCD BSM groups plan to deploy non-perturbative lattice methods to explore the implications of the partially conserved dilatation current (PCDC) relation when applied to four-dimensional BSM gauge models. The PCDC relation connects the dilaton mass m_σ and its decay amplitude f_σ with the non-perturbative gluon condensate. For discussion of the PCDC relation constraining the properties of the dilaton, standard arguments, like in [28, 31, 33] can be followed.

In strongly interacting gauge theories, like the sextet model under active USQCD BSM investigation [57, 61], a dilatation current $\mathcal{D}^\mu = \Theta^{\mu\nu} x_\nu$ can be defined from the symmetric energy-momentum tensor $\Theta^{\mu\nu}$. Although the massless theory is scale invariant on the classical level, from the scale anomaly the dilatation current has a non-vanishing divergence, $\partial_\mu \mathcal{D}^\mu = \Theta^\mu_\mu = \frac{\beta(\alpha)}{4\alpha} G^a_{\mu\nu} G^{a\mu\nu}$. While $\alpha(\mu)$ and $G^a_{\mu\nu} G^{a\mu\nu}$ depend on the renormalization scale μ, the trace of the energy-momentum tensor is scheme independent after renormalization. In BSM models, the massless fermions are in the fundamental representation, or in the two-index symmetric representation of the SU(N) color gauge group in most studies.

If we assume that the perturbative parts of the composite operators $G^a_{\mu\nu} G^{a\mu\nu}$ and Θ^μ_μ are removed, only the non-perturbative (NP) infrared parts have to be considered in $\partial_\mu \mathcal{D}^\mu$. The dilaton mass m_σ and its decay amplitude f_σ are defined by the vacuum to dilaton matrix element of $\partial_\mu \mathcal{D}^\mu$, $\langle 0|\partial_\mu \mathcal{D}^\mu(x)|\sigma(p)\rangle = f_\sigma m_\sigma^2 e^{-ipx}$. The important partially conserved dilatation current (PCDC) relation,

$$m_\sigma^2 \simeq -\frac{4}{f_\sigma^2} \langle 0| \Big[\Theta^\mu_\mu(0)\Big]_{NP} |0\rangle \,,$$

then connects m_σ and f_σ to the nonperturbative gluon condensate close to the conformal window [28].

Predictions for m_σ when close to the conformal window depend on the behavior of f_σ and the non-perturbative gluon condensate $G^a_{\mu\nu} G^{a\mu\nu}|_{NP}$. There are two different expectations about the limit of the gluon condensate to f_σ ratio when the conformal window is approached. In one interpretation, the right-hand side of the PCDC relation is predicted to approach zero in the limit, so that the dilaton mass $m_\sigma^2 \simeq (N_f^c - N_f) \cdot \Lambda^2$ would parametrically vanish when the conformal limit is reached. The Λ scale is defined where the running coupling becomes strong to trigger χSB. The formal parameter $N_f^c - N_f$ with the non-physical (fractional) critical number of fermions vanishes when the conformal phase is reached [28]. In an alternate interpretation the right-hand side ratio of the PCDC remains finite in the conformal limit and a residual dilaton mass is expected when scaled with $f_\sigma \simeq \Lambda$ [31, 33].

It is important to note that there is no guarantee, even with a very small β-function near the conformal window, for the realization of a light enough dilaton to act as the new Higgs-like particle. Realistic BSM models have not been built with parametric tuning close to the conformal window. For example, the sextet model is at some intrinsically determined position near the conformal window and only non-perturbative lattice calculations can explore the physical properties of the scalar particle.

The lattice determination of the non-perturbative gluon condensate is important in exploring the consequences of the PCDC relation. Power divergences are severe in the calculation of the lattice gluon condensate, because the operator $\alpha G^a_{\mu\nu} G^{a\mu\nu}$ has quartic UV divergences. If the gluon condensate is computed on the lattice from the expectation value of the plaquette operator U_P the severe UV divergences make the separation of the finite NP part difficult, or even suspect on theoretical grounds. If it turns out that the meaningful separation of the gluon condensate into perturbative and non-perturbative parts exists, as suggested by some recent developments, it will be very important to undertake these investigations in BSM gauge models with full fermion dynamics. One of the most interesting recent developments relates the energy from gradient flow to $G^a_{\mu\nu} G^{a\mu\nu}|_{NP}$ on lattice configurations at small flow times [62, 63]. The USQCD BSM groups plan to investigate this important alternative approach.

V. HIGGS AS A PSEUDO-GOLDSTONE BOSON

With the discovery of a new particle at 126 GeV with properties so-far consistent with the standard model Higgs boson, it is natural to explore strong dynamics where this Higgs is a scalar pseudo-Nambu-Goldstone boson (PNGB). This PNGB Higgs mechanism [64] plays a crucial role in little Higgs [65, 66] and minimal conformal technicolor [67] models. In little Higgs theory, an interplay of global symmetries is devised in an effective Lagrangian to cancel the quadratic divergences for the Higgs mass to one loop. This provides a weakly coupled effective theory with little fine tuning up to energies on order of 10 TeV. A large range of phenomenologically interesting models continues to be explored as weakly coupled extensions of the standard model with PNGB Higgs scalars.

Lattice field theory can explore realizations of this approach, starting with an ultraviolet complete theory for strongly interacting gauge theories with fermions in real or pseudo-real representations of the gauge group giving rise to Goldstone scalars. There are no ultraviolet divergences in this strongly interacting sector, and in the chiral limit the Higgs scalar is a member of set of Goldstone Bosons that includes the triplet of "techni-pions", required to give masses to the W and Z. Subsequently the mass of this Higgs is naturally light with its mass induced by small couplings to the weak sector. A central challenge to support this scenario for models based on effective phenomenological Lagrangian is to use lattice field theory to demonstrate that viable UV complete theories exist and to understand the impact of extending the standard model with this strong sector replacing the weakly coupled elementary Higgs.

A. Minimal PNGB Model

The minimal PNBG Higgs model consists of a $SU(2)$ color gauge theory with $N_f = 2$ fundamental massless fermions. Additional sterile flavors with $N_f > 2$ can be added [67] to drive the theory close to or into the conformal window. Because of the pseudo-real properties of $SU(2)$ color group, the conventional $SU(N_f)_L \times SU(N_f)_R$ vector-axial symmetry becomes a larger $SU(2N_f)$ flavor symmetry combining the $2N_f$ left/right 2-component chiral spinors. Most-attractive-channel arguments suggest that $SU(2N_f)$ will break dynamically to $Sp(2N_f)$. If explicit masses are given to $N_f - 2$ flavors, the remaining 2 massless flavors yield the $SU(4)/Sp(4)$ coset with 5 Goldstone Bosons: the isotriplet pseudo-scalars (or "techni-pions") to give mass to the W, Z, and two isosinglet scalars. With additional explicit breaking, these two extra Goldstone Bosons become massive. The lighter is a candidate for a composite Higgs and the heavier is a possible candidate for dark matter.

For the lattice formulation, it is convenient to use the standard four-component Dirac spinors for the vector-like theory. It can be shown that the most general mass matrix in the $SU(4)$ symmetric two-component chiral representation is equivalent to the up and down quark masses of the Dirac four- component theory, up to $SU(4)$ transformations plus a theta term to absorb the complex phase. (See Kogut et al [68] for the basic formalism required for this demonstration.) As in QCD one fixes the vacuum theta angle to zero to avoid CP violation. Standard lattice methods are suited to investigate this model at this stage. As a first step, lattice calculations for $N_f = 2$ have been performed [69] using Wilson fermions that give non-perturbative support to the breaking pattern, $SU(4) \to Sp(4)$, favored by the most attractive channel argument in the chiral limit.

However this is still not the correct vacuum alignment. With the fermion mass parameters, all 5 GBs are given a common mass. To lift the two scalars keeping the isotriplet pseudo-scalars massless, one must appeal to the weak interactions coupling the Higgs composite to the top quark. As demonstrated by the use of the chiral Lagrangian [66, 67] in the 5-Goldstone sector, the top quark loop destabilizes the vacuum alignment in $Sp(4)$ shifting to a new vacuum preserving the 3 pseudoscalars (π_T) but lifting the 2 scalars as required. The CP =1 scalar is identified as a composite Higgs ($\mathbf{h}$), and the CP = -1 scalar ($\mathbf{A}$) is a possible candidate for dark matter. The new alignment includes an alignment angle θ determined by the ratio of the top quark loop and the mass matrix that fixes the mass of the Higgs relative to the additional scalar. The alignment angle interpolates between a weakly coupled effective low energy theory and a strongly coupled composite Higgs depending on the ratio of the Higgs vacuum expectation value ($\mathbf{v}$) to the chiral Lagrangian coupling parameter $\mathbf{f}$. For $\mathbf{v}/\mathbf{f} = \sin\theta$ small, with the weak scale set by $\mathbf{v}$ = 246 GeV, after the "techni-pions" are "eaten" by W and Z, the "dark matter" scalar ($m_A^2 = m_h^2/\sin^2\theta$) along with all other composites are heavy at the new strong scale set by $\mathbf{f}$. The theory approximates the standard model following the methods of the little Higgs approach. For large $\mathbf{v}/\mathbf{f} = \mathbf{O}(\mathbf{1})$, the model is typical of strong coupling technicolor with the massive PNGB scalar merging with a rich technicolor composites. In between there is a range of finely tuned theories worthy of careful study.

It is important to confirm rigorously the above scenario as the correct description of the low energy effective theory in the context of *ab initio* non-perturbative lattice calculation. To include the effect of the top quark into the lattice simulations, one must add the four Fermi term induced by the top quark loop. The four Fermi term is then recast as a quadratic term coupled to a Gaussian auxiliary field. It is an open question whether or not the four-fermion operator requires an explicit cut-off or whether it becomes a relevant operator due to a large anomalous dimension. A central challenge is to use lattice field theory to demonstrate that UV complete theories are viable with PNGB scalars consistent with current experimental constraints.

