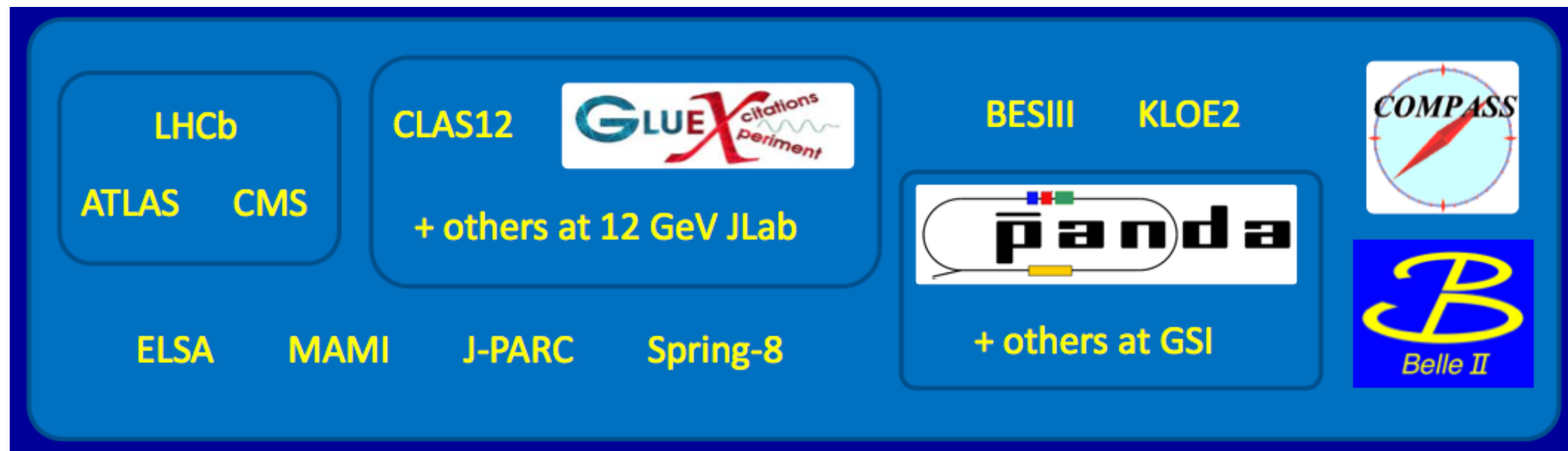


# Excited State Spectroscopy & QCD



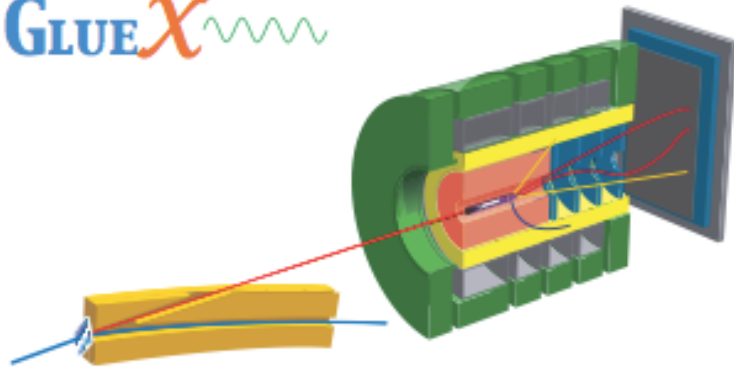
- Determination of hadron spectrum of QCD a central goal in NP
- Several experiments worldwide



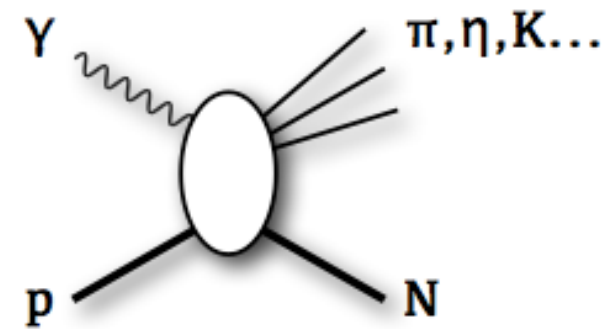
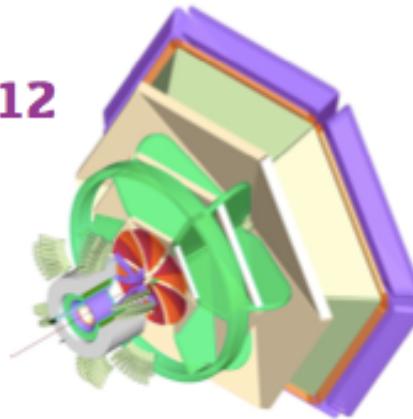
# Spectrum - light meson experiments

