Gluonic GPDs of the nucleon

Phiala Shanahan, Will Detmold

Nuclear Modification of Nucleon Structure

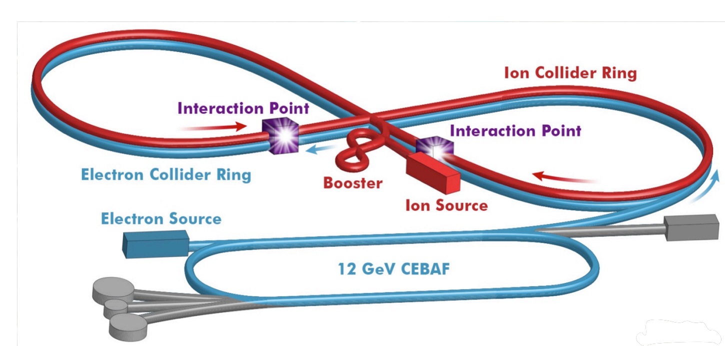
NPLQCD

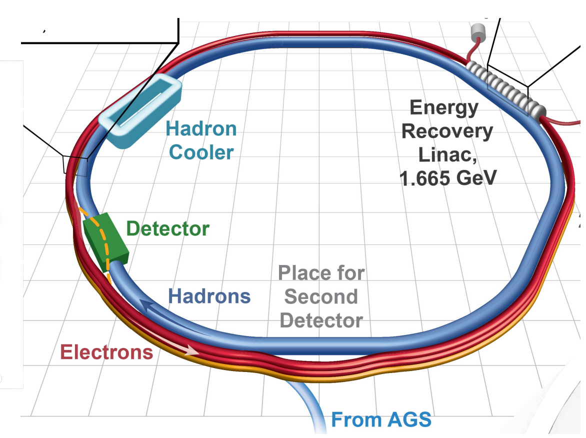
Silas Beane, Emmanuel Chang, Zohreh Davoudi, Will Detmold, Kostas Orginos, Assumpta Parreño, K.enny Roche, Martin Savage, Phiala Shanahan, Brian Tiburzi, Mike Wagman, and Frank Winter

Gluonic Structure

- Past 60+ years: detailed view of quark structure of nucleons
- Gluonic structure (beyond gluon density) relatively unexplored
- Electron-Ion Collider
  - Priority in 2015 long range plan
  - "Understanding the glue that binds us all"
- Propose: LQCD calculations to inform EIC development

Cover image from EIC whitepaper arXiv::1212.1701

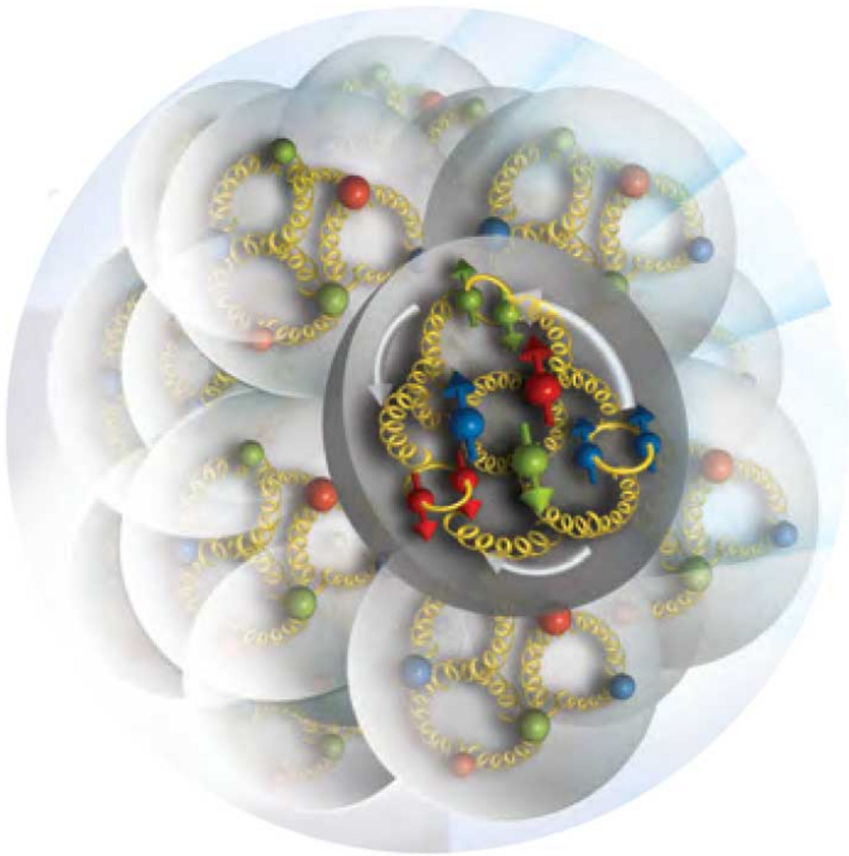




Gluonic Transversity

Double helicity flip structure function $\Delta(x,Q^2)$

- Purely gluonic observable: ideal goal for EIC
- No mixing with quark observables
- Hadrons: Gluonic Transversity
- Nuclei: Exotic Glue
  - fwd limit: gluons not associated with individual nucleons in nucleus
  - operator in nucleon = 0
    operator in nucleus ≠ 0



$$\langle p|\mathcal{O}|p\rangle = 0$$
$$\langle N,Z|\mathcal{O}|N,Z\rangle \neq 0$$

