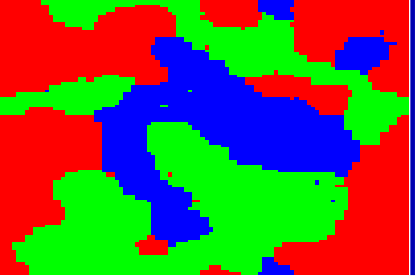
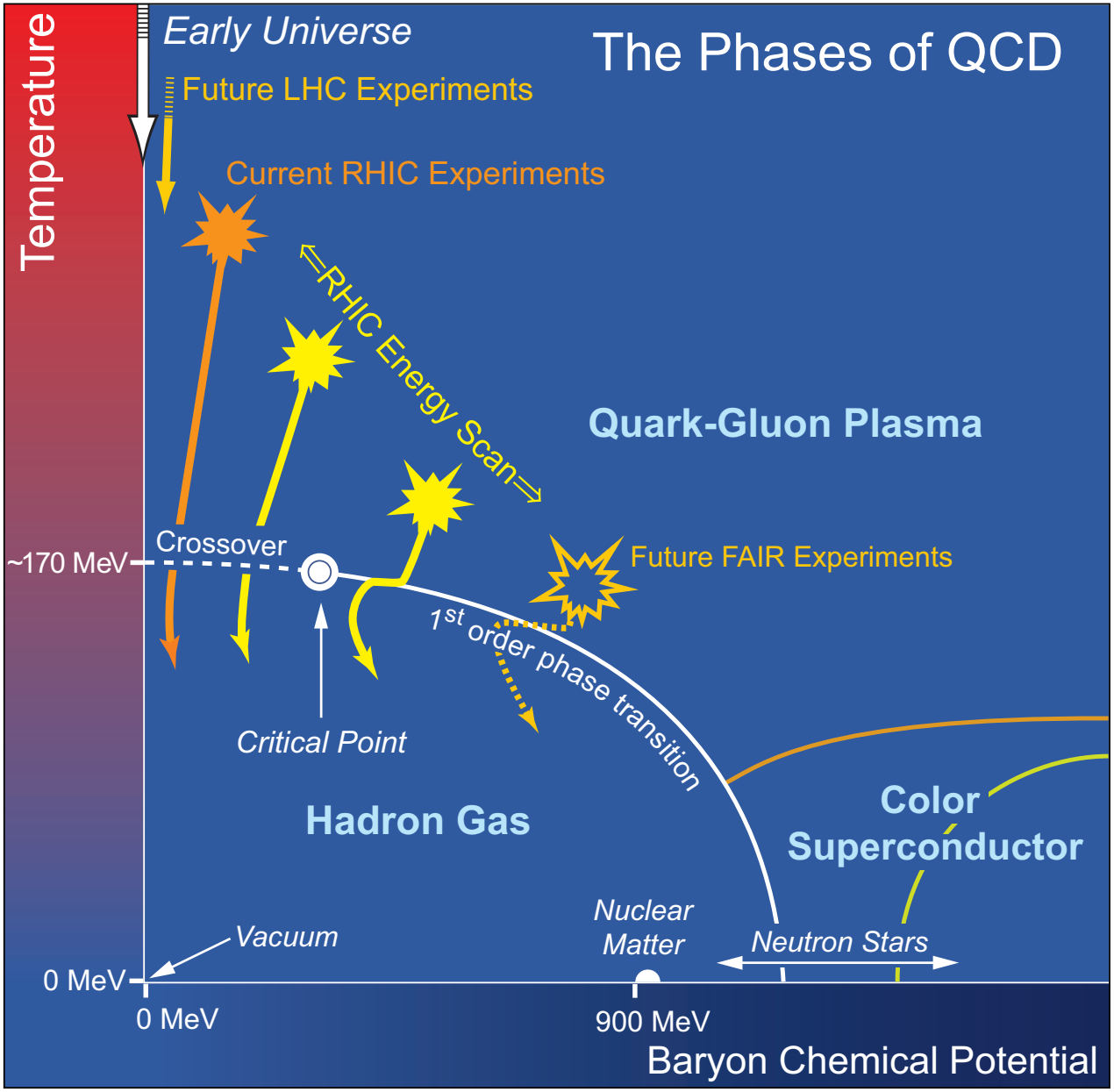


High Temperature/Density QCD

Frithjof Karsch, BNL and Bielefeld University



physics of the early universe

hot: $T \sim 10^{12} K$

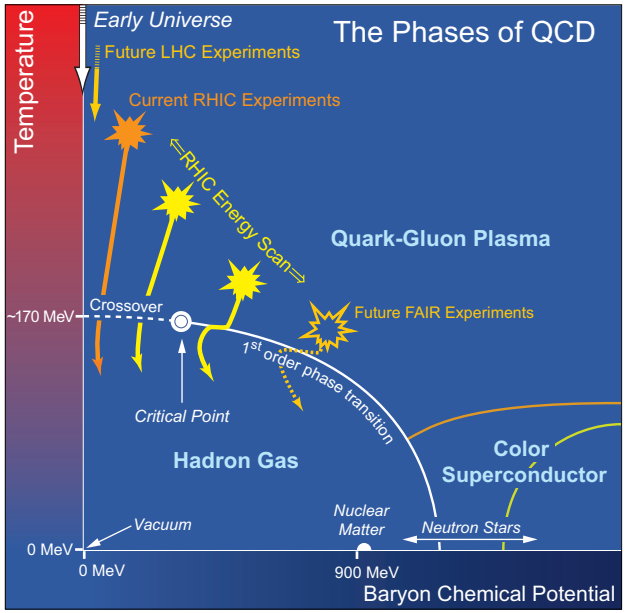
OUTLINE

- Overview
- Equation of State, T_c
- Transport Coefficients
- Quarkonium
- Charge Fluctuations
- Critical Point

properties of compact stars

dense: $n_B \sim 10 n_{NM}$

The Phases of Nuclear Matter
Key Questions (NP LRP 2007)



study properties of strongly interacting nuclear matter and elementary particles under extreme conditions

strongly interacting

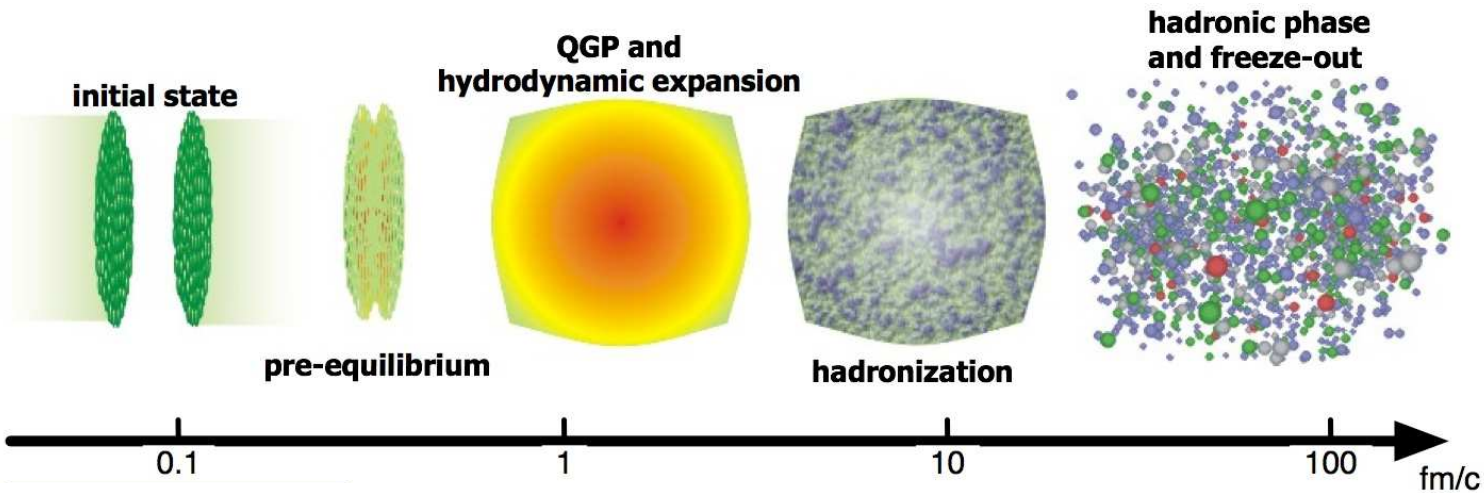
$\Rightarrow$ QCD = Quantum Chromo Dynamics

GOAL: learn about basic mechanisms that characterize QCD

chiral symmetry breaking; confinement; asymptotic freedom; axial anomaly

- What are the phases of strongly interacting matter, and what role do they play in the cosmos?
- What does QCD predict for the properties of strongly interacting matter?
- What governs the transition of quarks and gluons into pions and nucleons?

