

# Lattice QCD Calculations of Nucleon Properties at the Intensity Frontier

(USQCD Collaboration)  
(Dated: November 11, 2011)

We review recent successes of lattice QCD and outline how calculations of nucleon matrix elements will aid experiments with nucleons searching for effects beyond Standard Model. Our discussion focuses on three examples and includes assessments of current uncertainties and forecasts for improvement over the coming decade.

## I. INTRODUCTION

During the past decade, lattice QCD has made substantial progress in several areas that influence particle physics, nuclear physics, and astrophysics. Once enough computing and algorithmic power became available to treat virtual quark-antiquark pairs (the “sea” quarks) realistically, the results of lattice-QCD calculations rapidly reproduced a wide variety of hadron properties [1]. The same techniques then enabled genuine predictions of  $D$  meson semileptonic form factors,  $D$ - and  $D_s$ -meson leptonic-decay constants, and the mass of the  $B_c$  meson [2]. Lattice QCD now plays an important role in quark flavor physics, yielding indispensable results for neutral meson mixing and leptonic decay rates, and important results for semileptonic form factors [3, 4]. These results not only constrain the Cabibbo-Kobayashi-Maskawa (CKM) matrix but also enable indirect searches for new particles.

The success of lattice QCD is not confined to flavor physics alone. The nucleon mass, one of the original objectives, has been computed with a precision of about 2% [5]. Nambu’s ideas of spontaneous chiral symmetry breaking, once strong beliefs, have been verified via direct calculation from the QCD Lagrangian [6]. Connected to these developments are the only *ab initio* determinations of the light-quark masses [7]: the up-, down-, and strange-quark masses turn out to be small—about four, nine, and 180 times the electron mass, respectively. Lattice QCD meanwhile provides the most accurate determinations of the strong coupling  $\alpha_s$  [8] and competitive determinations of the bottom- and charm-quark masses. These results connect the QCD probed in high-energy processes with the QCD description of hadrons.

With matrix elements from flavor physics and the nucleon mass under control, a next step is to compute nucleon matrix elements [9]. These are helpful for interpreting experiments on the neutron electric dipole moment, nucleon  $\beta$  decay, and nucleon structure, as well as planning searches for proton decay. Another recent development is the calculation of virtual hadron properties, which influence electroweak parameters. One example is the evolution of QED’s fine structure constant from electronic to  $Z$ -pole scales. More prominent for the intensity frontier are related calculations of hadronic contributions to the muon’s anomalous magnetic moment [10, 11].

There are many other lattice-QCD calculations that are beyond the scope of this document, which we shall mention only briefly. Together with the CKM matrix, the

quark masses and  $\alpha_s$  constrain speculation about unification of the forces and other physics beyond the reach of accelerators. Calculations of the strangeness content of the nucleon are needed to understand dark-matter detection experiments [12, 13]. Studies of the QCD phase transition with lattice QCD have shown that the quark masses, though small, are just large enough to make the transition a crossover [14, 15]. Previously, research on the early universe assumed the transition was of first order, with phenomena like bubbles of hadrons; we now know that the universe did not cool this way. Calculations of hadron-hadron interactions help us understand the physics of neutron stars, particularly whether neutrons could dissociate into  $K\Lambda$  pairs [16]. Lattice gauge theories varying the number of colors and matter content are shedding light on the dynamics of technicolor models of electroweak symmetry breaking [17].

The rest of this document is organized as follows. Section II discusses three examples of nucleonic matrix elements that play a role in intensity-frontier experiments, including forecasts of future precision that we expect to obtain in these matrix elements. We conclude with some remarks of the broader role of lattice QCD at the intensity frontier in Sec. III. For similar information on quark-flavor physics and the muon anomalous magnetic moment, please consult documents submitted to the Heavy Quark and Charged Lepton WGs, respectively.

## II. LATTICE QCD AND PRECISION EXPERIMENTS WITH NUCLEONS

Nucleon matrix elements are generally more computationally demanding than mesonic matrix elements, because the statistical noise grows with Euclidean time  $t$  as  $e^{(M_N - 3M_\pi/2)t}$  for each nucleon in the system. Thus, results with high precision in the nucleon sector lag those in the meson sector. Furthermore, extrapolating to the physical light quark masses is more challenging for baryons, since chiral perturbation theory converges more slowly. The latter issue is likely to be brought under control in the near future, as ensembles of lattices begin to be generated with physical  $u$ ,  $d$  (and  $s$  and  $c$ ) quark masses [18–20]. This should greatly reduce the systematic uncertainties. Other systematics, such as finite-volume effects, renormalization and excited-state contamination can be systematically reduced by improved algorithms and by increasing the computational resources devoted to the calculations.

We highlight three areas in which lattice-QCD calcula-

tions will play an essential role in interpreting the results from nucleon experiments at the intensity frontier.