Gluonic Transversity

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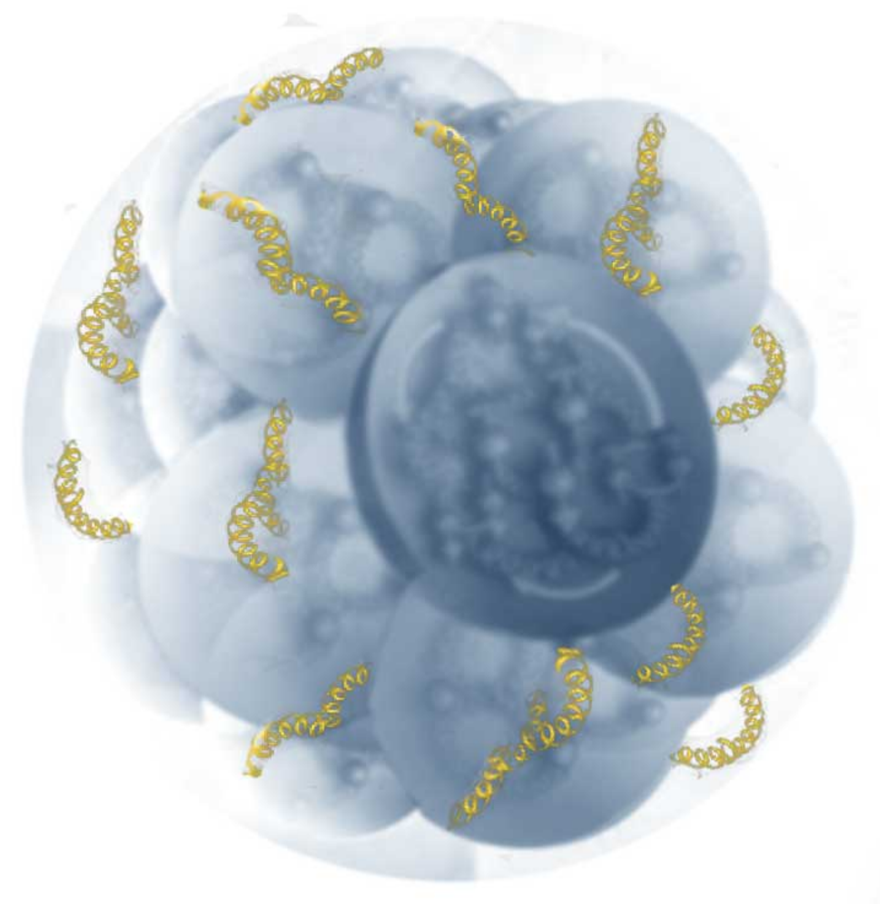


$$\langle p|\mathcal{O}|p\rangle = 0$$

$$\langle N,Z|\mathcal{O}|N,Z\rangle \neq 0$$

Proposed Calculations

Gluon transversity GFFs of the nucleon

To inform development of EIC (ca 2025), need completely quantified uncertainties on 5-year timescale

First calculation

- Need high statistics
- Expectation: access1-3 transversity generalised form factors for lowest moments of transversity GPD

- Uncertainties: 10% stat, 10% sys
- Different irreps: lattice spacing effects
- Two lattice volumes
- No pion mass dependence
- Perturbative renormalisation (talking to M. Constantinou)

β	a [fm]	m_π [MeV]	$L^3 \times T$	$N_{\rm cfg}$	$N_{\rm src}$	Cost/inversion [K20 hours]	Total Cost [K20 hours]
6.1	0.117(2)	450	$24^3 \times 64$	2000	256	0.33	1.69×10^5
6.1	0.117(2)	450	$32^3 \times 96$	2000	128	0.8	2.05×10^5

Preliminary Work

Gluon transversity in φ meson [W Detmold & PES PRD 94 (2016), 014507, 1703.08220]

- First moment in φ meson (simplest spin-1 system, ➡ nucleons, nuclei)
- Lattice details: clover fermions, Lüscher-Weisz gauge action

L/a	T/a	β	am_l	am_s
24	64	6.1	-0.2800	-0.2450
a (fm)	L (fm)	T (fm)	m_π (MeV)	m_K (MeV)
0.1167(16)	2.801(29)	7.469(77)	450(5)	596(6)
m_ϕ (MeV)	$m_\pi L$	$m_\pi T$	N_{cfg}	N_{src}
1040(3)	6.390	17.04	1042	10^5

- Many systematics not addressed (yet)!
  - Quark mass effects
  - Volume effects
  - Discretisation
  - Renormalisation

Gluonic Generalised Form Factors

Off-forward matrix elements are complicated

- Eg: moments of $\Delta(x,Q^2)$ related to many form factors

$$\begin{aligned}
&\left\langle p'E'\left| S\left[G_{\mu\mu_1}\overleftrightarrow{D}_{\mu_3}\ldots\overleftrightarrow{D}_{\mu_n}G_{\nu\mu_2}\right]\right| pE\right\rangle \\
&= \sum_{\substack{m\text{ odd}\\ m=3}}^{n}\Big\{ A^{(n)}_{1,m-3}(t,\mu^2)\, S\left[(P_\mu E_{\mu_1}-E_\mu P_{\mu_1})(P_\nu E'^*_{\mu_2}-E'^*_\nu P_{\mu_2})\Delta_{\mu_3}\ldots\Delta_{\mu_{m-1}}P_{\mu_m}\ldots P_{\mu_n}\right] \\
&\quad + A^{(n)}_{2,m-3}(t,\mu^2)\, S\left[(\Delta_\mu E_{\mu_1}-E_\mu\Delta_{\mu_1})(\Delta_\nu E'^*_{\mu_2}-E'^*_\nu\Delta_{\mu_2})\Delta_{\mu_3}\ldots\Delta_{\mu_{m-1}}P_{\mu_m}\ldots P_{\mu_n}\right] \\
&\quad + A^{(n)}_{3,m-3}(t,\mu^2)\, S\Big[\big((\Delta_\mu E_{\mu_1}-E_\mu\Delta_{\mu_1})(P_\nu E'^*_{\mu_2}-E'^*_\nu P_{\mu_2})-(\Delta_\mu E'^*_{\mu_1}-E'^*_\mu\Delta_{\mu_1})(P_\nu E_{\mu_2}-E_\nu P_{\mu_2})\big) \\
&\qquad\qquad \times\Delta_{\mu_3}\ldots\Delta_{\mu_{m-1}}P_{\mu_m}\ldots P_{\mu_n}\Big] \\
&\quad + A^{(n)}_{4,m-3}(t,\mu^2)\, S\left[(E_\mu E'^*_{\mu_1}-E_{\mu_1}E'^*_\mu)(P_\nu\Delta_{\mu_2}-P_{\mu_2}\Delta_\nu)\Delta_{\mu_3}\ldots\Delta_{\mu_{m-1}}P_{\mu_m}\ldots P_{\mu_n}\right] \\
&\quad + \frac{A^{(n)}_{5,m-3}(t,\mu^2)}{M^2} S\Big[\big((E\cdot P)(P_\mu\Delta_{\mu_1}-\Delta_\mu P_{\mu_1})(\Delta_\nu E'^*_{\mu_2}-E'^*_\nu\Delta_{\mu_2}) \\
&\qquad\qquad + (E'^*\cdot P)(P_\mu\Delta_{\mu_1}-\Delta_\mu P_{\mu_1})(\Delta_\nu E_{\mu_2}-E_\nu\Delta_{\mu_2})\big)\,\Delta_{\mu_3}\ldots\Delta_{\mu_{m-1}}P_{\mu_m}\ldots P_{\mu_n}\Big] \\
&\quad + \frac{A^{(n)}_{6,m-3}(t,\mu^2)}{M^2} S\Big[\big((E\cdot P)(P_\mu\Delta_{\mu_1}-\Delta_\mu P_{\mu_1})(P_\nu E'^*_{\mu_2}-E'^*_\nu P_{\mu_2}) \\
&\qquad\qquad - (E'^*\cdot P)\,(P_\mu\Delta_{\mu_1}-\Delta_\mu P_{\mu_1})(P_\nu E_{\mu_2}-E_\nu P_{\mu_2})\big)\,\Delta_{\mu_3}\ldots\Delta_{\mu_{m-1}}P_{\mu_m}\ldots P_{\mu_n}\Big] \\
&\quad + \frac{A^{(n)}_{7,m-3}(t,\mu^2)}{M^2}(E'^*\cdot E) S\left[(P_\mu\Delta_{\mu_1}-\Delta_\mu P_{\mu_1})(P_\nu\Delta_{\mu_2}-\Delta_\nu P_{\mu_2})\Delta_{\mu_3}\ldots\Delta_{\mu_{m-1}}P_{\mu_m}\ldots P_{\mu_n}\right] \\
&\quad + \frac{A^{(n)}_{8,m-3}(t,\mu^2)}{M^4}(E\cdot P)(E'^*\cdot P) S\left[(P_\mu\Delta_{\mu_1}-\Delta_\mu P_{\mu_1})(P_\nu\Delta_{\mu_2}-\Delta_\nu P_{\mu_2})\Delta_{\mu_3}\ldots\Delta_{\mu_{m-1}}P_{\mu_m}\ldots P_{\mu_n}\right]\Big\}
\end{aligned}$$