3

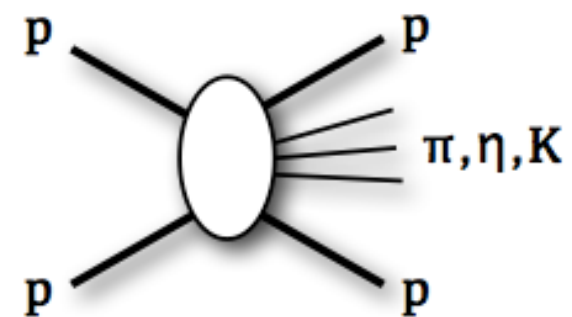
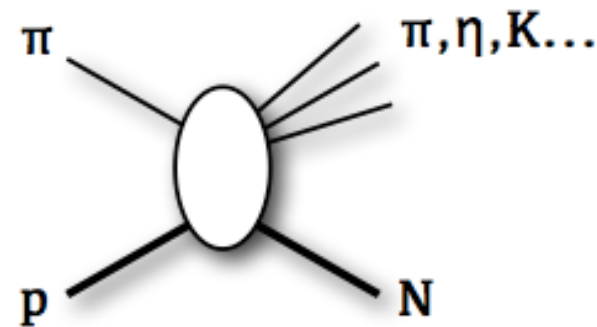
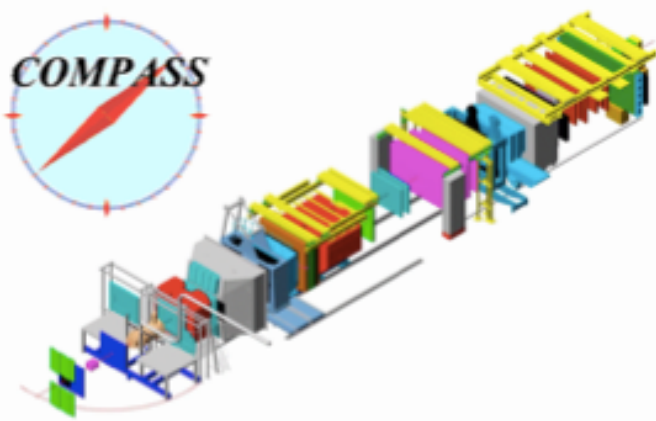
GLUEX



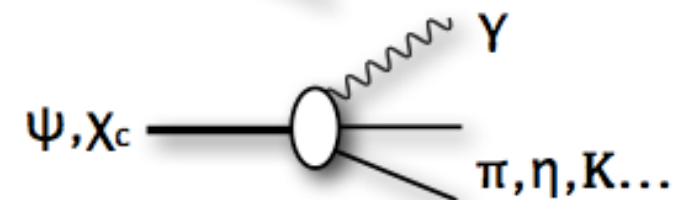
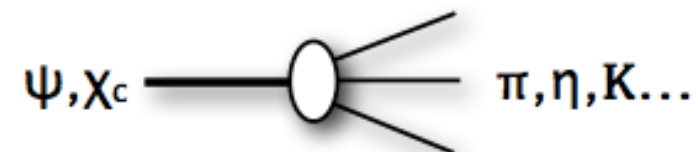
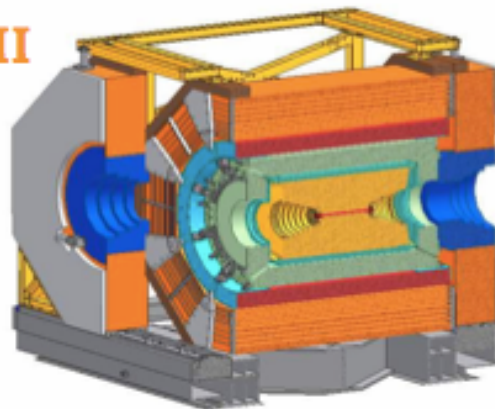
CLAS12