**Neutron electric dipole moment (NEDM):** A non-vanishing NEDM  $d_N$  requires time-reversal and parity violation. In the Standard Model (SM), there is a contribution from the CP-odd phase in the CKM matrix, but this leads to a prediction,  $\sim 10^{-30}$  e cm, that is far below the sensitivity of foreseeable experiments. A contribution from the  $\theta$  term is *a priori* much larger, but experiments set a tight limit  $|\theta| \lesssim 10^{-10}$ . This constraint is not known precisely, because one needs a nonperturbative method to calculate  $d_N/\theta$ ; this is what lattice-QCD calculations can provide.

The NEDM is a challenging calculation, but in the last few years significant progress has been made using three different methods: (i) directly adding a CP-odd term to the Lagrangian and studying the form factor  $F_3$  [21–23]; (ii) calculating the energy difference for two different spin states of the nucleon at zero momentum in the presence of an external static, uniform electric field [24]; (iii) examining the product of the anomalous magnetic moment of neutron and  $\tan 2\alpha$ , where  $\alpha$  is the nucleon-spin phase shift due to the  $\theta$ -term, in a CP-violating system [25]. Currently, statistical errors are at the level of 30% after chiral extrapolation. More precise calculations from various groups are currently in progress; within the next 5 years, lattice QCD should be able to make predictions at better than 10%, and one can hope that percent-level computations will be available on a 10-year timescale.

We note also that the NEDM plays an important role in constraining theories beyond the Standard Model (BSM). Many BSM models predict values that are higher than the experimental upper bound, and are thus ruled out. This includes parts of the parameter space for certain SUSY models. In some cases the detailed predictions of these models also require hadronic matrix elements that lattice QCD can provide.

**New physics of tensor and scalar contributions to neutron beta decay:** BSM physics at the TeV scale can be probed by searching for new scalar and tensor interactions in neutron beta decay. These interactions can be measured by experiments with ultra-cold neutrons, such as the UCNb/B experiment at LANL and the Nab and abBA experiments at ORNL. In order to constrain BSM theories, one needs to know the scalar and tensor charges of the nucleon. Quark models of QCD can provide only rough bounds on these charges, while lattice QCD can provide precise results.

Currently, there are only a few lattice-QCD calculations of  $g_T$  and one of  $g_S$ , with the errors being about 35% and 50%, respectively [9, 26–28]. Future calculations will substantially reduce the errors by improving the chiral extrapolation and providing non-perturbative estimates of the renormalizations constants for the lattice operators. We expect results with 5% errors on the

5- to 10-year timescale.

**The nucleon axial charge and  $|V_{ud}|$ :** The nucleon axial charge  $g_A$  is one of the fundamental measures of nucleon structure and is determined experimentally from neutron beta decay to high precision, about 0.2%. The calculation of  $g_A$  in lattice QCD is relatively straightforward, and the present uncertainty (including systematic errors) is around 6–10% [29–32]. The largest uncertainty comes from the chiral extrapolation and finite-volume effects. Simulations at, or close to, physical light-quark masses should greatly improve the accuracy, leading to percent-level calculations in five years. On this time scale the calculation of  $g_A$  will serve as a benchmark of the accuracy of lattice QCD calculations for nucleon matrix elements. In the longer term, if sub-percent calculations become feasible, lattice input on  $g_A$  together with neutron lifetime measurements will lead to a competitive extraction of  $|V_{ud}|$ , that is free of nuclear structure uncertainties.

### III. OUTLOOK

The intensity frontier complements high- $p_T$  physics in at least two ways. Observations discrepant with the Standard Model are discoveries in their own right. More generally, precise measurements offer constraints on the identity of high-mass particles, such as those that may be observed at the LHC.

The interpretation of precise experiments at the intensity frontier requires comparable precision in the corresponding theoretical calculations. In many experiments at the intensity frontier, hadrons are involved in an essential way, leading inevitably to the need to calculate hadronic properties, in particular matrix elements of operators that arise when integrating out short-distance SM or BSM particles. Even in some leptonic observables, the precision is such that virtual hadrons make a significant contribution. Lattice gauge theory provides a set of numerical methods for computing these hadronic properties, within a framework where uncertainties can be systematically reduced.

In the past several years, the right combination of algorithms, computing power and infrastructure, and collaboration structure has come together, leading to a plethora of results. Some of these results are quantitatively impressive and bode well for future experiments at the intensity frontier. Others are qualitatively interesting and connect to the energy and cosmic frontiers. We see special opportunities in quark flavor physics, nucleon matrix elements, and muon  $g - 2$ . With continued support, we look forward to the coming decade's interplay between experiment, theory, and lattice QCD.