Gluonic Generalised Form Factors

Off-forward matrix elements are complicated

- Eg: moments of $\Delta(x,Q^2)$ related to many form factors

$$\left\langle p'E' \middle| S\left[G_{\mu\mu_1}\overleftrightarrow{D}_{\mu_3}\ldots\overleftrightarrow{D}_{\mu_n}G_{\nu\mu_2}\right]\middle| pE\right\rangle$$

$$= \sum_{\substack{m\ \mathrm{odd}\\ m=3}}^{n} \Big\{ A^{(n)}_{1,m-3}(t,\mu^2)\, S\left[(P_\mu E_{\mu_1} - E_\mu P_{\mu_1})(P_\nu E'^*_{\mu_2} - E'^*_\nu P_{\mu_2})\Delta_{\mu_3}\ldots\Delta_{\mu_{m-1}}P_{\mu_m}\ldots P_{\mu_n}\right]$$

$$+ A^{(n)}_{2,m-3}(t,\mu^2)\, S\left[(\Delta_\mu E_{\mu_1} - E_\mu \Delta_{\mu_1})(\Delta_\nu E'^*_{\mu_2} - E'^*_\nu \Delta_{\mu_2})\Delta_{\mu_3}\ldots\Delta_{\mu_{m-1}}P_{\mu_m}\ldots P_{\mu_n}\right]$$

$$+ A^{(n)}_{3,m-3}(t,\mu^2)\, S\left[((\Delta_\mu E_{\mu_1} - \cdots)(P_\nu E'^*_{\mu_2} - E'^*_\nu P_{\mu_2}) - (\Delta_\nu E'^*_{\mu_2} - E'^*_\nu \Delta_{\mu_2})(P_\mu E_{\mu_1} - E_\mu P_{\mu_2}))\right.$$

$$\times \Delta_{\mu_3} \cdots \Delta_{\mu_m} \cdots$$

$$+ A^{(n)}_{4,m-3}(t,\mu^2)\, S\left[(\cdots E_{\mu_1} \cdots\right.$$

$$+ \frac{A^{(n)}_{5,m-3}(t,\mu^2)}{M^2} S\left[((E\cdot P)(P_\mu\Delta \cdots\right.$$

$$\left. + (E'^*\cdot P)(P \cdots P_{\mu_n}\right]$$

$$+ \frac{A^{(n)}_{6,m-3}(t,\mu^2)}{M^2} S\left[((E\cdot P)(P_\mu\Delta_{\mu_1} - \Delta_\mu P_{\mu_1})(P_\nu E'^*_{\mu_2} - E'^*_\nu P_{\mu_2})\right.$$

$$\left. - (E'^*\cdot P)\,(P_\mu\Delta_{\mu_1} - \Delta_\mu P_{\mu_1})(P_\nu E_{\mu_2} - E_\nu P_{\mu_2}))\,\Delta_{\mu_3}\ldots\Delta_{\mu_{m-1}}P_{\mu_m}\ldots P_{\mu_n}\right]$$

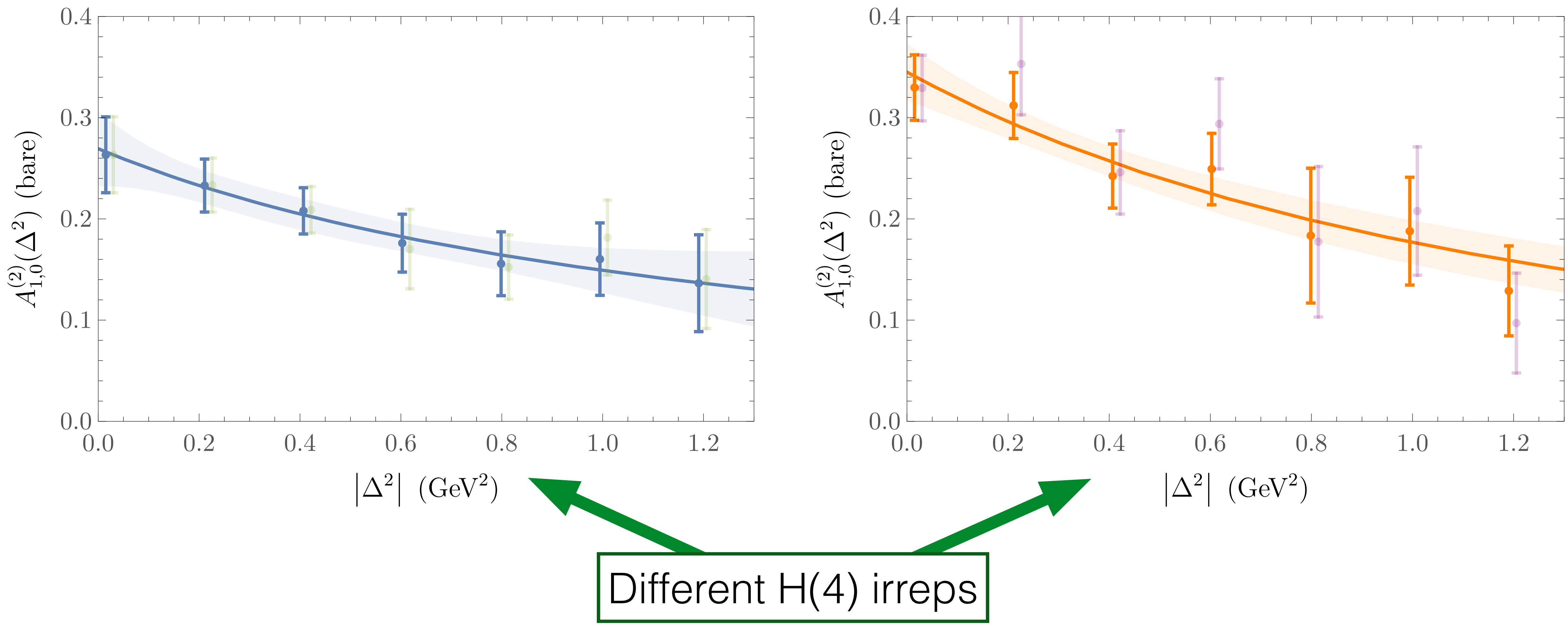
$$+ \frac{A^{(n)}_{7,m-3}(t,\mu^2)}{M^2}(E'^*\cdot E) S\left[(P_\mu\Delta_{\mu_1} - \Delta_\mu P_{\mu_1})(P_\nu\Delta_{\mu_2} - \Delta_\nu P_{\mu_2})\Delta_{\mu_3}\ldots\Delta_{\mu_{m-1}}P_{\mu_m}\ldots P_{\mu_n}\right]$$