Heavy Ion collisions and the QGP



simple Bjorken model $\sim$ 1-d hydrodynamic expansion

equation of motion: $\partial_\mu T^{\mu\nu} = 0$

energy density: $\frac{d\epsilon}{d\tau} + \frac{1}{\tau}(\epsilon + p) - \frac{1}{\tau^2}\left(\frac{4}{3}\eta + \theta\right) = \epsilon(\tau_0)\delta(\tau - \tau_0)$

$\Rightarrow$ understanding the time evolution requires knowledge of the equation of state and transport coefficients (bulk (θ) and shear (η) viscosity)

most cited US-Thermo papers 2008

1) M. Cheng et al (hotQCD Collaboration), The QCD equation of state with almost physical quark masses, Phys. Rev. D77, 014511 (2008) [63 citations]

5) M. Cheng et al (RBC-Bielefeld Collaboration), The Transition temperature in QCD, Phys. Rev. D74, 054507 (2006) [45 citations]

10) S. Datta et al (BNL-Bielefeld), Behavior of charmonium systems after deconfinement, Phys. Rev. D69, 094507 (2004) [39 citations]

20) H. B. Meyer (MIT), A Calculation of the shear viscosity in SU(3) gluodynamics, Phys. Rev. D76, 101701 (2007) [34 citations]

24) H. B. Meyer (MIT), A Calculation of the bulk viscosity in SU(3) gluodynamics, Phys. Rev. Lett. 100, 162001 (2008) [34 citations]

33) O. Kaczmarek and F. Zantow (Bielefeld-BNL), Static quark anti-quark interactions in zero and finite temperature QCD. I. Heavy quark free energies, running coupling and quarkonium binding, Phys. Rev. D71, 114510 (2005) [30 citations]

...some statistics from SPIRES...

- 50 top cited papers in hep-lat during each year in 2004-2008

year	# Thermo papers (WW)	# Thermo papers (US)	the top-cited Thermo (WW)	the top-cited Thermo (US)
2004	12	1	3 (finite-μ)	48 (Quarkonium)
2005	13	1	2 (finite-μ)	10 (Quarkonium)
2006	14	1	2 (MEM)	6 (Quarkonium)
2007	24	8	2 ((2+1)-f, T_c)	4 ((2+1)-f, T_c)
2008	20	6	1 ((2+1)-f, EoS)	1 ((2+1)-f, EoS)

- about 40% of the 50 top cited papers in hep-lat deal with QCD thermodynamics
- about 1/3 of these papers involve authors from the US (=USQCD). They gained increasing impact

Bulk thermodynamics

Goal: QCD thermodynamics with realistic quark masses in (2+1)-f QCD and controlled extrapolation to the continuum limit;

$\Rightarrow$ T_c, EoS,.. for $\mu_q \geq 0$

RBC-Bielefeld collaboration

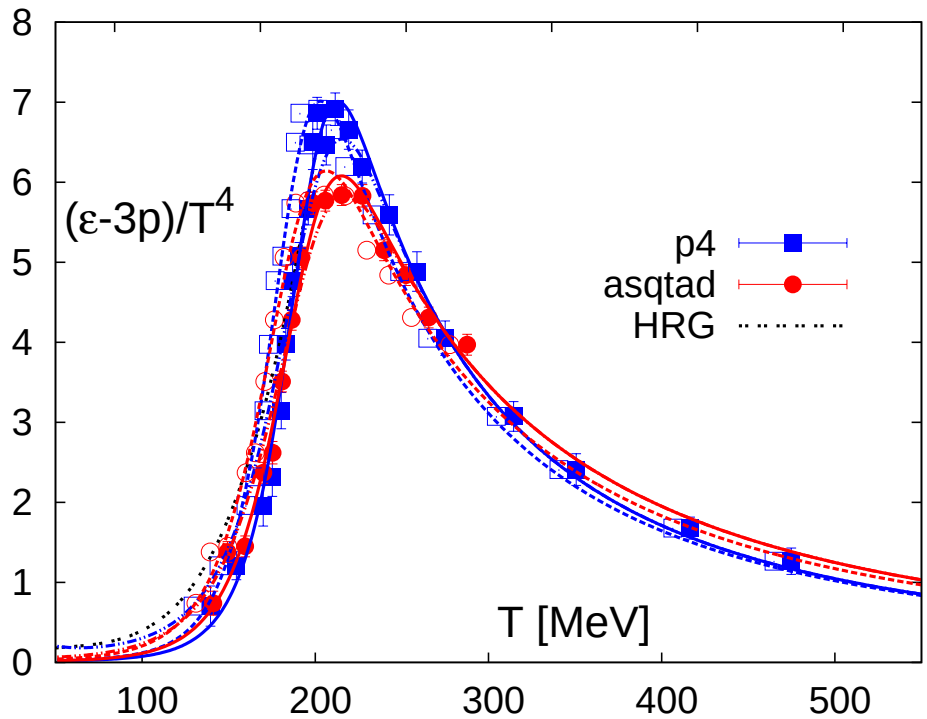
PRD77, 014511 (2008)

- $N_\tau = 4,\ 6$: bulk thermodynamics on a line of constant physics (LCP):
  (i) use $m_l = 0.1 m_s$, corresponding to $m_\pi \simeq 220$ MeV;
  (ii) tune m_s to physical strange quark mass using $m_K,\ m_{\bar{s}s}$ at all values of the cut-off
- analyze EoS in a wide T-range: $140\ \mathrm{MeV} \lesssim T \lesssim 800\ \mathrm{MeV}$
- extend analysis to $N_\tau = 8$; compare p4 and asqtad results: joint project of RBC, Bielefeld, MILC, LANL and LLNL
  $\Rightarrow$ hotQCD collaboration, arXiv:0903.4379

Parametrization of LGT-EoS: Input to hydrodynamic models

- easy to use analytic parametrization of LGT-EoS for hydrodynamic model calculations:
- matches to hadron resonance gas model at low temperature

$$\frac{\epsilon - 3p}{T^4} = \left(1 - \frac{1}{[1 + e^{(T-c_1)/c_2)}]^2}\right)\left(\frac{d_2}{T^2} + \frac{d_4}{T^4}\right)$$



A. Bazavov et al. (hotQCD Collaboration), arXiv:0903.4379 [hep-lat]

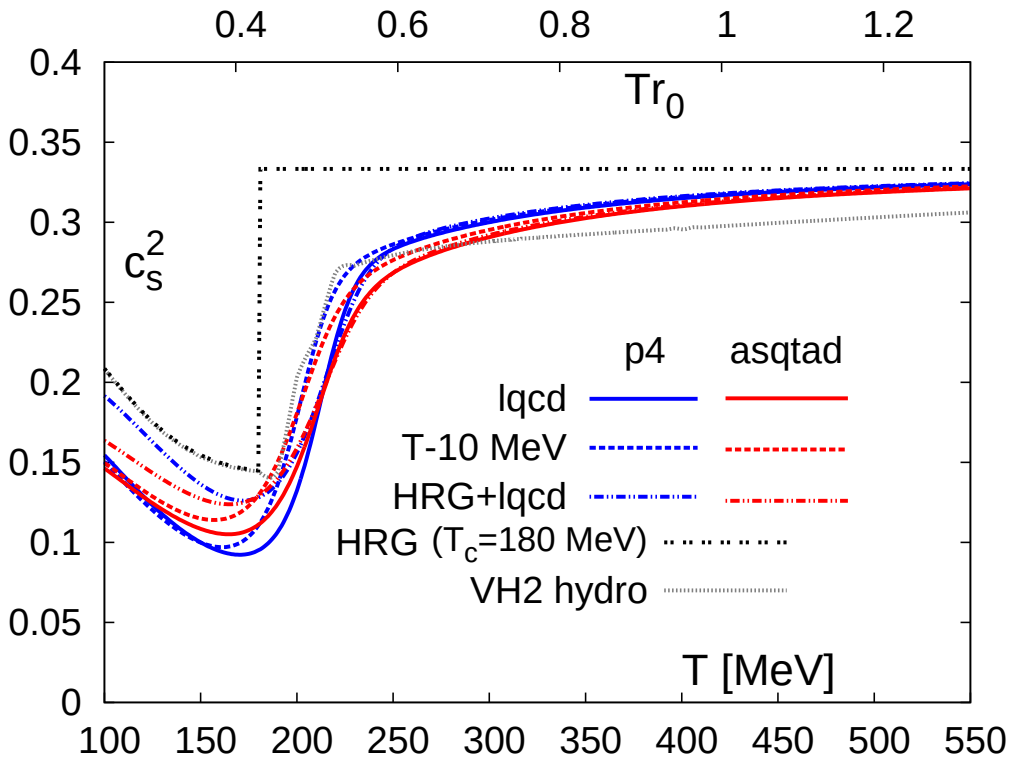
Data	d_2 [GeV]^2	d_4 [GeV]^4
p4-10 MeV	0.241(6)	0.0035(9)
HRG+p4	0.24(2)	0.0054(17)
asq-10 MeV	0.293(6)	0.00
HRG+asqtad	0.312(5)	0.00

Data	c_1 [GeV]	c_2 [GeV]
p4-10 MeV	0.1938(6)	0.01361(4)
HRG+p4	0.2073(6)	0.0172(3)
asq-10 MeV	0.1943(6)	0.01670(4)
HRG+asqtad	0.2048(6)	0.0188(4)

EoS and velocity of sound

- $p/\epsilon \Rightarrow$ velocity of sound:

$$c_s^2 = \frac{\mathrm{d}p}{\mathrm{d}\epsilon} = \epsilon \frac{\mathrm{d}(p/\epsilon)}{\mathrm{d}\epsilon} + \frac{p}{\epsilon} \equiv \frac{s}{c_V}$$



hydro-expansion:

$p/\epsilon < 1/3$

$\Rightarrow$ slows down expansion;
$\Rightarrow$ increases plasma lifetime

e.g.