B. Lattice Formulation

In the continuum there is a rather large range of models proposed for strongly interacting gauge theories with a candidate PNGB for the Higgs. However lattice formulations and numerical simulations for each new theory require a substantial investment in software and algorithmic development, so it is reasonable to start with the simplest example. Fortunately there is already considerable experience with this two-color, two-flavor lattice gauge theory example. In fact lattice gauge theories with SU(2) color have been studied extensively as toy models for QCD at finite chemical potential because, unlike SU(3) QCD, the chemical potential does not introduce a sign problem coming from the Fermion determinant. It is well known that these sign problems, when they occur, usually make lattice studies extremely difficult if not impossible.

A similar difficulty might present itself for this study. For the PNGB application, we must consider two new mass-like extensions to the the lattice action. One is the introduction of mass term for the diquark and the other is the four Fermi term induced by the top-quark loop. Remarkably, each has been studied to some degree. For example the diquark condensate is analyzed in Kogut et al [70], and Hands et al have demonstrated [71, 72] that adding the diquark term to the Wilson or staggered action does not cause a sign problem. In the case of the four-Fermi term, this must be recast via a Lagrange multiplier as local bilinear mass like term. Again, a similar term has already been considered as an chiral extension for QCD (χQCD) [73]. In χQCD, it was realized that the (mild) sign problem for SU(3) color is absent for SU(2) color. All these observations rely on the special properties of the pseudo-real nature of the SU(2) group. Still, there is more to consider in this application when one includes simultaneously both the diquark term and the χQCD-like extension.

When both the diquark and bilinear for the four Fermi term are included together, the measure is now a Pfaffian, or the square root of the fermionic determinant. Remarkably, it can still be proven that for this specific example of the four-Fermi coupling of χQCD, the fermionic measure for the Wilson action is again positive definite. Another example has been considered recently for staggered fermions by Catterall [74]. Clearly the extent of parameter space devoid of a severe sign problem needs further study. For example, the possibility that the breaking of custodial symmetry reintroduces a sign problem has not been fully explored. Most likely for some models, one will have to learn how to perturb away from examples with positive definite measure with methods similar to those used for QCD with small chemical potentials. Nonetheless, it appears that substantial progress can be made studying the vacuum alignment in strong coupling gauge theories by conventional lattice Monte Carlo methods.

The next issue is the choice of action: staggered, Wilson or domain wall? Our first choice is to start with the Wilson action while we investigate further possible advantages for the least expensive staggered case and more expensive domain wall case. The Wilson term breaks chiral symmetry but fortunately it aligns the vacuum consistent with the breaking of $SU(4)$ to $Sp(4)$. Research is underway to see if domain wall fermions at finite lattice spacing have, as expected, the full enhanced $SU(4)$ symmetry at infinite wall separation. If this is proven to hold, clearly the domain wall implementation, although more costly, does have theoretical advantages worth considering. As has been to be the case for lattice QCD research, we anticipate that each action has advantages for specific questions.

VI. STUDIES OF SUPERSYMMETRIC THEORIES ON THE LATTICE

One natural solution to the problems with the Standard Model Higgs is to incorporate supersymmetry. The Higgs is naturally light in such theories since it is paired with a fermonic superpartner whose mass is protected by chiral symmetries.

Supersymmetric theories have been extensively studied over many decades now both in the context of LHC phenomenology and string theory. However, for many years, significant barriers prevented serious study of such theories on the lattice. Recently this has changed; a series of studies has shown that $\mathcal{N} = 1$ super Yang-Mills can be studied successfully using domain wall fermions [75] and using these ideas one can even contemplate lattice study of super QCD theories.

A second recent development has been the construction of lattice theories which retain some exact supersymmetry at non zero lattice spacing. The prime example of such a theory is $\mathcal{N} = 4$ super Yang-Mills. One of the longstanding barriers to studying supersymmetric theories on lattices is the need to tune the bare couplings for large numbers of supersymmetry breaking operators to achieve a supersymmetric continuum limit. The exact supersymmetry present in the $\mathcal{N} = 4$ lattice theory we are studying saves the theory from this fine tuning problem and renders it possible for the first time to use lattice simulations to study the non-perturbative structure of this theory [76] - which lies at the heart of the AdS/CFT correspondence and hence to quantum gravity.

As part of its BSM physics program USQCD is engaged in extensive studies of supersymmetric lattice theories. Currently this is concentrated in two major directions: studies of $\mathcal{N} = 4$ super Yang-Mills using new formulations which possess exact supersymmetry and a long range effort to determine the non-perturbative features of super QCD using domain wall fermions.

The work on $\mathcal{N} = 4$ is exciting from a variety of perpectives; it potentially allows us to test and extend the AdS/CFT correspondence beyond the leading supergravity approximation and hence to learn about quantum gravity. But, in addition, it serves as a useful arena in which we can hope to develop the tools and techniques necessary to extract physics from four dimensional conformal field theories in general. In this way it is complementary to USQCD's effort to determine the conformal window and physical content of non-supersymmetric near conformal gauge theories. Indeed, in light of the recent discovery of a light Higgs at the LHC, it is important to understand whether a light Higgs might arise as a pseudo dilaton associated with spontaneous breaking of conformal symmetry. In this context, the flat directions generically encountered in supersymmetric theories, and those seen in $\mathcal{N} = 4$ in particular, may play an important role since they allow for just such a spontaneous breaking phenomenon to occur.

The work in super QCD is important for gaining a detailed understanding of the dynamical breaking of supersymmetry. This has important phenonenological implications since non-perturbative supersymmetry breaking in some high scale hidden sector generically feeds down to determine the structure of soft breaking terms in any low scale supersymmetric theory such as the MSSM. Indeed the values of many of these soft parameters are generically determined by just a handful of non-perturbative quantities in the high scale theory. An ab initio lattice calculation of such quantities in the high scale theory can thus provide valuable constraints on models of BSM physics. While super QCD is known to have no exact susy breaking vacua it does possess many metastable vacua whose lifetimes can be sufficiently large that they can play this role [77]. To facilitate such studies USQCD plans to extend its existing domain wall codes focused on $\mathcal{N} = 1$ super Yang-Mills to allow for the inclusion of the chiral multiplets needed to explore super QCD.

A. $\mathcal{N} = 4$ SYM

In recent years a new approach to the problem of putting supersymmetric field theories on the lattice has been developed based on orbifolding and topological twisting - see the recent Physics Report [78]. The central advantage of the new formulations is that they retain some exact supersymmetry for non-zero lattice spacing.

In the past year members of USQCD have been using these ideas to push forward a program to conduct a non-perturbative study of $\mathcal{N} = 4$ super Yang-Mills theory on the lattice. The lattice formulation that is being studied corresponds to a discretization of a topologically twisted form of the $\mathcal{N} = 4$ action. The constraints inherent in the construction pick out uniquely both the form of action, the distribution of fermions and bosons over the lattice, the structure of gauge covariant difference operators and even the type of lattice - in this case the so-called A_4^* lattice (a lattice whose fundamental basis vectors correspond to the fundamental weight lattice of $SU(5)$). This approach has the crucial advantage of leading to a lattice action which possesses a single exact supersymmetry at non zero lattice spacing. This theory may then be studied using the standard tools and algorithms of lattice QCD. The existence of a supersymmetric lattice formulation should dramatically reduce the usual fine tuning problem associated with supersymmetric lattice theories and makes it possible, for the first time, to attempt a serious non-perturbative study of the theory.

The availability of a supersymmetric lattice construction for this theory is clearly very exciting from the point of view of exploring the possible holographic connections between gauge theories and string/gravitational theories. For example, there is growing interest to relate the decay constant of the holographic techni-dilaton to important physics of the 126 GeV Higgs-like boson [23]. For the initial work on supersymmetric lattice contructions, see [79–84]. Lattice constructions allow new strategies to be employed such as strong coupling expansions and Monte Carlo simulation, which may lead to new insights and tools for extracting non-perturbative information. Preliminary results on the phase diagram of $\mathcal{N} = 4$ super Yang-Mills have been reported [85]. Initial results point to the existence of a single phase corresponding to a deconfined conformal field theory in the infrared for any value of the bare gauge coupling. Work is ongoing to develop a parallel code to extend this work to larger lattices.

In addition to the development of simulation code it is important to understand in detail how much tuning of the bare lattice parameters will be needed to recover full supersymmetry as the lattice spacing is sent to zero. Substantial analytic progress on this question has been obtained recently. A perturbative calculation has been completed and published [76] which contains several remarkable results; perhaps the most interesting of these is a calculation which shows that the effective potential of the lattice theory vanishes to all orders in perturbation theory in complete analogy to the continuum theory. The topological character of the twisted supersymmetry which is realized in the lattice theory plays a crucial role in this result. This result prohibits the appearance of any scalar mass terms as a result of quantum corrections. In addition, the authors were able to show that no fine tuning was needed at one loop to target the usual $\mathcal{N} = 4$ theory – a single wave function renormalization was the only correction needed to absorb all divergences of the lattice theory. Indeed, the authors give general arguments that the beta function of the lattice theory vanishes at one loop - affording the only known example of a non-trivial four dimensional lattice theory which is finite at 1-loop!

Beyond one loop there is potential for logarithmic running of various dimensionless couplings in the lattice action and numerical methods will likely be needed to proceed further. Work in this direction has already started with the elucidation of the action of all the additional twisted supersymmetries on the lattice fields. This analytic work must now be incorporated into the simulations to allow for numerical tests of various broken supersymmetric Ward identities.

In addition, a study has recently been completed on the question of whether the lattice theory exhibits a sign problem. The Pfaffian representing the effect of the fermion loops is generically complex in Euclidean space rendering the use of Monte Carlo methods potentially problematic. Members of USQCD have been examining this issue in some detail in the dimensional reduction of this model to two dimensions. The results reported in [86] are encouraging; the observed fluctuations in the Pfaffian phase as measured in the phase quenched Monte Carlo ensembles used for simulation are actually rather small allowing for the possibility of the use of reweighting techniques to compute expectation values. Indeed for small lattices the phase is sufficiently close to zero for gauge group $U(2)$ that it can be neglected in comparison with current statistical errors. Furthermore, there exist formal arguments based on the structure of the lattice moduli space and the topological character of the lattice partition function that also suggest that the sign problem is absent in this theory.

Finally there are efforts currently underway to study a variety of two point correlation functions in the theory. In principle on a sufficiently large lattice these are expected to exhibit power law rather than exponential behavior consistent with the conformal nature of the continuum theory. Some specific predictions for certain operators exist in the continuum literature; eg for the Maldecena loop or for the anomalous dimensions for various scalar operators like the Konishi and supergravity multiplet.