$$+ \frac{A^{(n)}_{8,m-3}(t,\mu^2)}{M^4}(E\cdot P)(E'^*\cdot P) S\left[(P_\mu\Delta_{\mu_1} - \Delta_\mu P_{\mu_1})(P_\nu\Delta_{\mu_2} - \Delta_\nu P_{\mu_2})\Delta_{\mu_3}\ldots\Delta_{\mu_{m-1}}P_{\mu_m}\ldots P_{\mu_n}\right] \Big\}$$

Many gluonic GFFs:
Extract from complicated systems

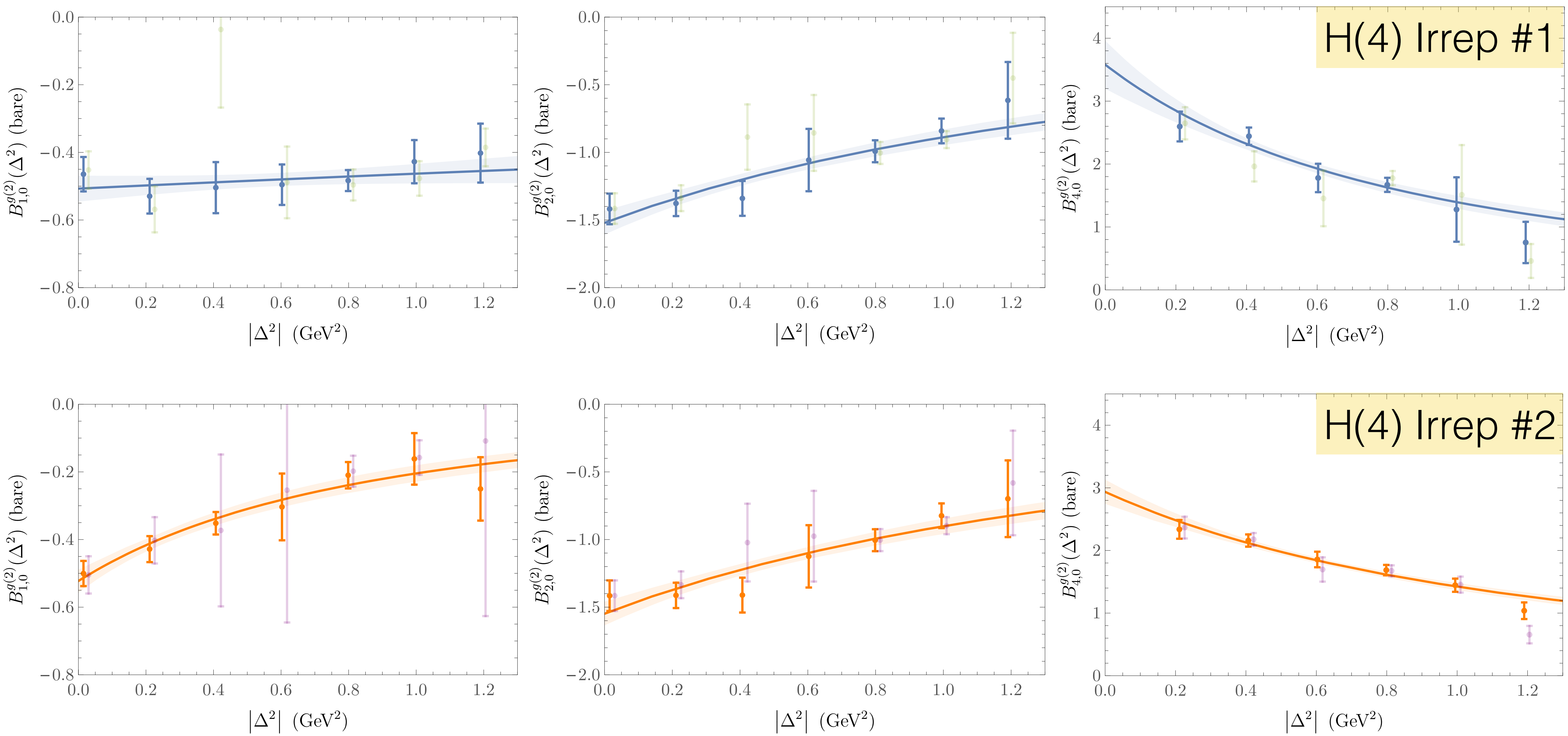
Gluon Transversity GFFs

One GFF can be resolved



Spin-Indep. Gluon GFFs

Three GFFs can be resolved (not safe from mixing with quark ops.)