- 
- [1] C. T. H. Davies *et al.* (HPQCD, MILC, and Fermilab Lattice), Phys. Rev. Lett. **92**, 022001 (2004), arXiv:hep-lat/0304004.
- [2] A. S. Kronfeld (Fermilab Lattice, MILC, and HPQCD), J. Phys. Conf. Ser. **46**, 147 (2006), arXiv:hep-lat/0607011.
- [3] J. Laiho, E. Lunghi, and R. S. Van de Water, Phys. Rev. **D81**, 034503 (2010), arXiv:0910.2928 [hep-ph], updates at <http://www.latticeaverages.org>.
- [4] G. Colangelo *et al.* (FLAG), Eur. Phys. J. **C71**, 1695 (2011), arXiv:1011.4408 [hep-lat].
- [5] S. Dürr *et al.* (BMW), Science **322**, 1224 (2008), arXiv:0906.3599 [hep-lat].
- [6] H. Fukaya *et al.* (JLQCD), Phys. Rev. Lett. **104**, 122002 (2010), arXiv:0911.5555 [hep-lat], Erratum: Phys. Rev. Lett. **105**, 159901 (2010).
- [7] J. Laiho (2011), arXiv:1106.0457 [hep-lat].
- [8] S. Bethke *et al.* (2011), arXiv:1110.0016 [hep-ph].
- [9] H.-W. Lin (2011), arXiv:1109.2542 [hep-lat].
- [10] T. Blum, Phys. Rev. Lett. **91**, 052001 (2003), arXiv:hep-lat/0212018.
- [11] Institute for Nuclear Theory Workshop (February 28–March 4, 2011) <http://www.int.washington.edu/PROGRAMS/11-47w/>.
- [12] D. Toussaint and W. Freeman (MILC), Phys. Rev. Lett. **103**, 122002 (2009), arXiv:0905.2432 [hep-lat].
- [13] J. Giedt, A. W. Thomas, and R. D. Young, Phys. Rev. Lett. **103**, 201802 (2009), arXiv:0907.4177 [hep-ph].
- [14] Y. Aoki, G. Endrödi, Z. Fodor, S. D. Katz, and K. K. Szabó, Nature **443**, 675 (2006), arXiv:hep-lat/0611014.
- [15] A. Bazavov *et al.*, Phys. Rev. **D80**, 014504 (2009), arXiv:0903.4379 [hep-lat].
- [16] S. R. Beane, W. Detmold, K. Orginos, and M. J. Savage, Prog. Part. Nucl. Phys. **66**, 1 (2011), arXiv:1004.2935 [hep-lat].
- [17] L. Del Debbio, PoS **LATTICE2010**, 004 (2010), arXiv:1102.4066 [hep-lat].
- [18] S. Aoki *et al.* (PACS-CS), Phys. Rev. **D81**, 074503 (2010), arXiv:0911.2561 [hep-lat].
- [19] S. Dürr *et al.* (BMW), Phys. Lett. **B701**, 265 (2011), arXiv:1011.2403 [hep-lat].
- [20] A. Bazavov *et al.* (MILC), PoS **LATTICE2010**, 320 (2010), arXiv:1012.1265 [hep-lat].
- [21] F. Berruto, T. Blum, K. Orginos, and A. Soni, Phys. Rev. **D73**, 054509 (2006), arXiv:hep-lat/0512004.
- [22] E. Shintani *et al.*, Phys. Rev. **D72**, 014504 (2005), arXiv:hep-lat/0505022.
- [23] S. Aoki *et al.* (2008), arXiv:0808.1428 [hep-lat].
- [24] E. Shintani, S. Aoki, and Y. Kuramashi, Phys. Rev. **D78**, 014503 (2008), arXiv:0803.0797 [hep-lat].
- [25] G. Schierholz, PoS **Lattice2011** (2011).
- [26] R. G. Edwards *et al.*, PoS **LAT2006**, 121 (2006), arXiv:hep-lat/0610007.
- [27] Y. Aoki *et al.*, Phys. Rev. **D82**, 014501 (2010), arXiv:1003.3387 [hep-lat].
- [28] T. Bhattacharya *et al.* (2011), arXiv:1110.6448 [hep-ph].
- [29] H.-W. Lin and K. Orginos, Phys. Rev. **D79**, 034507 (2009), arXiv:0712.1214 [hep-lat].
- [30] H.-W. Lin, T. Blum, S. Ohta, S. Sasaki, and T. Yamazaki, Phys. Rev. **D78**, 014505 (2008), arXiv:0802.0863 [hep-lat].
- [31] T. Yamazaki *et al.* (RBC+UKQCD), Phys. Rev. Lett. **100**, 171602 (2008), arXiv:0801.4016 [hep-lat].
- [32] J. D. Bratt *et al.* (LHPC), Phys. Rev. **D82**, 094502 (2010), arXiv:1001.3620 [hep-lat].