Unlike the study of the conformal window in non supersymmetric lattice gauge theories the exact supersymmetry prohibits any mass terms for the fermions and so the finite box size and lattice spacing constitute the only features that break conformality even at strong coupling. Furthermore, some of the leading cut-off effects stemming from tadpole terms in lattice perturbation theory are absent in the $\mathcal{N} = 4$ action since the gauge fields are valued in the algebra and not the group. It remains to be seen if these differences reduce lattice artifacts to levels where the power laws can be differentiated from exponentials on realistic lattice sizes. If not other approaches such as those based on exploiting the conformal mapping $R^4 \to S^3 \times R$ may need to be employed to make reliable measurements of quantities like anomalous dimensions.

Since the discovery of a light Higgs-like state at the LHC there there has been renewed interest in the possibility that the Higgs is a (pseudo) dilaton associated with spontaneous breaking of conformal invariance. Such a state is particularly simple to realize in a supersymmetric theory with flat directions such as $\mathcal{N} = 4$ Yang-Mills - the dilaton corresponding to translations along such a flat direction. The $\mathcal{N} = 4$ lattice action we are investigating possesses such flat directions and furthermore these flat directions are stable against quantum corrections due to the exact lattice supersymmetry. To realize the spontaneously broken state and the corresponding dilaton in the lattice theory then requires that a non-zero vacuum expectation value of one or more scalar fields be fixed and the Monte Carlo procedure modifed so that no integration over this mode is carried out. This procedure can be carried out in a straightforward manner and will be investigated in the near future.

B. Dynamical SUSY Breaking and Super QCD

It is straightforward to “supersymmetrize” the usual theories of particle physics; the Minimal Supersymmetric Standard Model (MSSM) is perhaps the most studied extension of the Standard Model of particle physics. In such models the Higgs is naturally light since it is accompanied by a fermionic partner whose mass is protected by chiral symmetries. A great deal of effort has been expended on predicting signals for SUSY/MSSM at LHC.

Of course the low energy world we inhabit is manifestly not supersymmetric; so a key component of any realistic theory of Beyond Standard Model physics must provide a mechanism for spontaneous supersymmetry breaking. In general a variety of no go theorems ensure that any such symmetry breaking must be non-perturbative in nature and hence inaccessible to most continuum calculations. As a consequence this breaking is usually handled by explicit soft breaking terms in the low energy theory. In general there are very many of these terms and their corresponding couplings (so-called soft parameters) which results in a large parameter space and a lack of predictivity in the low energy theory.

However, in general we might expect that these parameters are determined from dynamical breaking of SUSY at high energies in a “hidden sector”. A supersymmetrized version of QCD - super QCD - with N_c colors and N_f massive flavors is a natural candidate for this hidden sector. For $N_c + 1 \leq N_f < \frac{3}{2}N_c$ it is thought that super QCD has long lived metastable SUSY breaking vacua [77]. The lifetimes of these metastable vacua can exceed the age of the Universe and ensure that the physical vacuum breaks supersymmetry. Within such a vacuum state non-perturbative phenomena such as confinement and chiral symmetry breaking precipitate a breaking of supersymmetry. Furthermore, if the quark masses are small compared to the confinement scale these vacua have extremely long lifetimes. If the standard model fields are coupled to the hidden sector fields in an appropriate fashion then such non-perturbative dynamics arising in the broken phase of this theory can feed down to yield soft supersymmetry breaking terms in the low energy effective theory.

Thus a detailed understanding of the vacuum structure and strong coupling dynamics of super QCD can strongly constrain possible supersymmetric models of BSM physics; in some cases leading detailed predictions of the soft parameters in low energy BSM theories in terms of a handful of non-perturbative quantities obtained in the hidden sector super QCD theory. Lattice simulations of supersymmetric lattice QCD thus have the potential to play an important role in constraining the parameter space of supersymmetric models and in building realistic supersymmetric theories of BSM physics.

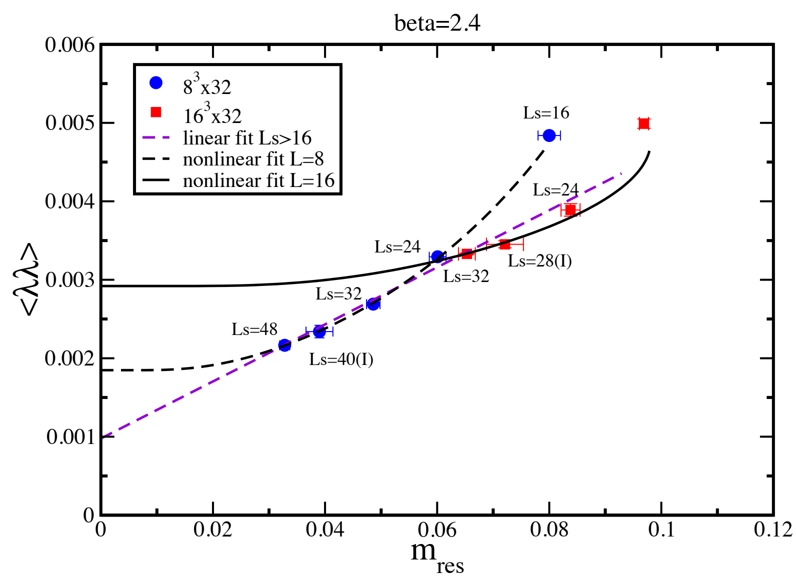

FIG. 3. Gaugino condensate vs residual mass for $SU(2)$ $\mathcal{N}=1$ super Yang-Mills regulated using a domain wall fermion action. The gauge coupling $\beta = 2.4$ while the bare fermion mass is set to zero. The data points correspond to different lattice volumes and L_s

In contrast to the $\mathcal{N}=4$ super Yang-Mills model discussed above the technical problem that must be immediately faced in studying generic supersymmetric lattice theories is that supersymmetry is broken by discretization and it is non-trivial to regain SUSY as the lattice spacing is sent to zero with conventional lattice fermion formulations. Luckily in the case of $\mathcal{N}=1$ super Yang-Mills theory, which contains both gluons and their fermionic super-partners gluinos, this problem can be avoided by the use of domain wall fermions (DWF). In this case the exact lattice chiral symmetry of the fermion action ensures that SUSY is *automatically* recovered in the chiral limit.

Preliminary work by USQCD has already revealed a non-zero gluino condensate in the SU(2) theory in agreement with theoretical expectations [75, 87]. A plot of the gaugino condensate vs residual mass is shown in Figure 3. However, to build a realistic theory capable of yielding the soft parameters of the MSSM one needs to add N_f quarks and their scalar superpartners (squarks), and additionally extend the gauge group to a larger number of colors N_c. To restore SUSY in the continuum limit now requires *both* use of DWF *and* tuning of parameters in the squark sector [88]. In principle this can be done by doing a series of runs over a grid in squark parameter space and using (offline) reweighting techniques in the scalar sector to tune to the supersymmetric point. Clearly such calculations are ambitious but constitute an important long term goal of USQCD's work on supersymmetric theories of BSM physics.

VII. METHODS AND PHENOMENOLOGICAL APPLICATIONS

The field of BSM lattice studies is still young, but it has already become clear that methods developed for lattice QCD are not always optimal and frequently not sufficient to investigate conformal or near-conformal systems. We continuously re-evaluate our methods and develop new techniques that are better suited to explore walking or IR-conformal dynamics that are expected to emerge near the conformal window. Conformal systems are particularly difficult to investigate as both the finite fermion mass and finite lattice volume break conformal invariance. The most promising lattice techniques attempt to use rather then fight this issue. A repeating theme of BSM model investigations is that it is essential to consider the systems from many different angles, use complementary and contrasting methods to fully understand infrared dynamics. Exciting phenomenological applications are beginning to emerge from these investigations.

A. Running Coupling and β-function

Running coupling studies connect the well understood perturbative regime to strong coupling conformal dynamics, or, near conformality, to chiral symmetry breaking. These are particularly important tasks close to the conformal edge. It is challenging to distinguish a chirally broken system with slowly running coupling from a strongly coupled conformal one. In both cases the weak coupling ultraviolet behavior is described by the asymptotically free gaussian fixed point. Below the conformal window the system is in the chirally broken phase and in the simplest theoretical scenario other phases in the continuum are not expected to emerge at strong coupling. Inside the conformal window, the coupling of the model runs from the UV phase of perturbation theory to conformal behavior in the infrared limit. This evolution along the renormalized trajectory of the critical surface connects the UV fixed point to the conformal infrared fixed point (IRFP). Lattice artifacts, which can only create other phases at finite cutoff, have to be identified and separated from the continuum analysis. The existence of the conformal phase and the critical exponents of the IRFP are universal, although the location of the IRFP in the multi-parameter action space depends not only on the lattice action but the specific renormalization scheme as well.

Most lattice running coupling calculations are done in the chiral limit with vanishing fermion masses as a function of the scale set by the finite lattice volume, thus avoiding the two most difficult aspects of lattice simulations. These studies aim to identify an IRFP from the flow of the gauge coupling. It is necessary to show in these studies that sufficiently strong continuum couplings were probed in large infrared volumes to capture the existence, or non-existence of the IRFP. Sometimes the emergence of spurious phase transitions due to discretization artifacts makes it challenging to reach continuum strong couplings.