Nuclear Modification of Nucleon Structure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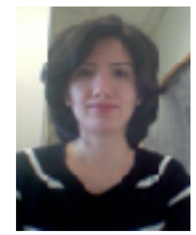
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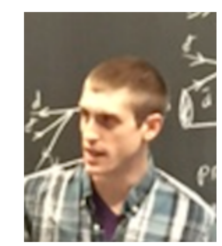
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Lowest Mellin moments of quark and gluon distributions in light nuclei

→ EMC effect

Nuclear Modification of Nucleon Structure

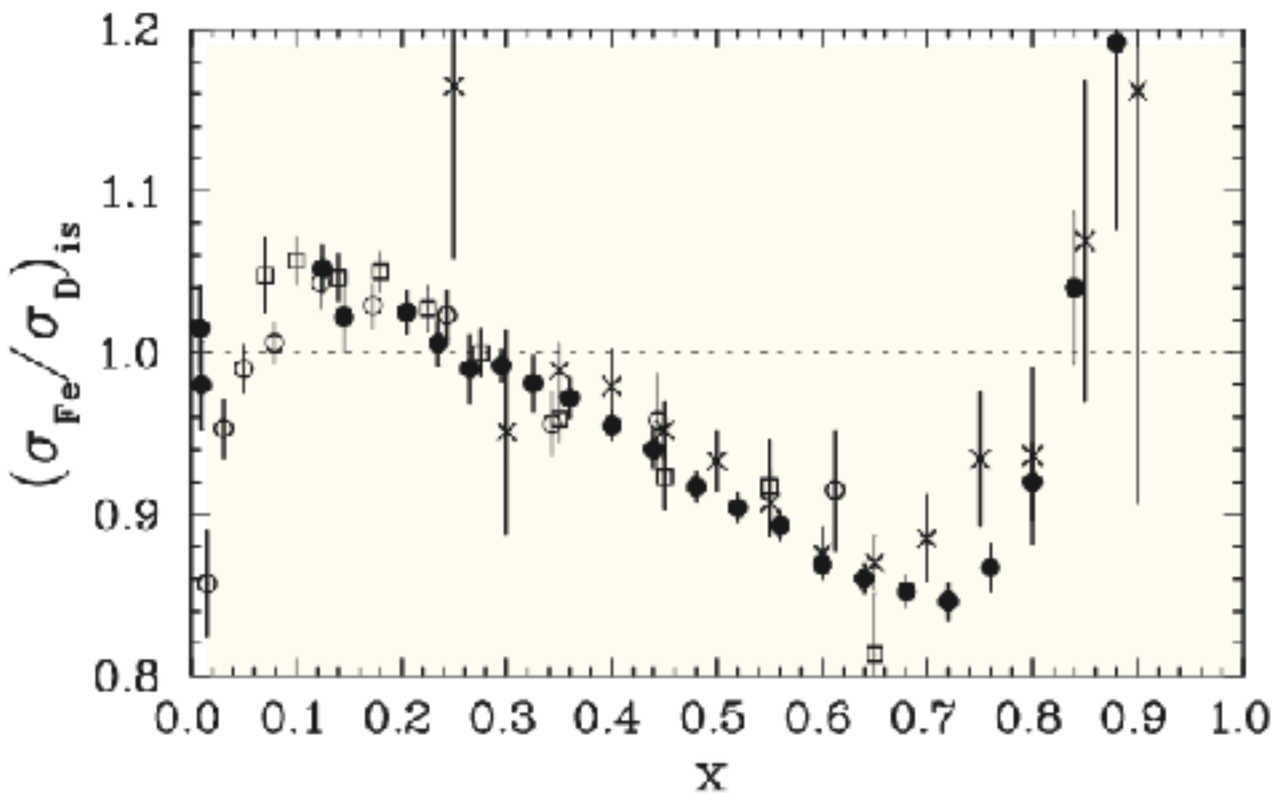
European Muon Collaboration (1983):

Modification of per-nucleon cross section of nucleons bound in nuclei

Precise understanding of nuclear targets essential for DUNE expt: extraction of neutrino mass hierarchy, mixing parameters

Ratio of structure function F2 per nucleon for iron and deuterium

$$\mathrm{F}_2(x, Q^2) = \sum_{\mathrm{q=u,d,s..}} x z_{\mathrm{q}}^2 \left[\mathrm{q}(x, Q^2) + \bar{\mathrm{q}}(x, Q^2)\right]$$



2016-2017 Highlights

Weak nuclear processes

- Matrix element determining

  $$pp \rightarrow de^{+}\nu$$

  fusion cross-section
  - Muon capture reaction (MuSun)
  - Neutrino breakup reaction (SnO)

- Gamow-Teller matrix element in tritium
  - Multi-body contributions to decay rates of nuclei

- Two-neutrino double-beta decay matrix element

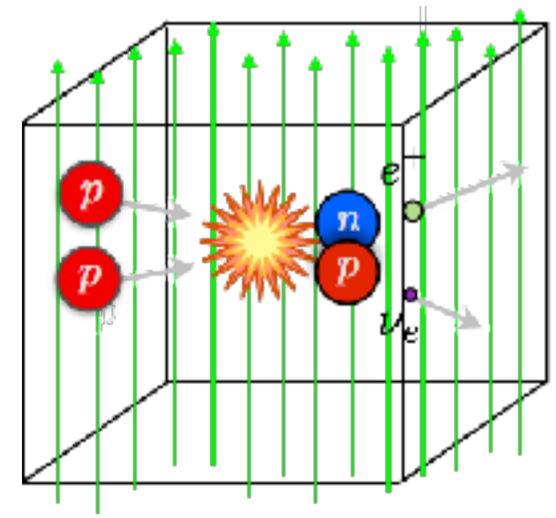
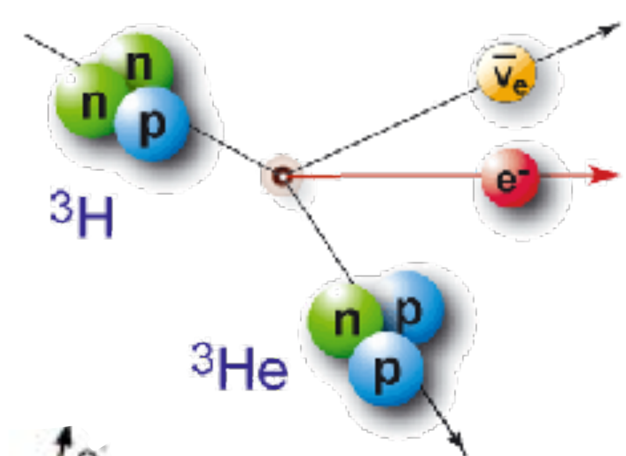
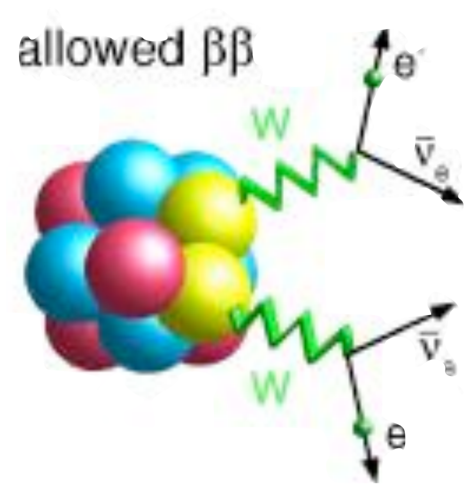