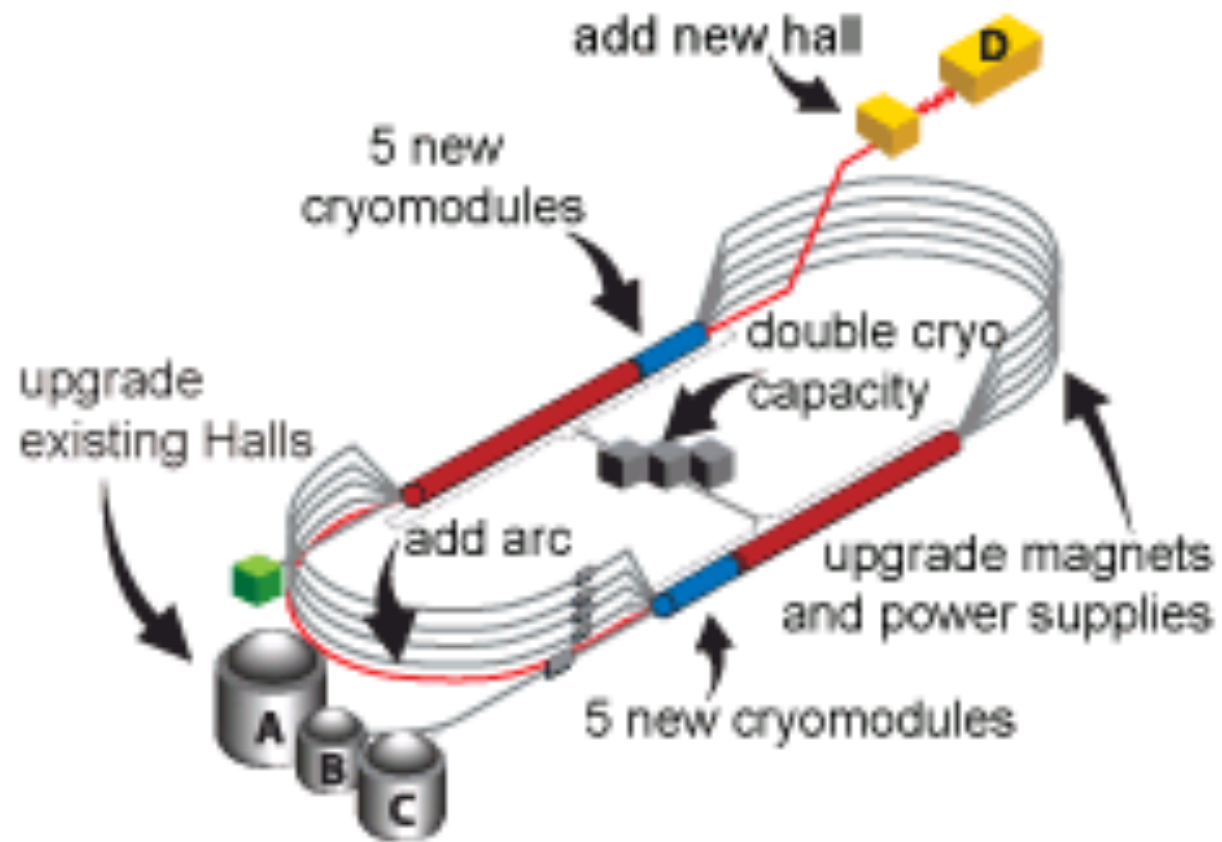
COMPASS



BES III



- JLab finishing a \$335M upgrade

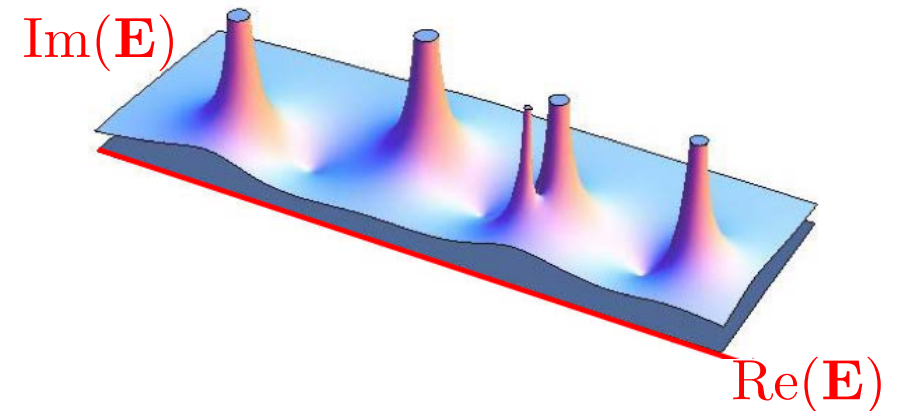
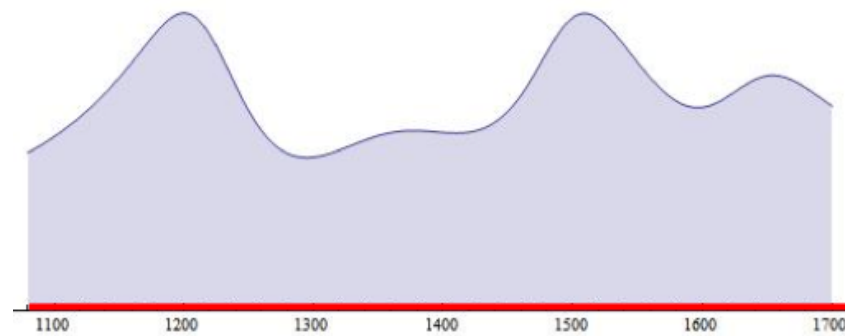