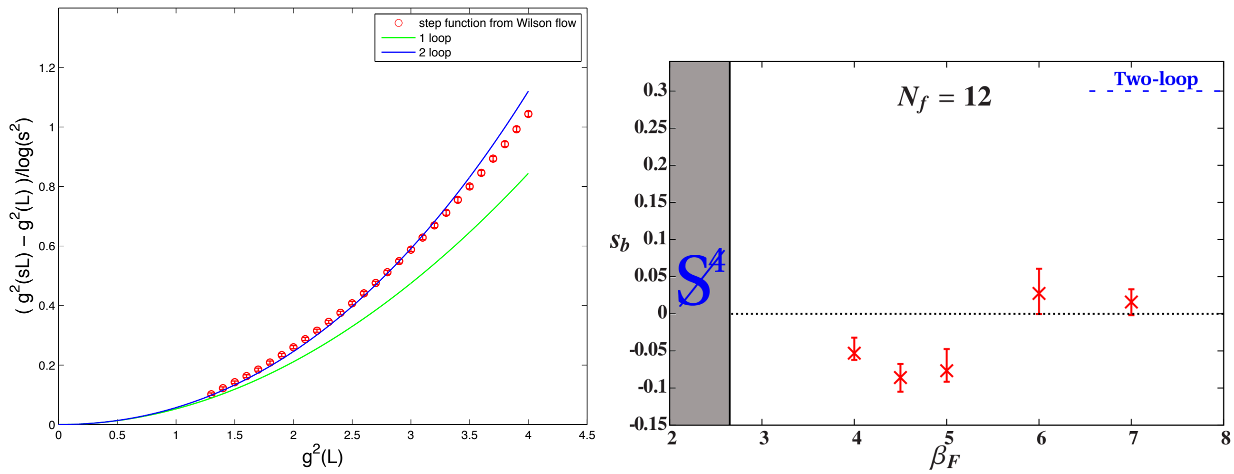

FIG. 4. Two complementary approaches are illustrated for the running coupling and its β-function. On the left plot, the discrete step β-function of SU(3) continuum gauge theory coupled to $N_f = 4$ flavors of massless fundamental fermions is shown for a scale change of s $= 3/2$. The coupling $g(L)$ runs with the linear size L of the finite continuum volume [89, 90]. The continuum extrapolated result is shown together with the 1-loop and 2-loop results of perturbation theory. On the right plot, preliminary results for the bare step-scaling function s_b is shown from Wilson-flowed MCRG two-lattice matching for $N_f = 12$ flavors [91]. $s_b = 0$ around $\beta_F = 5.5$ signals an infrared fixed point. The blue dashed line is the perturbative two-loop prediction for asymptotically weak coupling.

There are two complementary directions to follow which are illustrated in Figure 4. Methods that rely on the calculation of a renormalized coupling build on the properties of the gaussian fixed point to guide cutoff removal in the model study. These methods include Schrodinger functional methods, the twisted Polyakov loop, Creutz ratios, and static potential approaches [48, 92–94], and the promising new method based on the gradient flow of the Yang-Mills field [62, 89, 90]. The advantage of these methods is that they predict a well defined renormalized coupling in the continuum that can be connected to any renormalization scheme. It is difficult to control lattice artifacts as one takes the continuum limit by tuning the bare coupling towards zero even when investigating the properties of the system at strong renormalized coupling. The problem is particularly perplexing when a "backward flow" emerges in the running coupling as a potential signal for an infrared fixed point without complete control of the continuum limit. Theoretical and numerical studies continue work toward a resolution of these issues.

A complementary approach is to use bare quantities to determine a bare step scaling function, like the Monte Carlo renormalization group (MCRG) methods do [91, 95, 96]. MCRG methods are very effective in identifying phases and fixed points; their drawback is the lack of continuum coupling. Optimization of the block transformation of earlier MCRG calculations encountered difficulty in the determination of a well defined step scaling function. Progress to circumvent this problem has been made with the new Wilson-flow–MCRG approach [91]. In MCRG methods the renormalization flow has to reach the renormalized trajectory and simulations have to be repeated on increasingly larger volumes to ensure this. MCRG matching assumes that the flow is driven to and along a renormalized trajectory, which is problematic on the strong coupling side of the IRFP. Theoretical and numerical studies of the MCRG approach continue, in order to provide full control on the remaining important problems.

Running coupling investigations are rather unique in the study of conformal or near-conformal systems. There are many new approaches available that were designed for lattice BSM models. For the next five years plans include better control of systematic errors and an attempt to combine complementary methods.

B. Chiral Symmetry Breaking, Dirac Spectrum, and Anomalous Mass Dimension

The absence of spontaneous chiral symmetry breaking at small lattice spacing a, close to the continuum, is a clear indication for the position of the BSM lattice model inside the conformal window. The corresponding order parameter, the fermion condensate $\langle\bar{\psi}\psi\rangle$, can be measured directly in lattice simulations. The condensate is quadratically ultraviolet divergent at finite fermion mass m and vanishes in finite volumes for $m = 0$. Lattice calculations have to balance finite volume and finite fermion mass effects making the $m \to 0$ chiral extrapolation of the fermion condensate difficult. This makes it challenging to establish chiral symmetry breaking. An alternative and promising method is to extract the chiral condensate from the spectral density through the Banks-Casher formula $\Sigma = -\pi\rho(0)$ [97]. When the Dirac eigenvalue spectrum is calculated with vanishing valence fermion masses, Σ depends only on the sea fermion masses through the eigenvalue density $\rho(\lambda)$; the ultraviolet divergent parts of $\langle\bar{\psi}\psi\rangle$ are absent, making the chiral extrapolations more reliable. In the chiral limit both the direct and indirect measurements must give the same prediction for the chiral condensate, and we expect that in the next generation of lattice studies the two approaches will be used in a complementary manner.

The eigenvalue spectrum provides an excellent tool to study the anomalous mass dimension as well. For small mass deformations of the IRFP, the scheme independent critical exponent, associated with the anomalous mass dimension, can be determined directly from the mode density of the eigenvalue spectrum. In QCD-like theories with χSB the energy dependent anomalous mass dimension is calculated with a particular choice of renormalization scheme. In an infrared conformal theory the eigenvalue density scales as $\rho(\lambda) \propto \lambda^\alpha$ for small λ. The mode number $\nu(\lambda) = V \int \rho(\lambda')d\lambda'$ thus scales as $\propto V\lambda^{\alpha+1}$ and since $\nu(\lambda)$ is a renormalization group invariant quantity, the scaling exponent α is related to the mass anomalous dimension as $\gamma_m + 1 = 4/(1 + \alpha)$ [98–100]. The anomalous dimension depends on the energy scale, or when derived from the mode number, on λ. This energy dependence can be extracted by fitting the mode number in finite intervals with a λ-dependent exponent not only in conformal but also in chirally broken systems as well [101].

The scaling form $\rho(\lambda) \propto \lambda^\alpha$ is valid in the infinite volume chiral limit. In conformal systems or even in QCD-like systems in small volumes simulations are possible in the chiral limit.

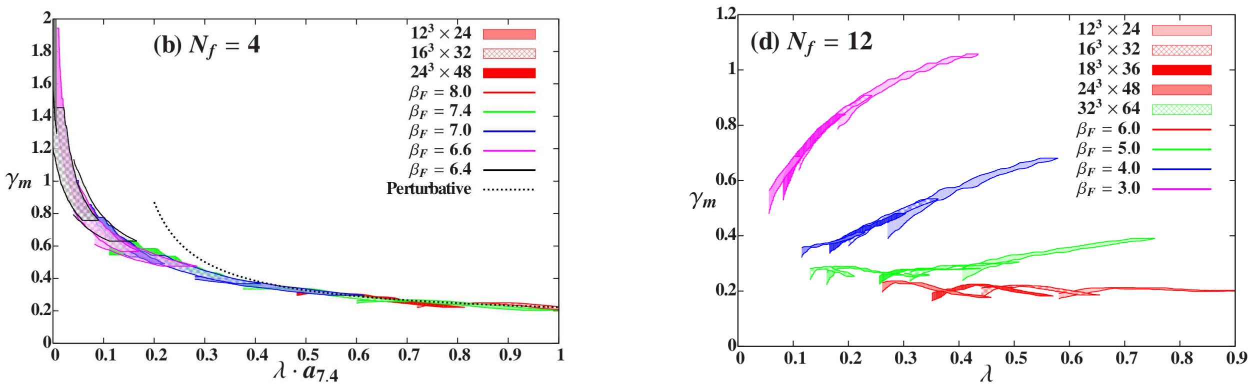

FIG. 5. The left panel shows the anomalous dimension as the function of the Dirac operator eigenvalues λ for the 4-flavor SU(3) system. The combination of different volumes allows an infinite volume extrapolation and rescaling the eigenvalues to a common lattice spacing predicts a universal curve connecting the perturbative and chirally broken regimes. The right panel is the anomalous dimension at several bare couplings for the 12-flavor system. The data are not consistent with ultraviolet dynamics governed by the gaussian fixed point nor with chiral symmetry breaking in the infrared limit.

Recent finite volume studies indicate that small volumes can be combined to approximate infinite volume results making it possible to map the scale dependent anomalous dimension in QCD-like system from the perturbative to the chirally broken regime. This is illustrated on the left panel of Figure 5 that shows the anomalous dimension as the function of λ for the 4-flavor SU(3) system. Over two orders of magnitude in energy scale is covered by combining different volumes and rescaling different gauge couplings to a common lattice spacing. At the large λ ultraviolet range the data is consistent with the 1-loop perturbative prediction (dashed curve) while in the infrared limit chiral symmetry breaking emerges. The right panel of Figure 5 shows the anomalous dimension for the 12 flavor system at several gauge coupling values. This data is very different form the $N_f = 4$ case. It is not possible to rescale the data to form a universal curve. At stronger couplings the ultraviolet scaling is the opposite of what is expected around an asymptotically free fixed point, and one can observe a qualitative change around $\beta = 5.0$. In this work, the data are consistent with an IRFP and an extrapolation to the $\lambda = 0$ limit predicts the conformal anomalous mass dimension as $\gamma_m = 0.32(3)$ [101].

The scaling of the mode number is a very promising approach to study the infrared dynamics of systems near the conformal window. Simulations on larger volumes and the calculation of more eigenmodes, possibly with the stochastic estimator method, will allow greater control of finite volume effects and a more reliable approximation to the infrared limit. The method described above can be generalized to Wilson fermions though it will require additional numerical tuning to the chiral limit.

C. Chiral Perturbation Theory and Condensate Enhancement

It has been known since the mid-1960's that the low energy behavior of pseudo-Nambu-Goldstone bosons (PNGBs) was governed by the decay constant F and the chiral condensate

$\sigma(\mu)$, up to corrections of $\mathcal{O}(m)$ and $\mathcal{O}(m \log m)$ in the fermion mass $m(\mu)$, as encoded in the GMOR relation [102]

$$M^2_{\rm PNGB} = \frac{2m(\mu)\Sigma(\mu)}{F^2}. \tag{1}$$

The notation $m(\mu)$ and $\Sigma(\mu)$ indicate that the numerical values for these quantities depend on the calculational scheme in which they are computed and all such schemes require the use of an arbitrary scale μ. Other quantities $M_{\rm PNGB}$ and F do not depend on μ so the combination $m(\mu)\Sigma(\mu)$ must be scale-independent. In the calculation, we typically define $\mu \equiv a^{-1}$, the inverse lattice spacing, and $m(a^{-1})$ is the bare lattice mass parameter.