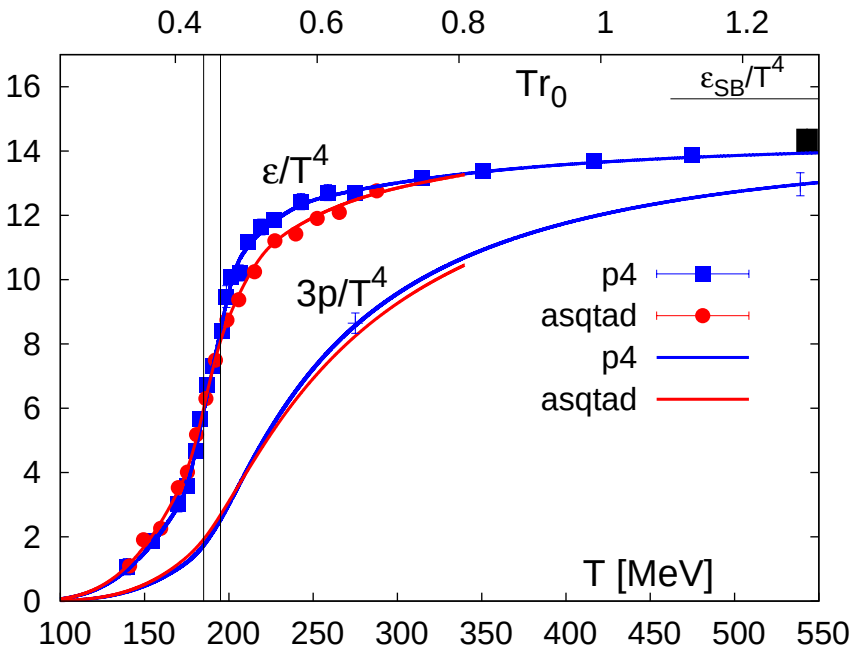
$1 \leq \epsilon$ [GeV/fm^3]≤ 10

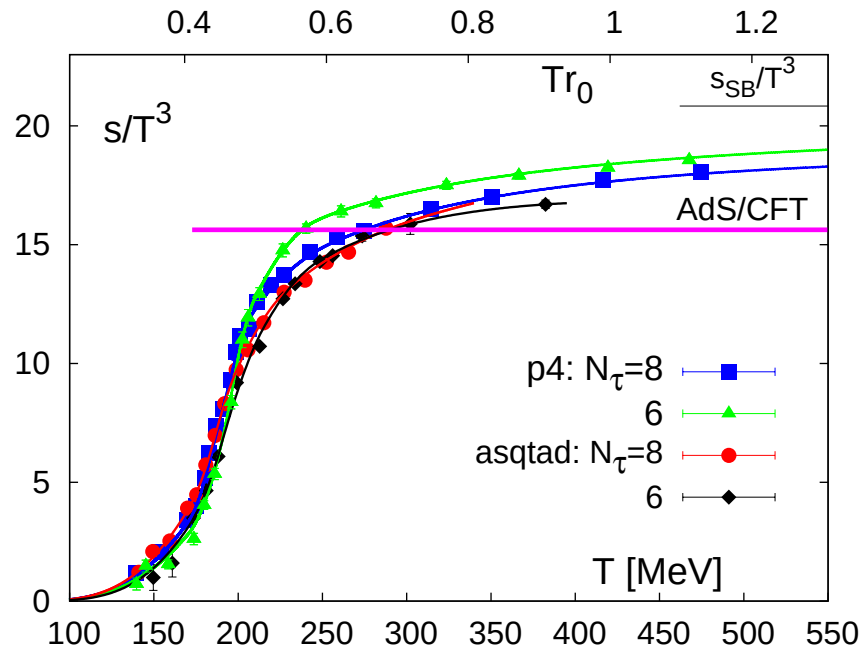
$\Rightarrow \Delta\tau \simeq$ 5.5 fm (ideal gas)
$\Rightarrow \Delta\tau \simeq$ 7 fm (LGT EoS)

hotQCD

Pressure, Energy and Entropy

- p/T^4 from integration over $(\epsilon - 3p)/T^5$;
  starting integration at $T = 0$ MeV with $p(0) = 0$;
  use hadron resonance gas at $T_0 = 100$ MeV to estimate systematic error:
  $[p(T_0)/T_0^4]_{HRG} \simeq 0.265$
- high-T region is well under control; significant deviations from conformal limit AND AdS/CFT

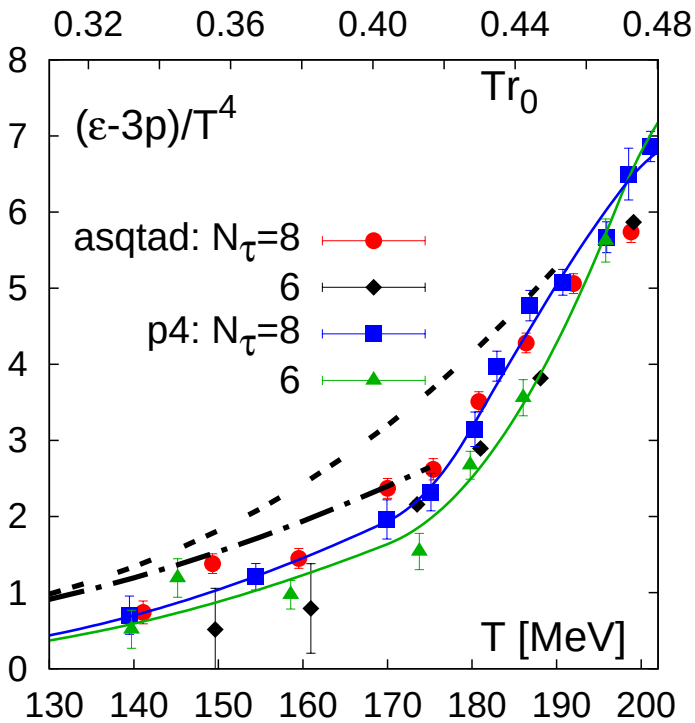




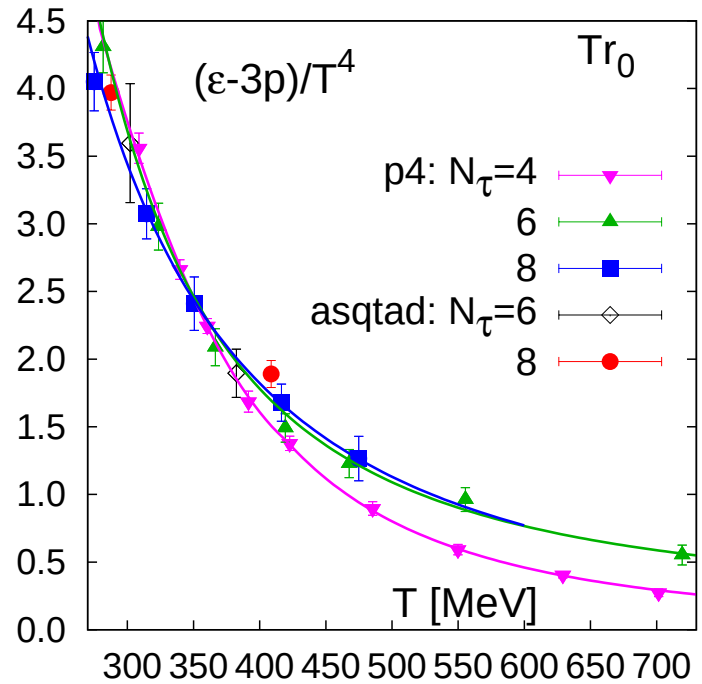
band:
$185 \text{ MeV} \leq T \leq 195 \text{ MeV}$

hotQCD: p4 vs. asqtad (arXiv:0903.4379)

Open Issues: Low and High-T asymptotics



LGT vs resonance gas



LGT vs. pert. theory

- approach to continuum limit
  $\longrightarrow$ $N_\tau = 6,\ 8$
  $\longrightarrow$ $\mathcal{O}(5MeV)$ shift of T-scale
- LGT below HRG for $T \lesssim 180$ MeV
  coarser lattice, larger cut-off effects
  but: Which HRG?
  M_{max} =1.5 GeV, 2.5 GeV,..

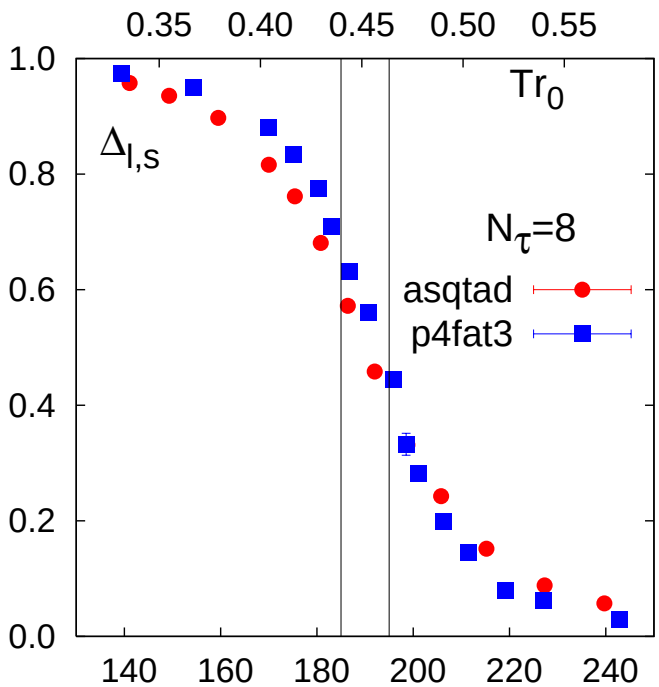
- approach to continuum limit
  $\longrightarrow$ $N_\tau = 6,\ 8$: no significant cut-off dependence for $T \gtrsim 300$ MeV
- strong deviations from conformal limit: find $(\epsilon - 3p)/T^4 \sim a/T^2 + b/T^4$ for $300\mathrm{MeV} \lesssim T \lesssim 700\mathrm{MeV}$