Fig: Z Davoudi





Background field approach

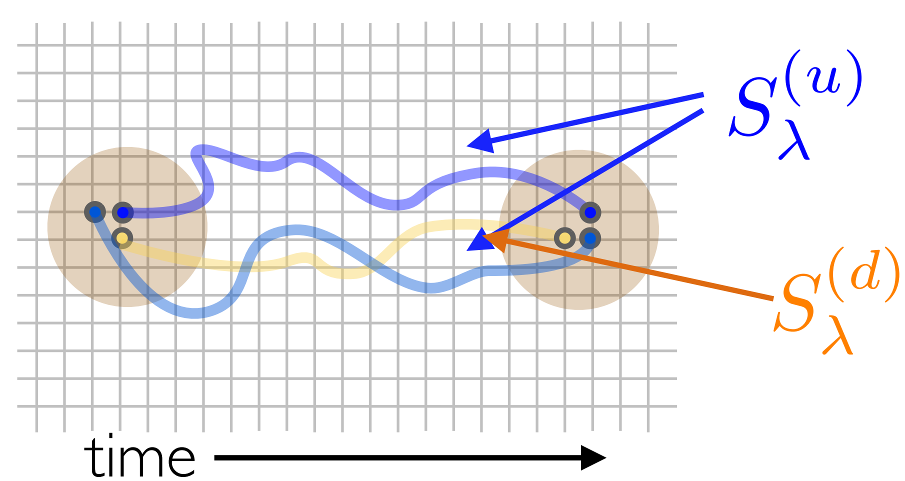
Fixed magnetic field → moments, polarisabilities
Fixed axial background field → axial charges, other matrix elts.

Construct correlation functions from propagators modified in field

compound propagator **constant**

$$\boxed{S^{(q)}_\lambda(x,y)} = S^{(q)}(x,y) + \boxed{\lambda_q} \int dz\, S^{(q)}(x,z)\, \gamma_3\gamma_5\, S^{(q)}(z,y)$$

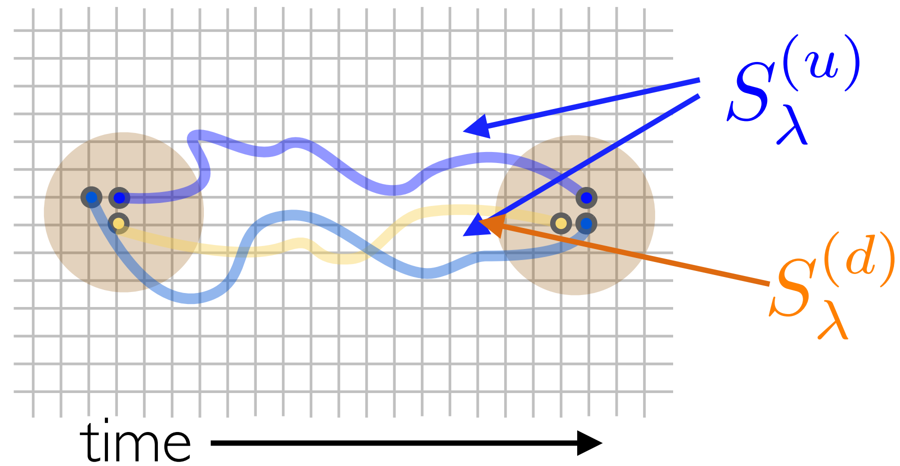
$$C_{\lambda_u;\lambda_d}(t) =$$



[Detmold and Savage, Nucl. Phys. **A743**, 170 (2004)]