What at first glance may seem to be a rather technical detail about scale-dependence in calculational schemes can be turned into a powerful probe of the relative sensitivity of low energy dynamics to high energy vacuum fluctuations in two different quantum field theories. We can study the rate of change of the dimensionless ratio $R(\mu) \equiv \Sigma(\mu)/F^3$ while varying μ as one measure of sensitivity to fluctuations of $\mathcal{O}(\mu)$. But, the numerical value of $\Sigma(\mu)$ at any fixed μ is somewhat arbitrary because it depends on the specific calculational scheme, so we compare the ratio computed in two different theories using the same scale μ and the same scheme.

One clear hallmark of a walking gauge theory is that the long-distance physics of the confining regime is dramatically modified by the presence of the approximately scale-invariant regime at higher energies. In a QCD-like theory $R(\mu) \sim \log(\mu)$ with increasing μ whereas in a walking gauge theory $R(\mu) \sim \mu^{\gamma_m}$, where the mass anomalous dimension $\gamma_m \approx 1$. This conjectured phenomenon is called *condensate enhancement*.

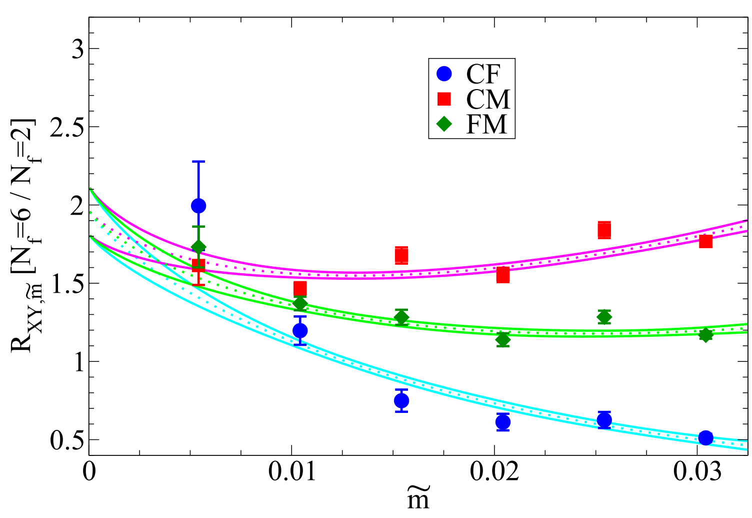

FIG. 6. The ratio of ratios $\mathcal{R}_{XY}$ described in the text, computed three different ways labeled by XY, for $N_f = 2, 6$ *vs.* geometric mean of the fermion masses $\widetilde{m}$. The chiral limit suggests a surprisingly large amount of condensate enhancement, but not enough to claim evidence for walking behavior.

Ideally, we can then calculate $R(\mu)$ in two different theories, say SU(3) gauge theory with different N_f flavors of light fermions, at two different $\mu = a^{-1}$ inverse lattice spacings. If the ratio of ratios $\mathcal{R} \equiv R(N_f)/R(N_f')$ grows like $(\mu_1/\mu_2)^{\gamma_m}$ and $\gamma_m \approx 1$, then this is clear evidence that the theory with N_f fermions exhibits walking behavior between the scales

$\mu_1 > \mu > \mu_2$. In preliminary calculations performed at a single inverse lattice spacing, a reasonable conjecture can be employed that $\mathcal{R} \sim 1$ at $\mu = 4\pi F$ in which case $\mathcal{R}$ can be compared to $(\mu/4\pi F)^{\gamma_m}$ [103]. Figure 6 shows an example of this kind of calculation where the intercept is $\mathcal{R}$ and should be compared to $1/a4\pi F \sim 4$. Since $\mathcal{R} \approx 2$ it gives a clear indication the SU(3) $N_f = 6$ theory is not a walking theory.

Inside the conformal window we have to change our approach. The finite size scaling of bound state masses at a conformal fixed point can be used to predict the mass anomalous dimension, but there are systematic effects that have to be investigated carefully. An important issue that has not been considered in depth yet is the control of the region in gauge coupling where scaling according to the infrared fixed point (as opposed to the gaussian fixed point) can be expected. This could be a serious problem if the gauge coupling changes slowly with the energy as indicated by some of the other methods. Once the dependence on the irrelevant gauge coupling is understood one still has to find the region in the fermion mass where the IRFP is close enough to guarantee scaling. It is likely that lack of scaling, both due to the gauge coupling or the fermion mass, will show up as scaling violations in any FSS analysis. One has to wonder if that is the main reason for controversial results in SU(3) gauge theory with twelve flavors.

D. S-parameter

The S parameter was introduced [104] in the early 1990s in analyses of oblique corrections, that is, the effects of new physics on the vacuum polarizations of electroweak gauge bosons. Along with the Higgs-like particle recently discovered at the LHC, S remains an important experimental constraint on models of electroweak symmetry breaking through new strong dynamics.

Experimentally, S is determined from a global fit to a variety of electroweak measurements, including Z boson partial decay widths and asymmetries, neutrino scattering cross sections, and atomic parity violation. Since we define $S = 0$ for the standard model with Higgs boson mass m_h, the experimental value depends on m_h; with $m_h = 126$ GeV, $S = 0.03 \pm 0.10$, completely consistent with the standard model [105].

However, QCD-like new strong dynamics would predict $S \gtrsim 0.3$; this was originally obtained by scaling up experimental QCD data for vector (V) and axial-vector (A) spectral functions to the electroweak scale [104], and has been confirmed by lattice QCD calculations [106–108]. The discrepancy between experiment and QCD-like technicolor is worsened by the absence of light states in QCD: replacing $m_h = 126$ GeV with a typical technihadronic scale $M_H^{(ref)} \sim 1$ TeV shifts the experimental value to $S \approx -0.15 \pm 0.10$ (the shift is logarithmic in $M_H^{(ref)}$).

To see how non-QCD-like dynamics may change the situation, consider

$$S = 4\pi N_D \lim_{Q^2 \to 0} \frac{d}{dQ^2} \Pi_{V-A}(Q^2) - \Delta S_{SM}.$$

There are three important ingredients in this expression:

2. ΔS_{SM} accounts for the three Nambu–Goldstone bosons (NGBs) eaten by the W and Z bosons, and also sets $S = 0$ for the standard model.

1. N_D is the number of doublets with chiral electroweak couplings; its presence corresponds to the intuition that S measures the "size" of the sector that hides electroweak symmetry.

2. ΔS_{SM} accounts for the three Nambu–Goldstone bosons (NGBs) eaten by the W and Z bosons, and also sets $S = 0$ for the standard model.

3. $\Pi_{V-A}(Q^2)$ is the transverse component of the difference between vector and axial-vector vacuum polarization functions, and can also be related to a dispersive integral of spectral functions, $4\pi\Pi'_{V-A}(0) = \frac{1}{3\pi}\int_0^\infty \frac{ds}{s}\left[R_V(s) - R_A(s)\right]$.

The near restoration of chiral symmetry or parity doubling with strong cancellations between $R_V(s)$ and $R_A(s)$ will therefore reduce S. Such dynamics may be expected near to the conformal window.

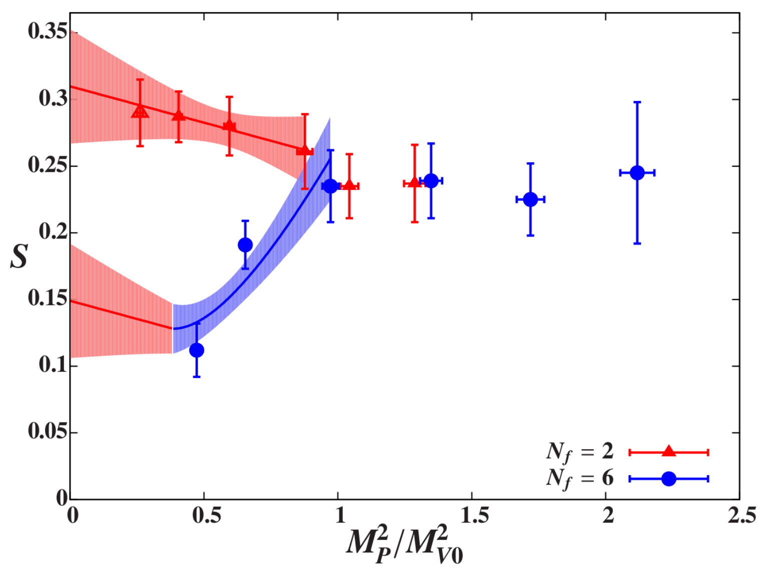

FIG. 7. The S parameter for SU(3) gauge theories with $N_f = 2$ and 6 fundamental fermions, from Ref. [109]. M_P is the pseudoscalar mass, while M_{V0} is the vector meson mass in the chiral limit. The $N_f = 6$ theory has 35 pseudo-NGBs, and here we imagine that 32 of them have mass $\approx 0.6 M_{V0}$.

On the lattice, we measure $\Pi_{V-A}(Q^2)$ from V and A two-point current correlation functions. Existing calculations use overlap or domain wall fermions, for which good chiral and flavor symmetries lead to large cancellations of lattice artifacts in the V–A difference, and to equal renormalization factors $Z_V = Z_A$ [106, 107, 109]. These formulations also allow direct connection to continuum chiral perturbation theory, where the S parameter corresponds to the low-energy constant L_{10}.

So far few lattice studies of S have been completed: Refs. [106–108] investigated QCD-like technicolor, and relative to those results Ref. [108] observed a significant reduction in S with $N_f = 6$ light fermions (Figure 7). Ref. [110] reported some qualitative investigations of $\Pi_{V-A}(Q^2)$ for fermions in the sextet representation of SU(3), and the USQCD BSM Collaboration is currently working to compare mixed-action and staggered measurements.