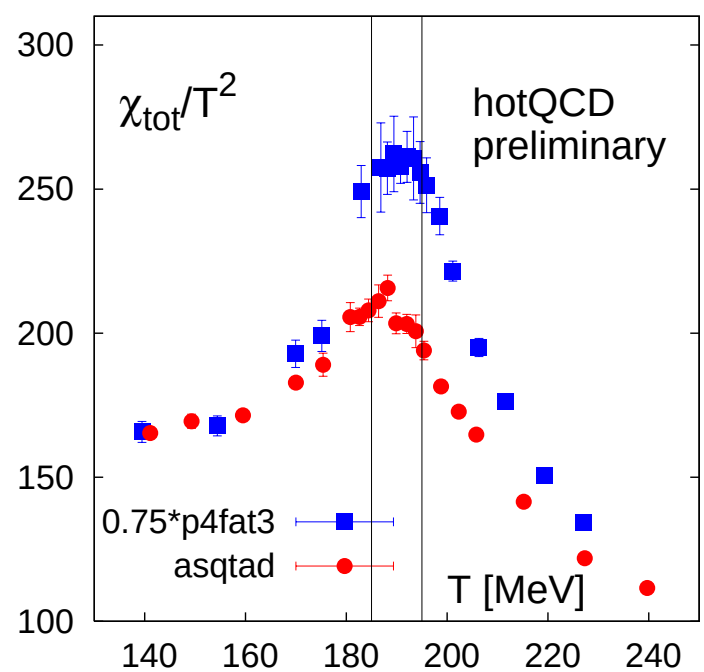
Chiral symmetry restoration: χ-condensate and susceptibility

- sudden change in chiral condensate is, of course, related to peaks in the (singlet) chiral susceptibility

$$\frac{\langle\bar\psi\psi\rangle_{l,T} - \frac{m_l}{m_s}\langle\bar\psi\psi\rangle_{s,T}}{\langle\bar\psi\psi\rangle_{l,0} - \frac{m_l}{m_s}\langle\bar\psi\psi\rangle_{s,0}}$$

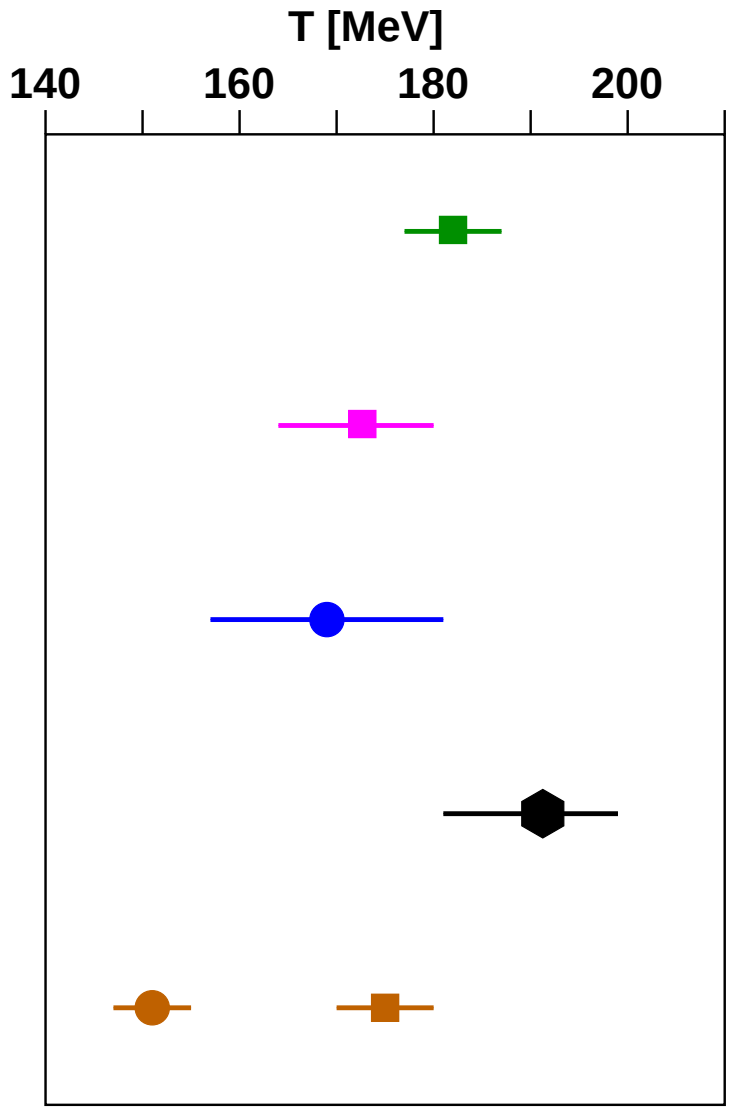
$$\begin{aligned}\chi_{tot}/T^2 &= \frac{1}{T^2}\frac{d\langle\bar\psi\psi\rangle_{l,T}}{dm_l} \\ &= 2\chi_{dis}/T^2 + \chi_{con}/T^2\end{aligned}$$





p4 and asqtad: hotQCD

Transition temperature in 2- and (2+1)-flavor QCD



use T=0 scale: r0=0.469fm

Nf=2:

V.G. Bornyakov et al, POS Lat2005, 157 (2006)
(improved Wilson, Nt=8, 10; input: r0=0.5 fm)
(added Nt=12, Lattice'07) (rescaled to r0)

Y. Maezawa et al., hep−lat/0702005 (QM'2006)
(improved Wilson, Nt=4, 6; input: m−rho)
(no cont. exp. yet)

Nf=2=1:

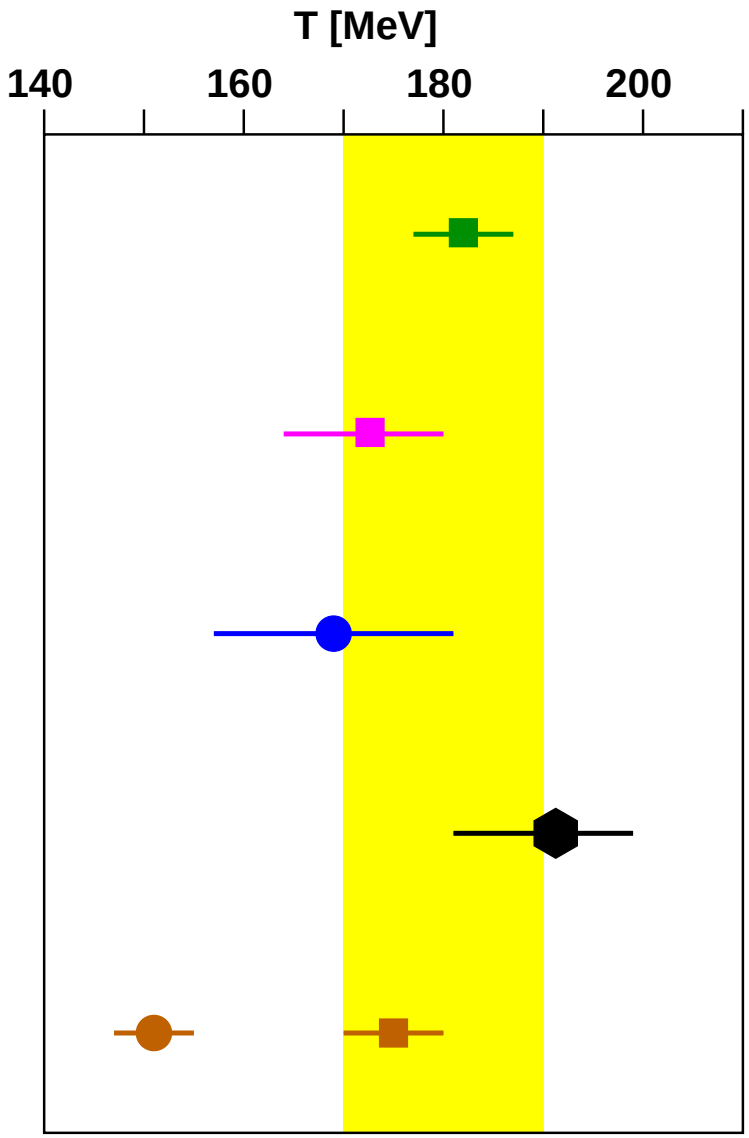
C. Bernard et al., Phys.Rev. D71, 034504 (2005)
(improved staggered (asqtad), Nt=4,6,8, input r1)
(rescaled to r0)

M. Cheng et al., Phys.Rev D74, 054507 (2006)
(improved staggered (p4), Nt=4,6; input r0)

Y. Aoki et al., Phys. Lett. B643, 46 (2006), arXiv:0903.4155
(staggered (stout), Nt=4,6,8,10,12; input fK)
(converted to r0)

● chiral ■ deconfinement ⬢ chiral+deconfinement

Transition temperature in 2- and (2+1)-flavor QCD



chiral deconfinement chiral+deconfinement

Known shortcomings:

too few data: 3-parameter fit to 4 data points for T_c determined at $m_\pi > 550$ MeV