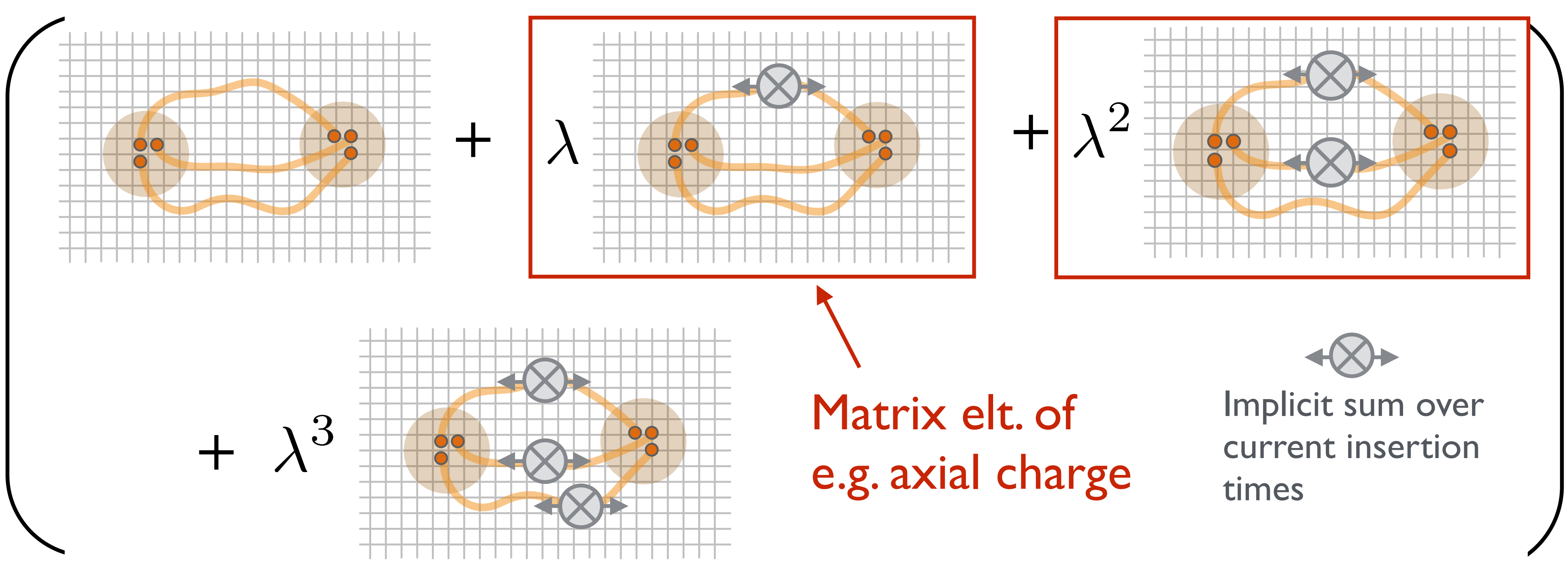
Background field approach

$$C_{\lambda_u;\lambda_d}(t) =$$

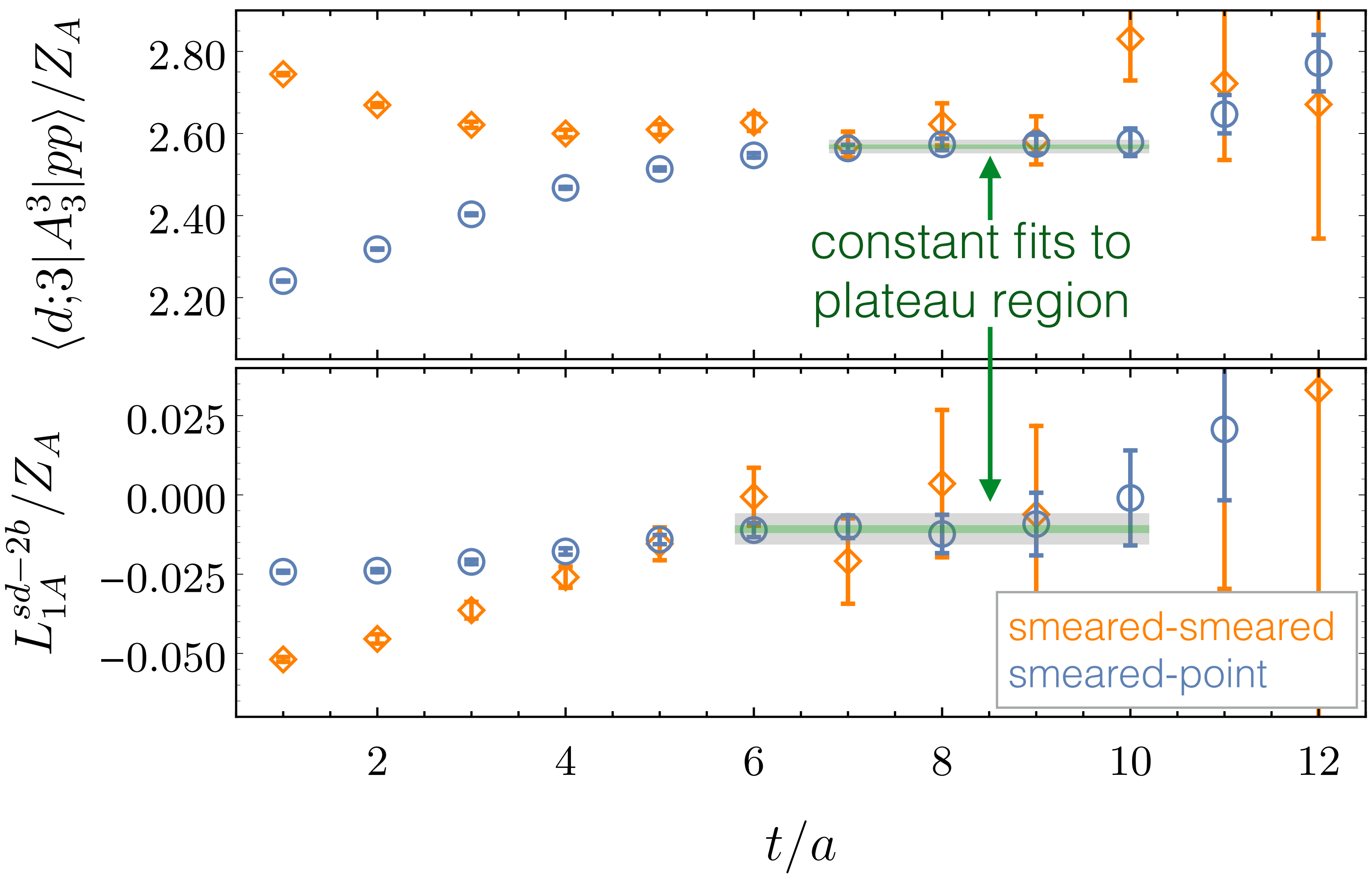


Second-order weak interactions

$$C_{\lambda_u;\lambda_d}(t) =$$



Proton-proton fusion



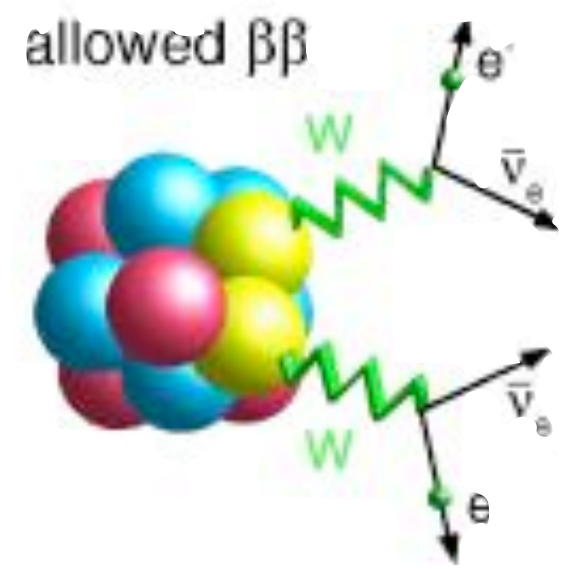
EFT parameter dictating fusion rate

$$\frac{L_{1,A}^{sd-2b}}{Z_A} = -0.011(1)(15)$$

Extrapolate, predict physical cross-section

Second order weak interactions

NPLQCD arXiv:1701.03456, 1702.02929



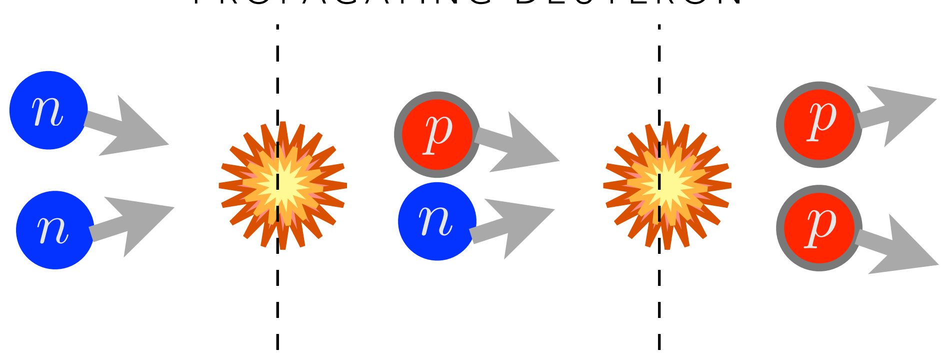
- Background axial field to second order
  - nn→pp transition matrix element

$$M^{2\nu}_{GT} = 6 \int d^4x d^4y \langle pp | T \left[J_3^+(x) J_3^+(y) \right] | nn \rangle$$

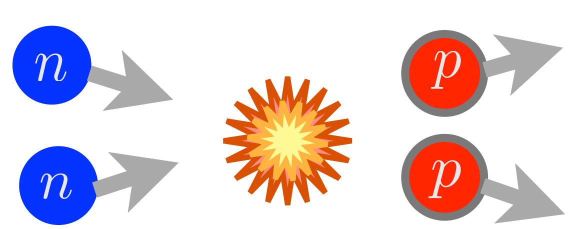
many technical LQCD complications
similar to RBC $K_L - K_S$ mixing work 1406.0916