Some important targets for future work include better understanding and controlling finite-volume effects (which may lead to spurious parity doubling), as well as improving the $Q^2 \to 0$ extrapolation of $\Pi_{V-A}(Q^2)$. The latter issue has recently attracted a great deal of interest in the context of hadronic contributions to $(g-2)_\mu$, which are sensitive to $\Pi_V(Q^2)$ at very small Q^2.

E. WW Scattering on the Lattice

The recent discovery of the Higgs boson, or of some other new physics which would replace it, was widely expected by particle physicists. A theory including all other known particles and interactions, but no Higgs boson, is well-known to be mathematically inconsistent; the scattering rate of W and Z bosons grows as the energy, eventually occurring with probability greater than 1. The addition of scattering processes involving an intermediate Higgs boson cuts off the growth in this rate at high energy, solving the problem. Even so, the scattering of W and Z gauge bosons can be directly sensitive to the presence of deviations from the Standard Model in the electroweak symmetry-breaking sector. Experimentally, high-energy W/Z scattering is quite clean but occurs with very low rate, so that a detailed study will require many years of data at the upgraded LHC.

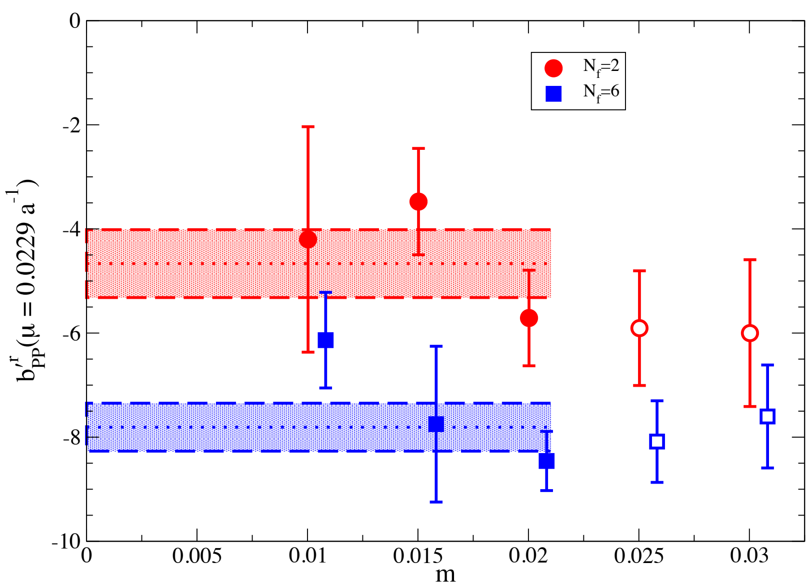

FIG. 8. From [111], comparison of chiral parameter b'^r_{PP} obtained from maximal-isospin $\pi - \pi$ scattering in SU(3) gauge theories with $N_f = 2$ (red) and $N_f = 6$ (blue) light fermion species. The parameter b'^r_{PP} is sensitive to a combination of low-energy constants which would be relevant for WW scattering, hinting that the scattering rate may be enhanced in theories of electroweak symmetry breaking constructed with larger N_f.

Even with the Higgs boson added, there remain some theoretical questions in the Standard Model, in particular the issue of "naturalness"; the strong sensitivity of the Higgs boson mass to quantum corrections makes it puzzling that its observed mass should be so close to the electroweak scale. Composite Higgs theories provide one possible solution, with the quantum corrections effectively cut off at the scale of compositeness. Within such a theory, there will be many observable composite states aside from the Higgs, but they can be quite heavy and thus difficult to produce directly. The first hints of new physics may therefore come from indirect observation of effects at low energies.

Considering W/Z scattering at relatively low energies (compared to the scale at which new physics becomes strongly coupled) allows us to "integrate out" the heavier states and work in an effective field theory resembling the Standard Model [112–114]. The first deviations would appear in two-, three-, and four-point functions of the electroweak gauge bosons, corresponding to oblique corrections [104, 108], anomalous triple-boson vertices, and W/Z boson scattering. In principle, all such corrections (known as "low-energy constants") are predicted by the dynamics of the underlying strongly-coupled gauge theory under which the composite Higgs is bound together. It can be noted that LHC experiments using 7 and 8 TeV data [115–117] are already becoming sensitive to the possibility of anomalous couplings consistent with LEP and Tevatron limits. Lattice gauge theory provides an ideal non-perturbative approach for studying the values of these low-energy constants and their relations to one another as the underlying strong dynamics is changed.

A recent lattice study has focused on the low-energy constants relevant for WW scattering for three-color gauge theories with $N_f = 2$ and 6 light fermion species [111]. In the strongly-coupled gauge theory, the relevant physical process to determine these low-energy constants is π-π scattering (these "pions" will become the longitudinal modes of the W bosons in a composite Higgs theory.) In this initial work, only the extraction of a combination of the relevant low-energy constants was possible; calculation of scattering lengths in other channels and the use of partially-quenched fermion masses may be necessary to separate the individual constants. Still, a promising trend was shown in the initial study, with the measured combination of low-energy constants showing a significant enhancement from $N_f = 2$ to $N_f = 6$ (see Figure 8). If this trend continues, theories with greater fermion content may give rise to greatly enhanced WW scattering rates, which could be observed at the LHC with a relatively modest amount of data.

F. Composite Dark Matter

Although only the gravitational effects of dark matter have been observed to date, the existence of other interactions between the dark matter and standard model (SM) particles is well-motivated in many models. In the standard thermal relic scenario, such interactions keep the dark matter in equilibrium with the thermal bath of SM particles. Asymmetric production is an alternative mechanism [118, 119], in which the dark matter and ordinary baryons possess particle-antiparticle asymmetries which are connected e.g. by non-perturbative electroweak interactions. In either case, the presence of interactions connecting the dark sector to the SM is a central ingredient. However, constraints on such interactions from dark matter direct-detection experiments have strengthened by many orders of magnitude in recent years. A fine balance between the presence and absence of such DM-SM interactions when constructing models is therefore needed.

Composite dark matter models provide an attractive mechanism for realizing this balance, and can have unique phenomenological signatures. In these models, an electroweak-neutral

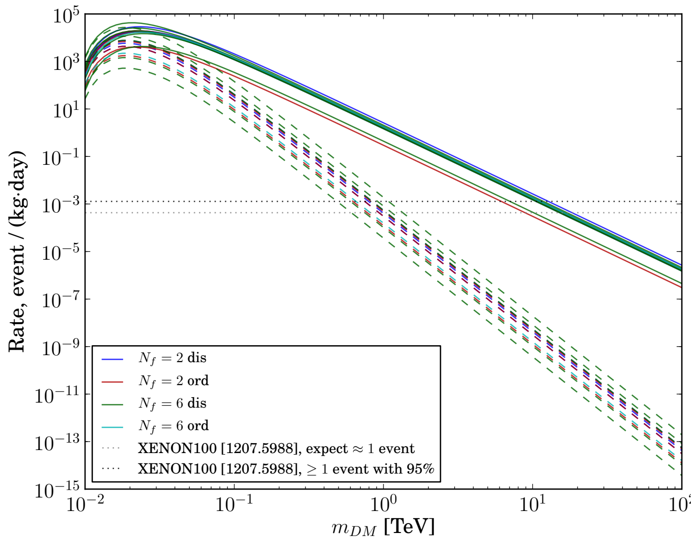

FIG. 9. From [122], calculated Xenon100 event rates based on simulation results for the charge radius (dashed) and magnetic moment (solid) of baryonic dark matter in three-color gauge theories with $N_f = 2$ and $N_f = 6$ light fermion species. Bounds arising from the magnetic moment interaction are quite strong, of order 10 TeV based on the latest Xenon100 data [123].

composite dark matter candidate can be formed as a bound state of electroweak-charged fundamental particles, bound together by a new strong gauge force. The charged constituents give rise to the necessary interactions in the early universe, but the interactions of the neutral state with ordinary matter in the present Universe will be greatly suppressed. In scenarios of dynamical electroweak symmetry breaking, in which the Higgs boson itself is a composite state, the presence of additional bound states which can play the role of dark matter is natural and the couplings to the electroweak sector are particularly well-motivated [118, 120, 121].

The properties of a strongly-coupled dark sector can be roughly estimated from our knowledge of QCD, but for theories which differ significantly from QCD, such estimates are unlikely to be reliable. Lattice field theory is thus the only systematically controlled approach to calculation in such new strongly-coupled theories. In the context of composite dark matter, there are many observables of interest, but one set of observables which are directly relevant to experiment are the electromagnetic form factors, which determine the electroweak interaction strength of the dark matter with ordinary matter. Recently, a lattice calculation of the magnetic moment and charge radius has been carried out in three-color gauge theories ($N_c = 3$) with $N_f = 2$ and 6 light fermion species. The former theory corresponds to QCD, while the presence of extra fermions in the $N_f = 6$ theory modifies some of its dynamical properties [103]. Results for the charge radius and magnetic moment, converted to direct-detection constraints for the recent Xenon100 experimental release, allow the exclusion of composite dark matter based on these models for masses up to 10 TeV [122] - see Figure 9.

This bound is dominated by the magnetic-moment operator, which exists naturally in any theory with odd N_c. Motivated by this initial study, a natural next step is the exploration of theories with even N_c, where the bounds will be much weaker. Work is currently underway in this direction, requiring code development, theoretical study, and exploration of the parameter space through simulation. In particular, the present focus is on 4-color simulations, where the baryon is bosonic but not a light pseudo-Goldstone state as in the $N_c = 2$ theory. Generation of the necessary lattices to address the open questions of the $N_c = 4$ theory, including study of the baryonic charge radius and polarizabilities, will require large-scale machine resources. The Blue Gene/Q architecture is ideal for dealing with the large matrices required for this problem, and simulations are estimated to require on the order of 24 rack-years on the BG/Q for completion.

VIII. NEW METHODS FOR BSM LATTICE FIELD THEORY

Lattice field theory for Beyond the Standard Model physics presents new challenges to formulating optimal algorithms and rapidly developing high performance software.