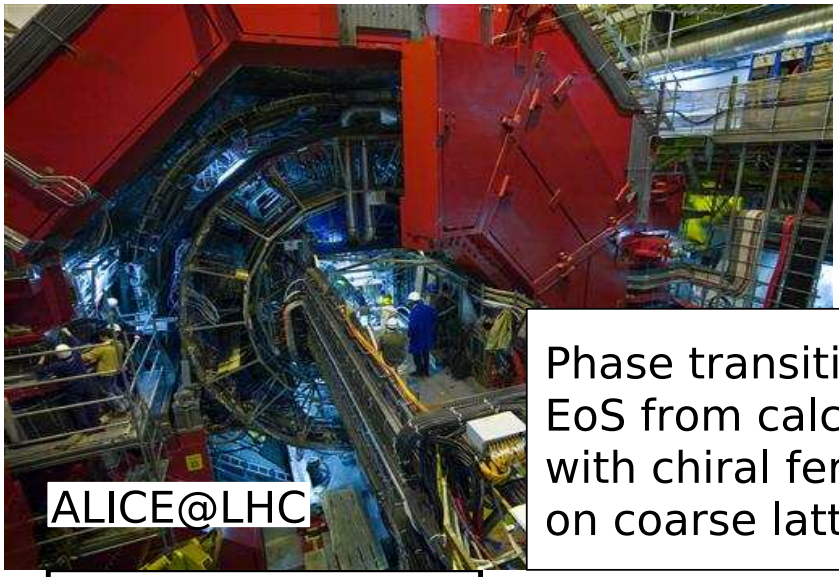
no continuum extrapolation attempted yet; would like to see results in units of r_0

$N_\tau = 4,\ 6,\ 8$, but small spatial volume for larger N_τ; still large statistical errors on individual data points

only $N_\tau = 4$ and 6

only one quark mass; T_c determination partly based on determination of inflection points

Future EoS calculations

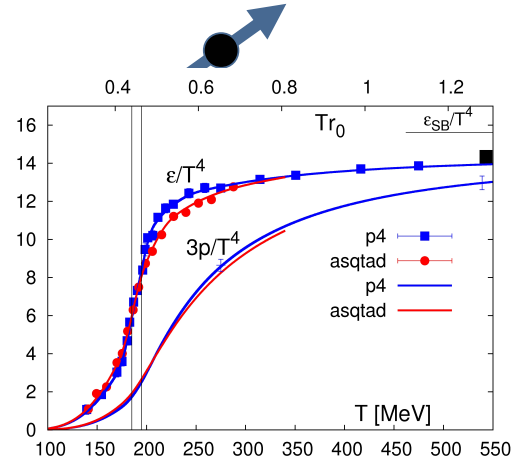


- Continuum extrapolated results for EoS, Tc
- universal properties of QCD at non-zero temperature

Phase transition and EoS from calculations with chiral fermions on coarse lattice

High temperature limit of QCD equation of state

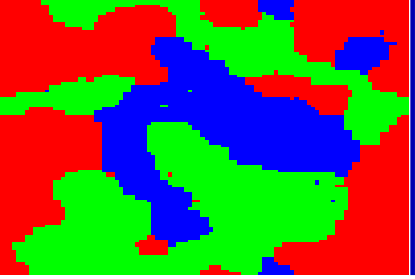
Analysis of chiral properties of QCD transition using highly improved staggered fermions



10x tera **100x tera** **peta** **10x peta** **100x peta** **exa**

-flop year sustained

Extreme Scale Computing Workshop



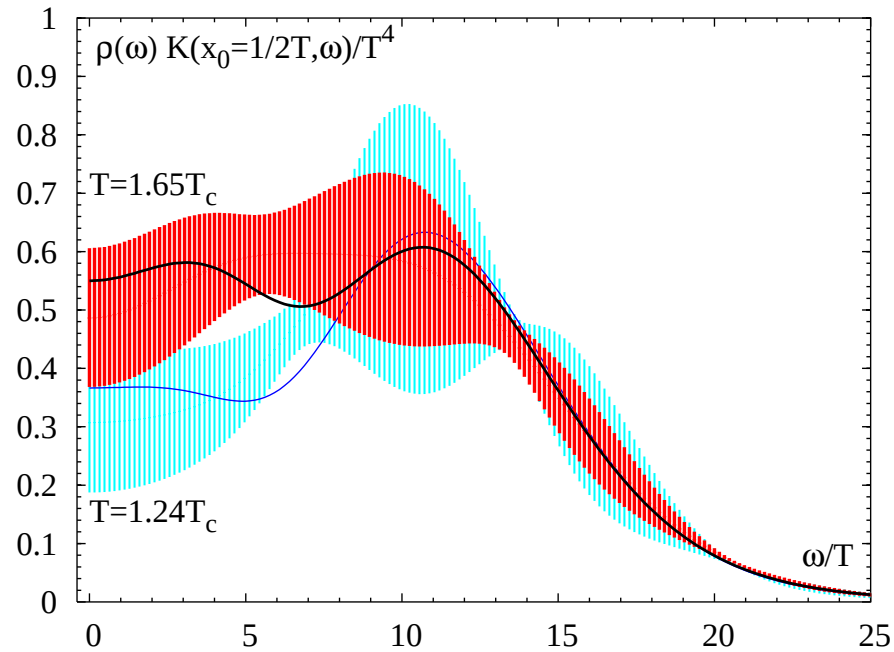
Viscosity and elliptic flow at RHIC

- asymmetric interaction region $\Rightarrow$ asymmetric pressure gradients
  $\Rightarrow$ asymmetric flow velocities: elliptic flow
- hydrodynamic modeling of flow pattern requires
  small shear viscosity over entropy: η/s
- lattice calculations give small η/s: $\eta/s \simeq 0.1$ for $T/T_c \sim (1.2 - 1.7)$

spectral representation of energy-momentum correlator

$$\langle T_{12}(0) T_{12}(t) \rangle = \int_0^\infty d\omega \; \rho(\omega) \; \frac{\cosh(\omega(t - 1/2T))}{\sinh(\omega/2T)}$$

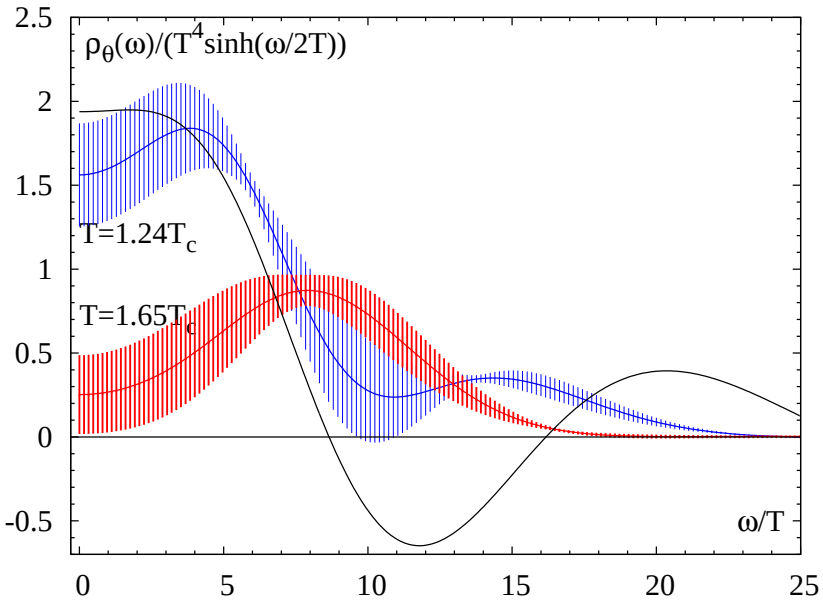
$\Rightarrow$ shear viscosity $\eta = \lim_{\omega \to 0} \frac{\rho(\omega)}{\omega}$



H. Meyer, Phys. Rev. D76, 101701 (2007)

Transport coefficients

QGP: a perfect fluid? $\Leftrightarrow$ (??) $\eta_{shear}/s \simeq 1/4\pi$ η_{bulk} large at T_c



$$G_{XX}(\tau,T) = \int_0^\infty \mathrm{d}\omega \rho_X(\omega) \frac{\cosh(\omega(\tau - 1/2T)}{\sinh(\omega/2T)}$$

$$X = P,\ \epsilon,\ (\epsilon - 3P)$$

$$\rho(\omega) \simeq \frac{9}{\pi}\zeta(T)\omega\frac{\omega_0^2}{\omega^2+\omega_0^2} \quad , \quad \tau_R \equiv 1/\omega_0$$

spectral analysis of SU(3) gauge theory

$\Rightarrow$ bulk viscosity

H. Meyer, Phys. Rev. Lett. 100, 162001 (2008).

SU(2): 2nd order transition:

$\tau_R \to \infty \Leftrightarrow \omega_0 \to 0$

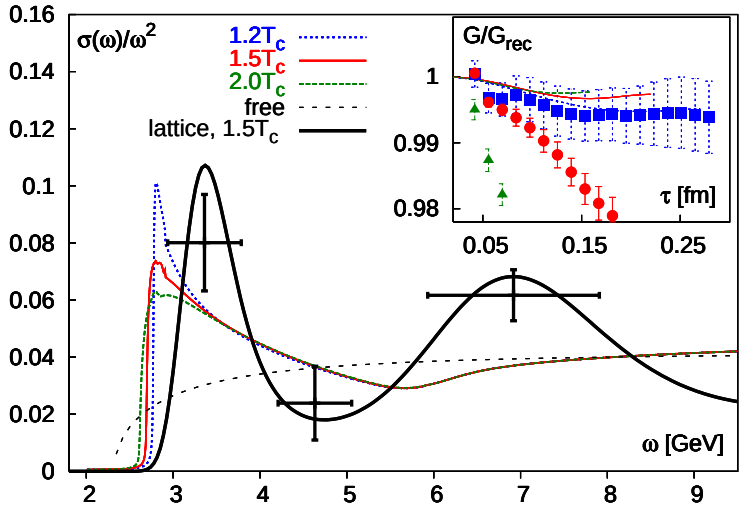
$G_{pp}(\tau,T) \simeq 9T\zeta(T)\omega_0(T) + ...$