- Non-negligible deviation from long distance deuteron intermediate state contribution

PROPAGATING DEUTERON



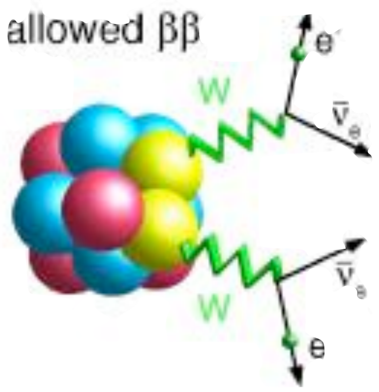
LONG-DISTANCE PIECE



SHORT-DISTANCE PIECE
(C.F. HADRONIC SCALE)

Second order weak interactions

NPLQCD arXiv:1701.03456, 1702.02929

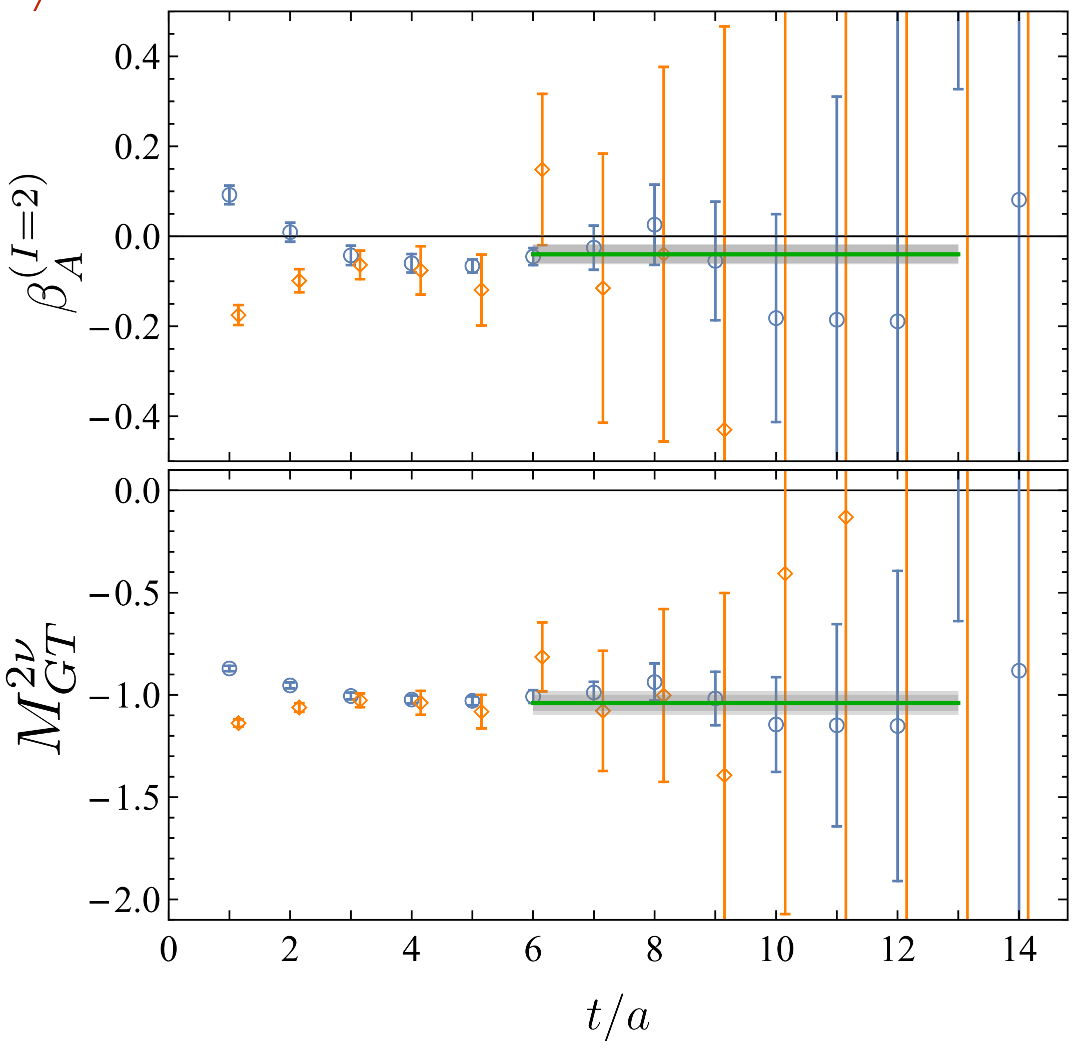


- Non-negligible deviation from long distance deuteron intermediate state contribution

Isotensor axial polarisability

$$M_{GT}^{2\nu} = -\frac{|M_{pp\to d}|^2}{E_{pp} - E_d} + \beta_A^{(I=2)}$$

➡ Potentially significant previously-neglected contribution



- TBD: connect to EFT for larger systems

Proposed Calculations

Lowest moments of isovector quark and gluon distributions in light nuclei (~15% uncertainty at m_π ~ 800 MeV and m_π ~ 450 MeV)

- Background field technique with twist-2 operator insertions
- Show how EMC effect emerges from interactions between nucleons
- Re-use two point functions, negligible additional cost:
  Predict gluonic analogue of EMC effect: benchmark for EIC program

TASK	Lattice Dimensions	β	m_π (MeV)	# of sources	# of calls	KNL Time [KNL-Hrs]	CPU Time [J/ψ core-Hrs]
A: Inversions	$32^3 \times 96$	6.1	450	1.3×10^5	$18 \times 1.3 \times 10^5$	2.03×10^5	-
A: Block Production	$32^3 \times 96$	6.1	450	1.3×10^5	$103 \times 1.3 \times 10^5$	2.82×10^5	-
A: Contractions	$32^3 \times 96$	6.1	450	1.3×10^5	$103 \times 1.3 \times 10^5$	-	6.9×10^6
B: Inversions	$32^3 \times 48$	6.1	806	1.3×10^5	$18 \times 1.3 \times 10^5$	1.24×10^5	-
B: Block Production	$32^3 \times 48$	6.1	806	1.3×10^5	$103 \times 1.3 \times 10^5$	2.82×10^5	-
B: Contractions	$32^3 \times 48$	6.1	806	1.3×10^5	$103 \times 1.3 \times 10^5$	-	6.9×10^6
Total Request:						$\mathbf{8.91 \times 10^5}$	$\mathbf{13.8 \times 10^6}$