First, unlike QCD, there is a wide variety of strongly interacting theories of potential interest. The traditional software and algorithms must be generalized to accommodate new gauge groups, fermion representations and large scale separations. Extensive and on going research is needed to discover and implement new methods. In some cases, just formulating the lattice theory requires considerable new mathematics and physical insight. Putting supersymmetric field theories on the lattice discussed above is one dramatic example requiring new lattice actions based on orbifolding and topological twists. Even for non-SUSY theories new gauge groups and fermion representations require substantial code development. SciDAC-3 is developing FUEL (Framework for Unified Evolution of Lattices) written with a LUA front-end to accelerate the adaptation of code for new theories and to control and autotune them for high performance.

However the most serious new challenge is a larger range of scales compared to those encountered to date in QCD. As described above, BSM models must not only have zero mass fermions to implement the Higgs mechanism, but also are very likely to exhibit near conformal behavior in the infrared to satisfy precision electro-weak experimental constraints. Nonetheless the problems of scale separation are not entirely new to lattice field theory so there are some traditional methods that are available to address them, such as Wilson real-space renormalization group, step scaling, finite size analysis, multigrid solvers etc. Their role in BSM lattice research will be briefly described. However it is also important to explore entirely new algorithmic methods. One such idea consists of replacing the standard Euclidean hypercubic lattice by one suited to Radial Quantization.

A. Traditional Multi-scale Tools

Several methods to confront scale separation are well known but are just now being refined for BSM lattice field theory. The first class is the use of Wilson real-space renormalization group, step scaling and finite size effects. These methods start by acknowledging that with large scale separations, no single lattice is large enough to accommodate all the important scales. So one compares lattices with different scales set by the bare coupling to interpolate gradually from small to large scales. Some of these have been described above but it is clear that BSM lattice investigation will inevitably refine and develop them further.

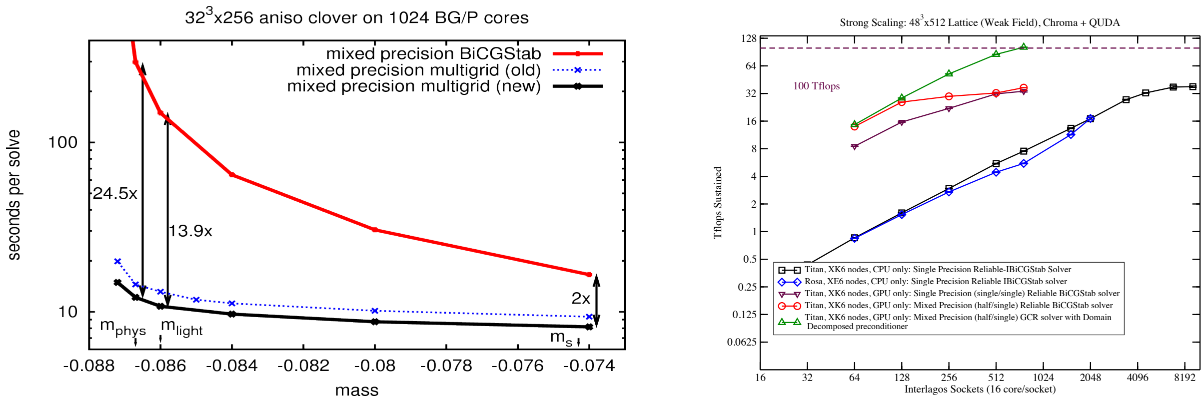

FIG. 10. On the left is a comparison of multigrid performance relative to the best Krylov solver on the BlueGene/P for the Wilson Dirac solver as the quark mass is decreased to its physical value. On the right is the benefit of using a block Jacobi domain decomposition preconditioner to reduce inter GPU communication on the Titan Cray/NVIDIA system.

A second class directly exploits multiple scales on each lattice to optimize convergence of solvers or to accommodate communication constraints on hierarchal parallel architectures. For example new multi-grid methods have recently been discovered that remove the divergent computational cost for the Wilson Dirac solvers in the chiral limit [124]. These multigrid algorithms are beginning to demonstrate more than an order of magnitude improvement in production codes, when applied to the Wilson Dirac solver near to physical mass for the light up/down quarks as illustrated in Figure 10. Soon codes for multigrid algorithms will be available on GPUs decreasing the cost of the Dirac solver by two orders of magnitude or more. Future work to be done on these codes includes generalization to domain wall solvers [125], integration into standard evolution codes, and tuning to the chiral limit within specific BSM lattice theories.

A second role of separating short distances from long distance is to decrease the need for internode communication relative to local computation. This approach will investigate Schwarz domain decomposition methods for solvers. As illustrated in Fig 10, even the simplest example of Jacobi block decomposition [126, 127] is enabling improved performance on the Titan GPU machines and good scaling up to O(1000) GPUs. This will be increasingly important on future hierarchical memories anticipated for future Exascale computing platforms.

B. Radial Quantization

One new approach at the very early stages of research is worth mentioning as an illustration of a more radical reformulation of lattice simulation [128]. Here one abandons the traditional Euclidean lattice in favor of one suited to *Radial Quantization*. Indeed this idea has an ancient history starting with the observation that the early covariant quantization of the 2-d conformal string action indeed was given as a radial quantized system with the L_0 Virasoro operator replacing the Hamiltonian. In 1979 Fubini, Hanson and Jackiw [129] suggested radial quantization of field theory in higher dimensions and later in 1985 Cardy suggested lattice implementations for a "strip" in general dimensions [130].

For an exactly conformal field theory, the idea is straight forward. The flat metric for any Euclidean field theory on $\mathbb{R}^D$ can obviously be expressed in radial co-ordinates,

$$d^2s = dx^\mu dx^\mu = r_0^2 e^{2t}(dt^2 + d\Omega_{D-1}^2) \, , \tag{2}$$

where $t = \log(r/r_0)$, introducing an arbitrary reference scale r_0, and where $d\Omega_{D-1}^2$ is the metric on the $\mathbb{S}^{D-1}$ sphere of unit radius. However in the case of an exactly conformal field theory, a local Weyl transformation will also remove the conformal factor, $\exp[2t]$, from the Lagrangian of the quantum theory. The resultant theory is mapped from the Euclidean space $\mathbb{R}^D$ to a D dimensional cylinder, $\mathbb{R} \times \mathbb{S}^{D-1}$. Now one introduces a "uniform" radial lattice in $\tau = \log(r)$ and lattice in angles on concentric spheres $\mathbb{S}^d$ with fixed radius R with the potential advantage that a finite lattice in $log(r)$ grows exponential long relative to the conventional Euclidean lattice. This lattice is a discrete approximation to dilatation $\tau \to \tau + a_t$, inversions $\tau \to -\tau$ and discrete rotations in $O(d-1)$. The full conformal group, including Poincare invariance, should be realized in the continuum limit. A preliminary test of this idea has been made for the Wilson-Fisher fixed point in the 3D Ising model [128] on a lattice discretization of $\mathbb{R} \times S^2$.

The study of lattice field theories for BSM models does call for new theoretical and algorithmic tools many of which are almost certain to significantly strengthen methods even for lattice QCD. For lattice field theory, like many areas of computational physics, the advances in computer hardware at the rate of Moore's law has generally been matched by equal contributions due to fundamental theoretical and algorithmic advances. This should extend the range of practicable BSM projects beyond those suggested in the next section.

IX. RESOURCES FOR STUDIES AT THE ENERGY FRONTIER

In this section we discuss the computational resources needed to reach the scientific goals set out above. At present, members of USQCD are making use of dedicated hardware funded by the DOE through the LQCD-ext Computing Project, as well as a Cray XE/XK computer, and IBM Blue Gene/Q and Blue Gene/P computers, made available by the DOE's INCITE Program. During 2013, USQCD, as a whole, expects to sustain approximately 300 Tflop/s on these machines. USQCD has a PRAC grant for the development of code for the NSF's petascale computing facility, Blue Waters, and expects to obtain a significant allocation on this computer during 2013. Subgroups within USQCD also make use of computing facilities at the DOE's National Energy Research Scientific Computing Center (NERSC), the Lawrence Livermore National Laboratory (LLNL), and centers supported by the NSF's XSEDE Program. In addition, RBC Collaboration, has access to dedicated Blue Gene/Q computers at Brookhaven National Laboratory and the University of Edinburgh. For some time, the resources we have obtained have grown with a doubling time of approximately 1.5 years, consistent with Moore's law, and this growth rate will need to continue if we are to meet our scientific objectives.

The software developed by USQCD under our SciDAC grant enables us to use a wide variety of architectures with very high efficiency, and it is critical that our software efforts continue at their current pace. Over time, the development of new algorithms has had at least as important an impact on our field as advances in hardware, and we expect this trend to continue, although the rate of algorithmic advances is not as smooth or easy to predict as that of hardware. Consequently while algorithmic improvements are expected to play an important role in advancing the USQCD BSM research program, the sustained Teraflops years in the tables below are based solely on the performance of our current code base.

In our work at the energy frontier, members of USQCD are currently using two choices of fermion actions, critically important to achieve physics goals: highly improved staggered fermions and domain wall fermions. However for the pseudo-Nambu-Goldstone boson project, we are developing two color gauge code using Wilson fermions both because they offer a first level project less computationally demanding than domain wall fermions, but also because we are rapidly implementing the Wilson multigrid algorithm into evolution code for both the BlueGene and GPU clusters, which has the potential to reduce the cost substantially. Similar multigrid procedures for domain wall code is under active study [125]. For each subfield, we pick a project that is in our current plans for the next two years, although with the rapid experimental and theoretical developments in BSM physics priorities should clearly be reevaluated beyond this time frame.

A. Scalar Spectrum and χSB Project

Spectroscopy and the determination of the chiral condensate are essential to evaluate the experimental viability of many BSM phenomenological models. In view of the recent discovery at the LHC, the scalar spectrum with Higgs quantum numbers is particularly pressing and becomes a significant part of the measurement time. The main goal is to determine whether a composite dilaton-like particle or light Higgs can emerge in the scalar spectrum of the model [57]. While the generic mechanism for a light dilaton may well be model independent, the detailed parameters are not. We chose a representative model with a fermion flavor doublet in the two-index symmetric (sextet) representation of the SU(3) color gauge group which was introduced in Section 3 [57]. The resource estimates are based on theses simulations which provided the first results consistent with near-conformal behavior, and benchmarks for future studies of the sextet model. The scaling of resources with the fermion mass and the lattice volume does not follow closely the pattern of lattice QCD.