K. Hübner, FK, C. Pica, PR D78, 094501 (2008)

- the role of relaxation times (width of the sound peak) needs to get better controled

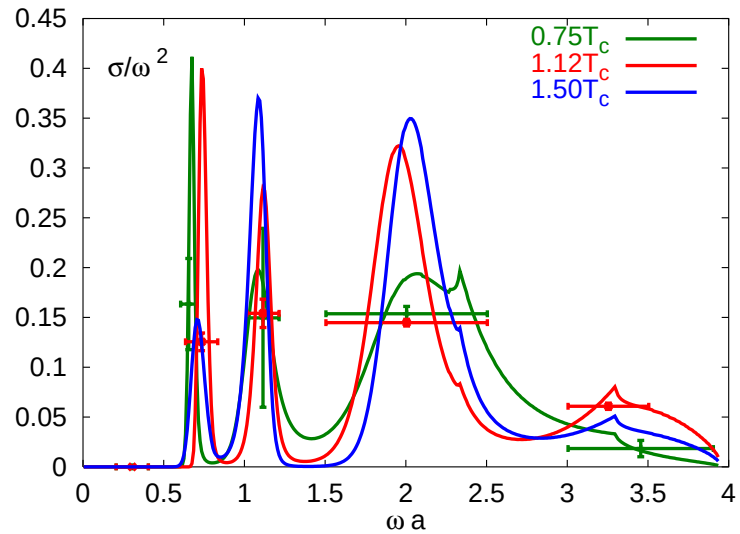
Heavy quark spectral functions and correlation functions

spectral function from potential model using lattice free energy



A.Mocsy, P. Petreczky, PRD77, 014501 (2008)

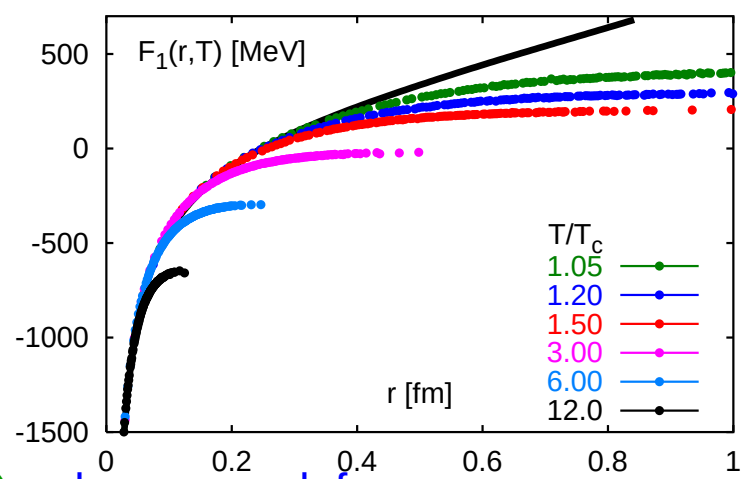
reconstructed spectral functions using the Maximum Entropy Method



S. Datta et al., PRD69, 094507 (2004)

$$G_{J/\psi}(\tau,T) = \int_0^\infty \mathrm{d}\omega \rho_{J/\psi}(\omega) \frac{\cosh(\omega(\tau - 1/2T)}{\sinh(\omega/2T)}$$

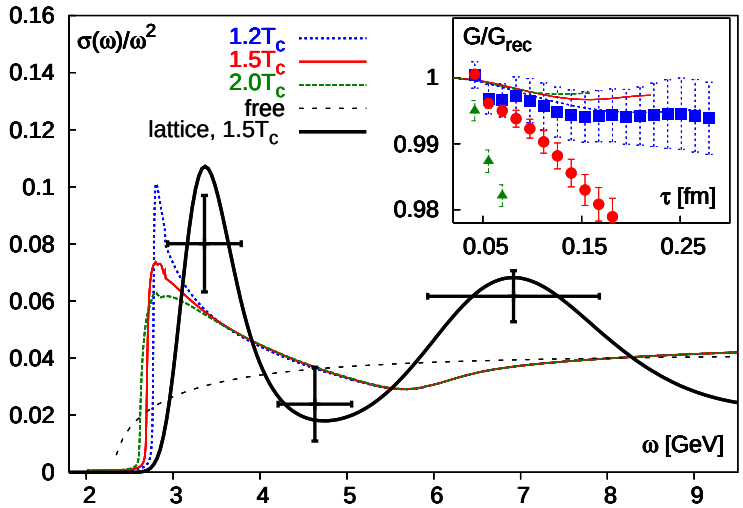
O.Kaczmarek, F. Zantow, PRD71, 114510 (2005)



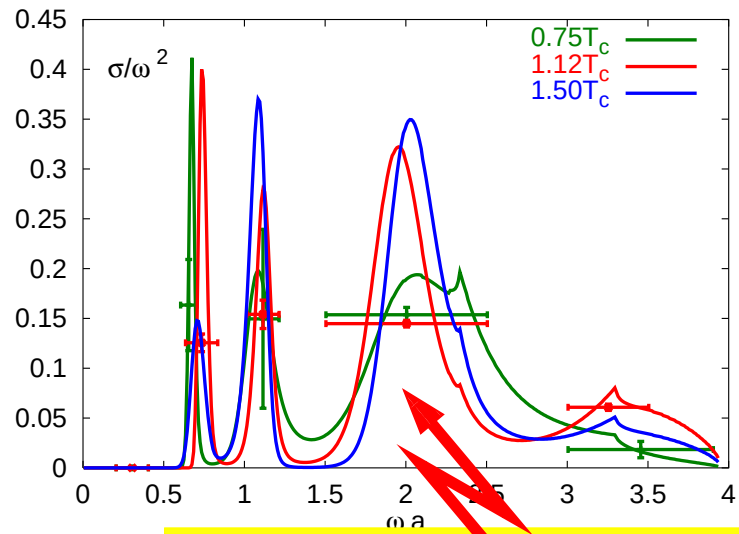
heavy quark free energy

Heavy quark spectral functions and correlation functions

spectral function from potential model using lattice free energy



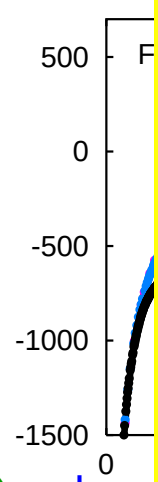
reconstructed spectral functions using the Maximum Entropy Method



need to get better control over ultra-violet cut-off effects (Wilson-doublers)

use better fermion actions
- overlap fermions
- domain wall fermions
- (truncated) perfect actions...

$$G_{J/\psi}(\tau,T) = \int_0^\infty \mathrm{d}\omega \rho_{J/\psi}(\omega) \frac{\cosh(\omega(\tau - 1/2T)}{\sinh(\omega/2T)}$$



O.Kaczmarek, F. Zantow, PRD71, 114510 (2005)

heavy quark free energy

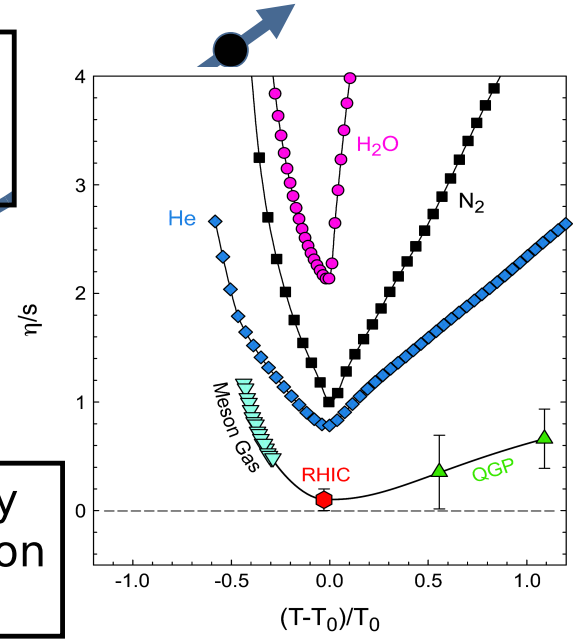
Future transport and quarkonium spectroscopy calculations

- Melting temperatures and broadening of heavy quark bound states in QCD
- Transport properties of hot, strongly interacting matter

Transport properties of QCD with light quarks on coarse lattices

Quarkonium spectroscopy and transport coefficients in quenched QCD

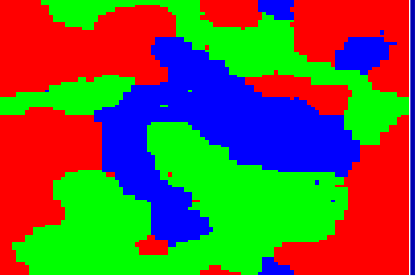
Quarkonium spectroscopy in QCD with light quarks on coarse lattices



10x tera **100x tera** **peta** **10x peta** **100x peta** **exa**

-flop year sustained

Extreme Scale Computing Workshop



Extending the phase diagram to non-vanishing chemical potential

non-zero baryon number density: $\mu > 0$

$$Z(V,T,\mu) = \int \mathcal{D}\mathcal{A}\mathcal{D}\psi\mathcal{D}\bar{\psi}\ \mathrm{e}^{-S_E(V,T,\mu)}$$
$$= \int \mathcal{D}\mathcal{A}\mathcal{D}\ det\ M(\mu)\ \mathrm{e}^{-S_E(V,T)}$$

$\Uparrow$complex fermion determinant;