The resources are shown in Table I in teraflop/s-years (TF-Years) to generate the required gauge configurations using the staggered fermion action with stout smeared gauge links and with a minimal number of fermion masses necessary to extrapolate to the chiral limit at two different lattice spacings with appropriate scaling in the lattice volume. The measurements include the scalar spectrum with 0^{++} quantum numbers, the spectroscopy of composite states in the TeV region, and the extrapolation of the chiral condensate to vanishing fermion mass (one TF-Year is defined to be the number of floating point operations produced in a year by a computer sustaining one teraflop/s). The required resource is 82 TF-Years for the project.

Other model studies with the SU(3) color gauge group would require similar resources near conformality. A great deal of effort will be needed to narrow down the model choices to the most promising scenario to capture BSM physics close to the conformal window. The required resources presented in Table I are large compared with lattice QCD. The physical

lattice spacing a (in fermi)	fermion mass (in a units)	lattice volume $V \times T$	config generation (TF-Years)	spectrum and χSB (TF-Years)
2.25×10^{-5}	0.003	$48^3 \times 96$	5	5
2.25×10^{-5}	0.004	$48^3 \times 96$	4.8	5
2.25×10^{-5}	0.005	$48^3 \times 96$	4.4	5
1.75×10^{-5}	0.0023	$64^3 \times 128$	9	9
1.75×10^{-5}	0.0030	$64^3 \times 128$	8.6	9
1.75×10^{-5}	0.0035	$64^3 \times 128$	8.2	9

TABLE I. Resources for SU(3) two flavor sextet project. The first column is the approximate lattice spacing a in fermi, as set from the measured VEV in the chiral limit at fixed gauge coupling using the physical value v = 246 GeV for $\mathrm{a} \cdot \mathrm{v} = 0.029$ on the coarser lattice. The second column is the fermion mass of the run and the third is the lattice volume with twice the lattice points in the temporal direction. The fourth column gives the resources needed to generate 1,000 configurations from 10,000 MD time units. The fifth column shows the required resources to determine the full spectrum of the model based on the generally well-tested balance between configuration generation and measurements.

point of BSM models is at vanishing Goldstone pion mass making the chiral extrapolation more difficult and more costly. The small finite Goldstone masses required for extrapolation to the physical point is a significant demand on resources. Not unrelated, increased finite volume effects and strong chiral condensate enhancement near the conformal window are new and costly challenges compared with lattice QCD.

B. PNGB SU(2) Color Higgs Project

The investigation of strongly coupled theories with a composite Higgs as a pseudo-Goldstone boson has received considerable attention in the last few years. Lattice field theory is needed to give a definitive non-perturbative realization of this mechanism. We propose to undertake a careful study of the dynamics of the minimal composite PNGB Higgs model: SU(2) gauge theory with $N_f = 2$ fermions in the fundamental representation, which lies at the core of many phenomenological extensions. In much the same manner as described above for the technicolor example, the low energy effective chiral parameters, F and $\Sigma(\mu)$ can be found, the S parameter determined and the masses for higher spin resonances (e.g. ρ, a_1 mesons, *etc.*) computed.

Over the past few years, the USQCD SciDAC software libraries (QLA, QDP/C, QDP++, FUEL, Chroma) has been extended to support SU(2) gauge theories for both Wilson and domain wall fermions. Several exploratory calculations have started for SU(2) wiht $N_f = 2$ and 6 fundamental matter fields. This phase has focused on mapping out the bare parameter space using the finite temperature phase transition and the Schrödinger Functional running coupling.

The next step is to move gradually to larger volumes at zero temperature with Wilson fermions to study the low energy spectrum nearer to the chiral limit. The resources are give

min. M_H (GeV)	lattice volume $V \times T$	MD trajectory (time units)	config generation (TF-Years)
650	$32^3 \times 64$	10000	1.2
520	$40^3 \times 80$	10000	8.6
433	$48^3 \times 96$	10000	44
371	$56^3 \times 112$	10000	180

TABLE II. Resources to generate gauge configuration ensembles in SU(2) gauge theory with $N_f = 2$ fermions in the fundamental representation. The inverse lattice spacing is held fixed at $a^{-1} = 5$ TeV. The first column gives the minimum Higgs mass that can fit in the volume assuming $LM_H \geq 4$ and the second column gives the corresponding lattice volume. The fourth column gives the resources in teraflop/s-years (TF-Years) needed to generate 10,000 molecular dynamics time units (1,000 equilibrated gauge configurations) for each ensemble for the Wilson fermions. (One TF-Year is defined to be the number of floating point operations produced in a year by a computer sustaining one teraflop/s).

in Table II. The estimates are based on the benchmarks of the USQCD software system for SU(2) gauge theories and scaling experience from SU(3) lattice QCD with 2 flavors. The first priority of this project is to use Wilson fermions for investigating the most important features of the PNGB mechanism. The initial inverse lattice spacing will be $(aF)^{-1} \approx 20$ or $a^{-1} \approx 5$ TeV since $F \approx 250$ GeV is the Higgs vev. This will enable study of Higgs masses in the range $M_H \sim 650$–1000 GeV, well above the physical Higgs mass of 126 GeV. As in the case of lattice QCD for the past two decades, we will have to perform calculations where the Higgs mass is heavier than the physical Higgs mass and extrapolate our results using chiral perturbation theory. Also it should be noted that the experimental mass of the Higgs maybe reduced substantially by perturbative contributions when the this strong dynamics composite Higgs is coupled to the electro-weak sector of the standard model.

In addition to the proposed Wilson simulations, exploratory study will be continued with domain wall fermions. While both the Wilson and domain wall formulations have an exact Sp($2N_f$) global symmetry at finite lattice spacing, only domain wall fermions have the larger SU($2N_f$) global symmetry in the chiral limit at finite lattice spacing. The enhanced chiral symmetries of domain wall fermions maybe important to rigorously establish the precise nature of the vacuum alignment mechanism of the model and in chiral extrapolations to the physical point, which for BSM models requires an exactly vanishing techni-pion mass. However the prices of domain wall fermions, as for lattice QCD, is at least an order of magnitude increase in computational cost. Consequently production using domain wall fermions for large lattice volumes is out of reach in the next five years unless ongoing algorithmic optimizations for the Möbius DW action [128, 131] and multigrid solvers [124, 125] are able to reducing their cost substantial.

C. Supersymmetric Yang-Mills theories

Supersymmetry is interesting both for its potential for phenomenological extensions of the standard model with light scalars, and for fundamental properties of field theory and their gravity duals in the AdS/CFT correspondance. We discuss resource needs for both of these programs in the context of 1.) $\mathcal{N} = 1$ Yang-Mills and super QCD and 2.) $\mathcal{N} = 4$ Yang-Mills. Resource estimates are based on investigations of $\mathcal{N} = 1$ supersymmetric Yang Mills theory (gauge bosons and gauginos) with phyics results of the gluino condensate and string tension in these theories.

1. $\mathcal{N} = 1$ *Yang-Mills and super QCD*

Current simulations of $\mathcal{N} = 1$ super Yang-Mills using domain wall fermions the trajectory takes 30 mins on a $16^3 \times 32 \times L_s = 24$ lattice on a single rack of the BG/L. Since our initial work has revealed that the residual mass is larger than typically encountered in QCD and falls off slowly as a function of L_s, we will also implement the Möbius DWF improvement [128, 131] which falls of like $1/L_s^2$.

lattice volume $V \times T$	wall separation L_s	bare coupling $\beta = 4/g_0^2$	trajectory. (time units)	config generation (TF-Years)
$16^3 \times 32$	16	2.3	10000	2.3
$16^3 \times 32$	24	2.3	10000	3.5
$16^3 \times 32$	16	2.4	10000	3.6
$16^3 \times 32$	24	2.4	10000	5.5
$16^3 \times 32$	48	2.4	10000	11.0
$24^3 \times 48$	24	2.4	10000	41.8
$24^3 \times 48$	48	2.4	10000	83.6
$24^3 \times 48$	48	2.5	10000	200

TABLE III. Resouces needed for DWF simulation of $SU(2)$ $\mathcal{N} = 1$ Yang-Mills theory. As in previous studies, we set the bare fermion mass $m_f = 0$ for these estimates. Residual masses fall in the range 0.02-0.1 for these values of the parameters using Shamir (non-Möbius) domain wall fermions. Using two lattice volumes, three lattice spacings and several values of L_s should allow for careful extrapolation to the chiral continuum limit while maintaining control over finite volume effects

The spectrum of $\mathcal{N} = 1$ super Yang-Mills requires the computation of disconnected correlators. Using stochastic estimators and dilution techniques these can be computed in a time comparable to the time needed to generate configurations. Table III indicates the scale of resources that will be needed to compute the condensate and low lying spectrum of $\mathcal{N} = 1$ super Yang-Mills. These calculations can be completed in principle within a two year timeframe on leadership class machines like the BG/Q.

While the spectrum and structure of $\mathcal{N} = 1$ Yang-Mills is clearly very interesting this work also forms the basis for the much more difficult study of super QCD. Super QCD will require significantly more resources; a series of DWF simulations must be performed to tune the scalar (squark) sector of the theory for a fixed value of the gauge coupling and gaugino mass. In addition it is necessary to simulate for larger numbers of colors and flavors to access the metastable susy breaking vacua. The simplest system which is expected to exhibit metastable vacua corresponds to four colors and five flavors. If we model the dependence on N_f and N_c via the naive formula $T \sim N_f \times (N_c^2 - 1)$ we find that a single DWF simulation on a $16^3 \times 32 \times 24$ lattice will require approximately 137 Teraflop-years. Clearly super QCD will require Petascale resources and a longer time frame to complete.

2. $\mathcal{N} = 4$ *Yang-Mills*

Another high priority is to push into large scale simulation of $\mathcal{N} = 4$ Yang-Mills. Preliminary work has focused on surveying the phase diagram of the theory with lattices of size $8^3 \times 16$ over a wide range of gauge coupling and scalar mass. The current parallel code (based on the FermiQCD communication libraries) is capable of producing 1 trajectory in 1.25 hrs CPU time on a single node of the Fermilab Ds cluster (32 cores). More realistic simulations on a $16^3 \times 32$ lattices correspond to a 5.8 Teraflop-yr calculation. Long term goals would require simulating lattices as large as $32^3 \times 64$ and would clearly require Petascale resources.

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