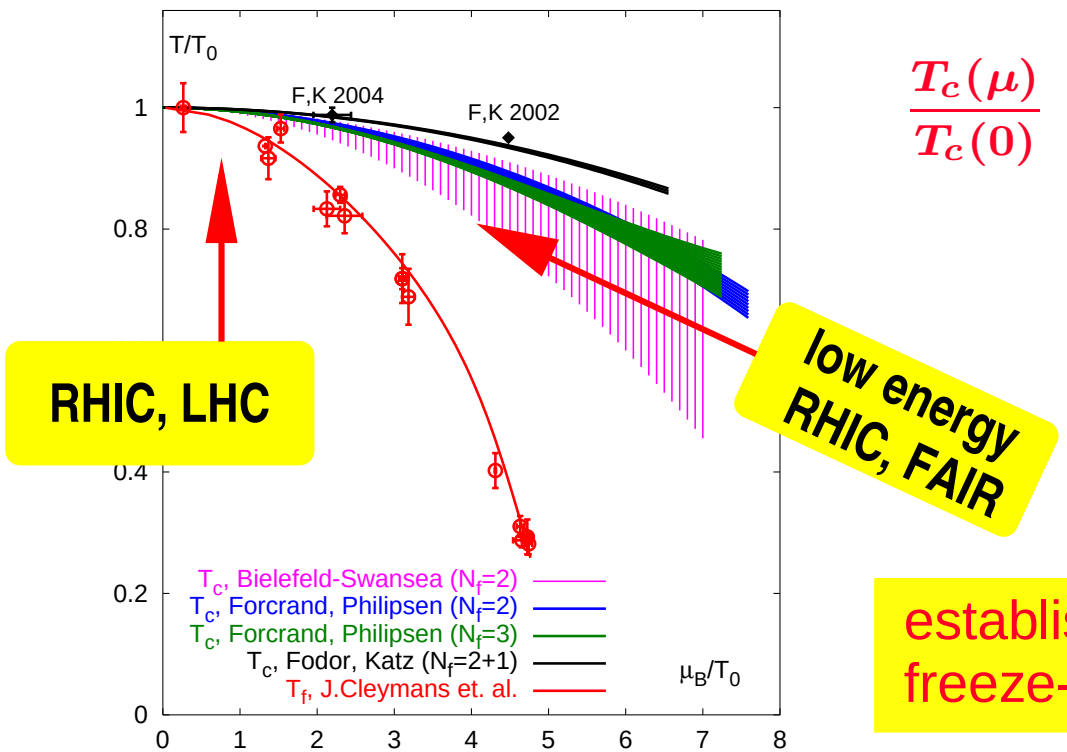
long standing problem

$\Rightarrow$ three (partial) solutions for large T, small μ

Extending the phase diagram to non-vanishing chemical potential

non-zero baryon number density: $\mu > 0$

$$Z(V,T,\mu) = \int \mathcal{D}\mathcal{A}\mathcal{D}\psi\mathcal{D}\bar{\psi}\ \mathrm{e}^{-S_E(V,T,\mu)}$$
$$= \int \mathcal{D}\mathcal{A}\mathcal{D}\ det\ M(\mu)\ \mathrm{e}^{-S_E(V,T)}$$



$\frac{T_c(\mu)}{T_c(0)}$: $1 - 0.0056(4)(\mu_B/T)^2$

deForcrand, Philipsen (imag. μ)

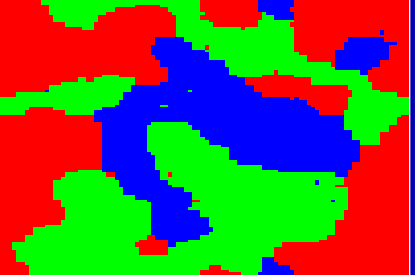
$1 - 0.0078(38)(\mu_B/T)^2$

Bielefeld-Swansea

($\mathcal{O}(\mu^2)$ reweighting)

search for critical point

establish relation between freeze-out and critical behavior



Quark number susceptibility... ...and its susceptibility

- rapid change in quark/baryon/strangeness number susceptibility reflects change in mass of the carrier of these quantum numbers ⇔ DECONFINEMENT
- quark number susceptibility feels nearby singular point just like the energy density

scaling field: $t = \left|\frac{T - T_c}{T_c}\right| + A\left(\frac{\mu_q}{T_c}\right)^2 \quad , \quad \mu_{crit} = 0$

singular part: $f_s(T, \mu_q) = b^{-1} f_s(t b^{1/(2-\alpha)}) \sim t^{2-\alpha}$

Y. Hatta, T. Ikeda, PRD67 (2003) 014028

$$c_2 \sim \chi_2^q \sim \frac{\partial^2 \ln \mathcal{Z}}{\partial \mu_q^2} \sim t^{1-\alpha} \quad , \quad c_4 \sim \chi_4^q \sim \frac{\partial^4 \ln \mathcal{Z}}{\partial \mu_q^4} \sim t^{-\alpha} \qquad (\mu = 0)$$

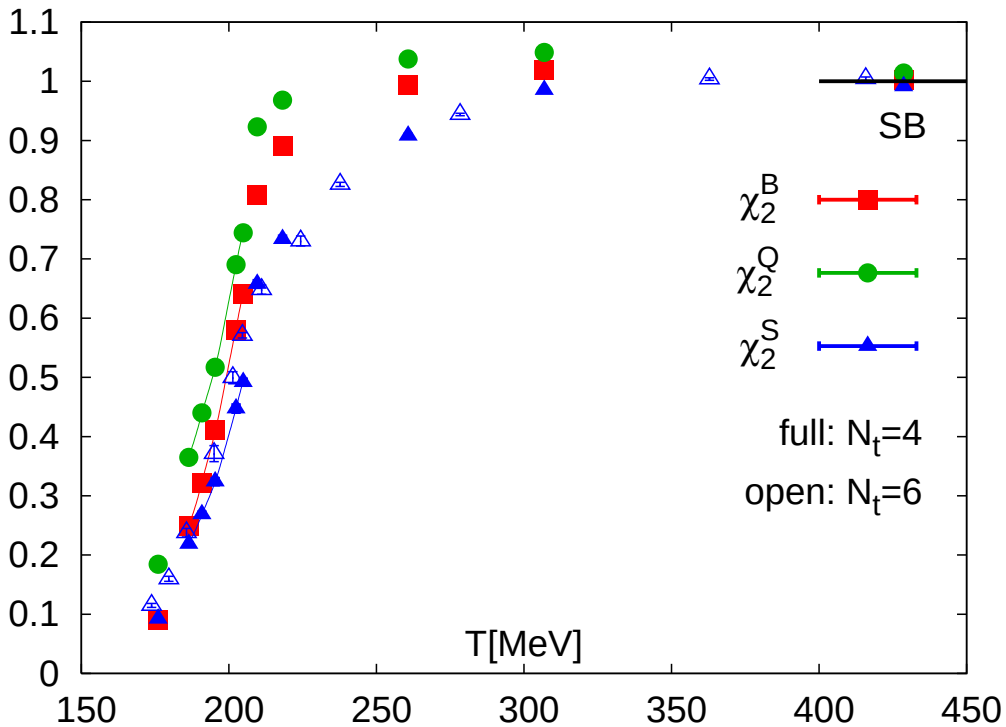
$$\epsilon \sim \frac{\partial \ln \mathcal{Z}}{\partial T} \sim t^{1-\alpha} \quad , \quad C_V \sim \frac{\partial^2 \ln \mathcal{Z}}{\partial T^2} \sim t^{-\alpha} \qquad (\mu = 0)$$

$\Rightarrow 2^{nd}$ derivative w.r.t μ_q "looks like energy density"

$\Rightarrow 4^{th}$ derivative w.r.t μ_q "looks like specific heat"

Quadratic fluctuations of baryon number, charge & strangeness

vanishing chemical potentials: RBC-Bielefeld, Phys. Rev. D79, 074505 (2009)



$$\chi_2^Q = \frac{1}{VT^3}\langle Q^2\rangle$$

$$\chi_2^B = \frac{1}{VT^3}\langle N_B^2\rangle$$

$$\chi_2^S = \frac{1}{VT^3}\langle N_S^2\rangle$$

rapid approach to SB limit

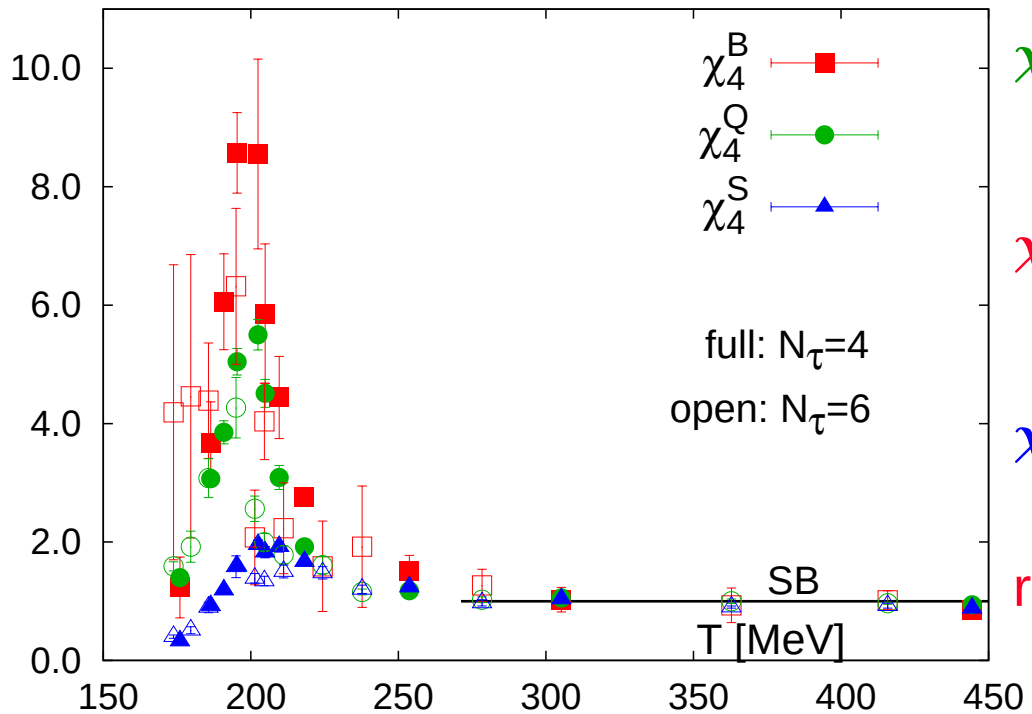
$\Rightarrow$ smooth change of quadratic fluctuations across transition region

chiral limit: $\chi_2^B,\ \chi_2^Q \sim |T - T_c|^{1-\alpha} + \mathrm{regular}$

Quartic fluctuations of baryon number, charge & strangeness

vanishing chemical potentials:

RBC-Bielefeld, Phys. Rev. D79, 074505 (2009)



$$\chi_4^Q = \frac{1}{VT^3}\left(\langle Q^4\rangle - 3\langle Q^2\rangle^2\right)$$

$$\chi_4^B = \frac{1}{VT^3}\left(\langle N_B^4\rangle - 3\langle N_B^2\rangle^2\right)$$

$$\chi_4^S = \frac{1}{VT^3}\left(\langle N_S^4\rangle - 3\langle N_S^2\rangle^2\right)$$

rapid approach to SB limit

$\Rightarrow$ large light quark number & charge fluctuations across transition region

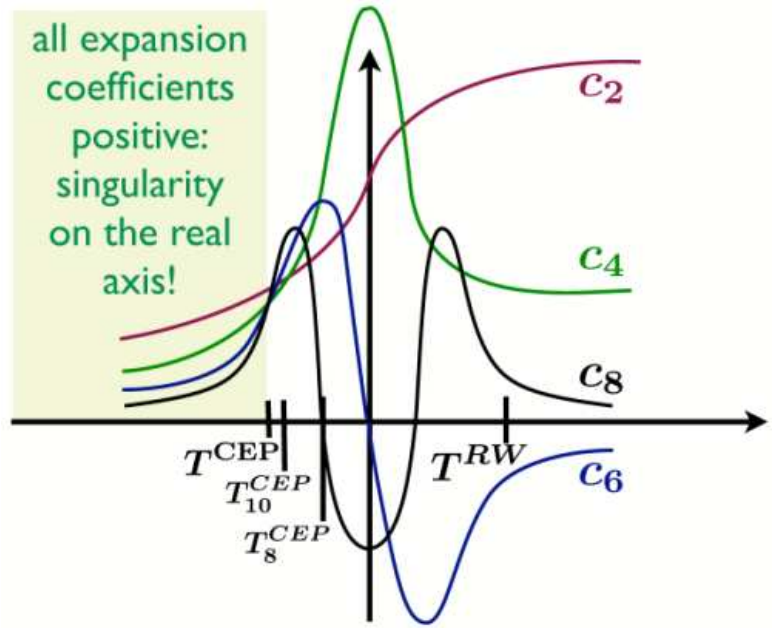
chiral limit: $\chi_4^B,\ \chi_4^Q \sim |T - T_c|^{-\alpha} + \mathrm{regular}$

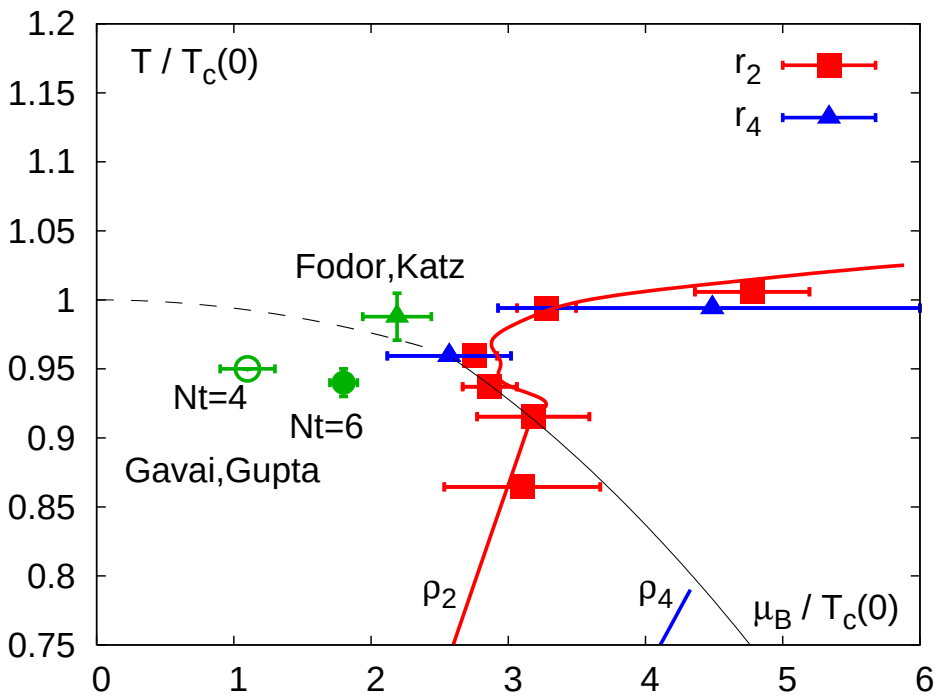
Estimating the location of the critical point: (T^{CEP}, μ_c)

i) T^{CEP}: find the largest temperature for which all c_n stay positive

ii) μ_c/T^{CEP}: estimate $\lim_{n\to\infty} r_n$

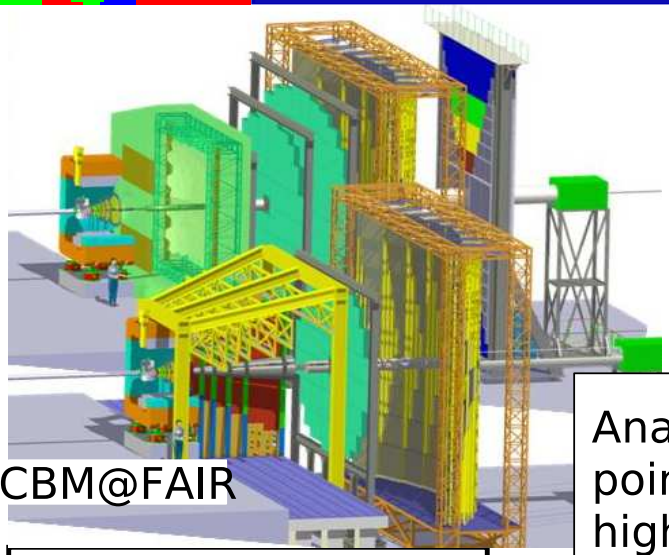
$$r_n \equiv \left(\frac{\mu_c}{T^{CEP}}\right)_n = \sqrt{\frac{c_n}{c_{n+2}}}$$





- $\mathcal{O}(a^2)$ improved action; slight quark mass dependence; weak cut-off dependence
- first non-trivial estimate of T^{CEP} requires 8th order for c_n: $\Rightarrow$ r_6
- already $\mathcal{O}(c_6)$ requires more statistics

Future finite density calculations

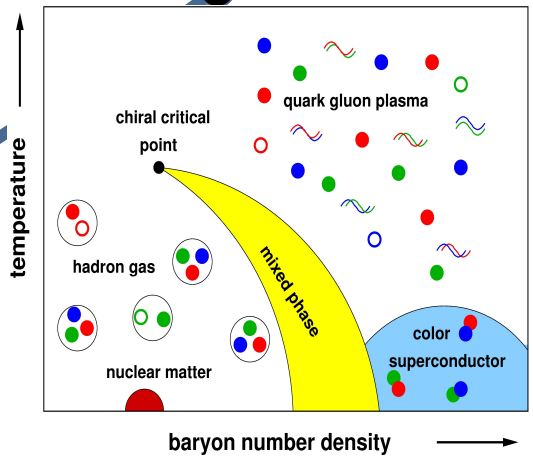


- Results on the existence/non-existence of a critical point in the QCD phase diagram
- Analysis of first order transition line using canonical simulations

Analysis of critical point/surface with highly improved staggered fermions

Critical surface from calculations with unimproved staggered fermions

Critical point estimates from Taylor expansions on coarse lattices



10x tera **100x tera** **peta** **10x peta** **100x peta** **exa**

-flop year sustained

Extreme Scale Computing Workshop

Conclusions

- bulk thermodynamics with almost physical quark masses seems to be well under control; exploitation of USQCD resources allowed a world-wide leading, unique study of bulk thermodynamics with improved staggered fermions;

  low temperature region requires finer lattices to control systematics and allow a comparison with hadron gas phenomenology;

  high-T regime requires calculations on larger lattices to establish contact to (resummed) perturbation theory
- calculations of transport coefficients in quenched QCD yield results consistent with interpretation of heavy ion experiments, but require more work to control systematics
- studies of finite density QCD do not yet give a conclusive answer on the existence or non-existence of a critical point in the QCD phase diagram

Critical Point and Onset of Deconfinement

5th International Workshop • June 8–12, 2009

Brookhaven National Laboratory, Long Island, New York, USA

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