- Doubled beam energy
- Added new Hall D (GlueX)



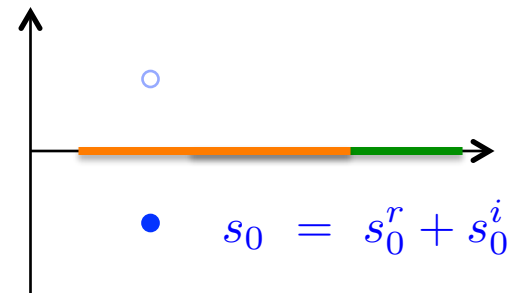


- Most hadrons are resonances
  - E.g.,  $\pi N \pi N$

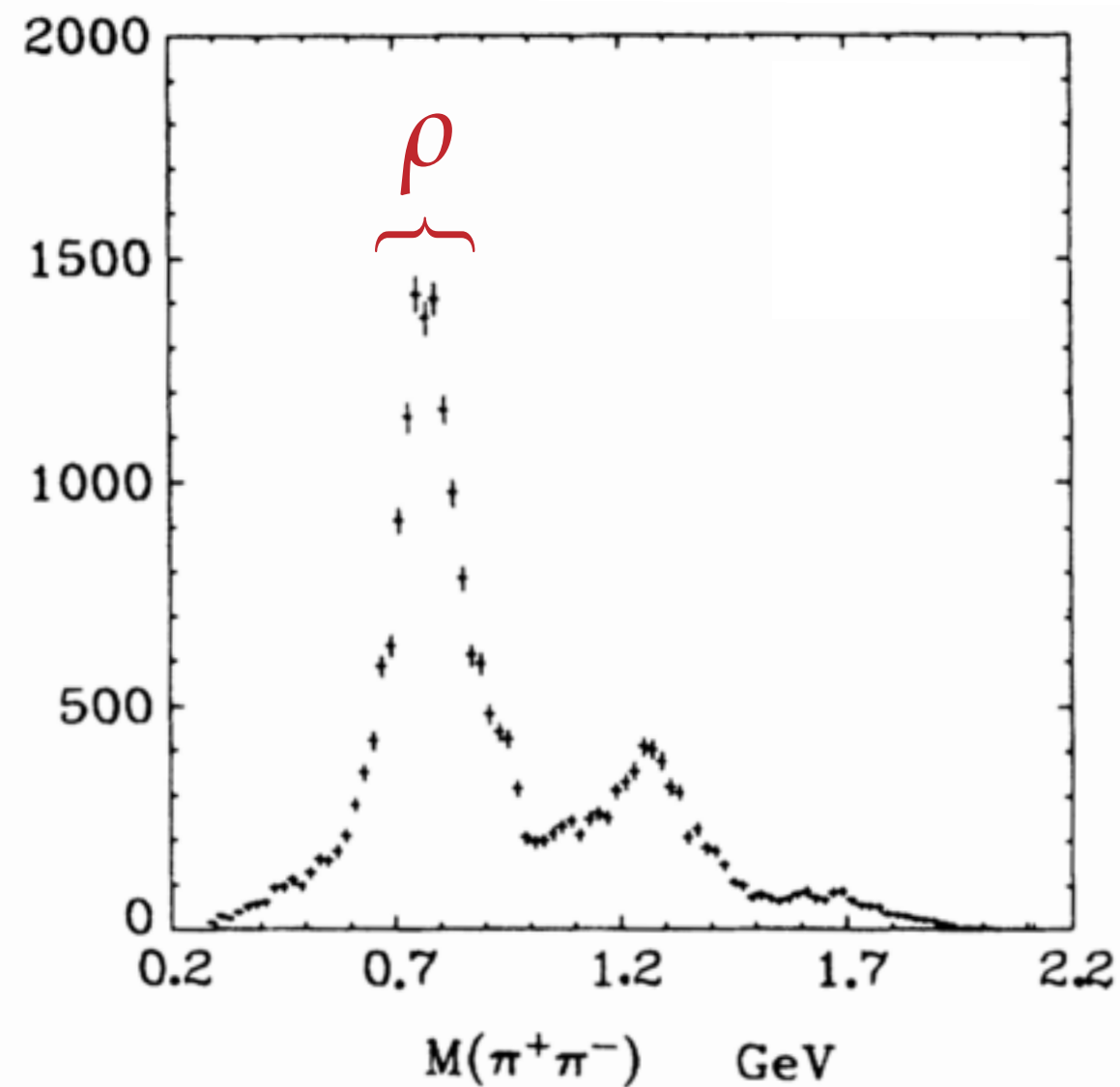


- Formally defined as a pole in a partial-wave scattering amplitude

$$t_l(s) \sim \frac{R}{s_0 - s} + \dots$$



- Different channels should have same pole location
  - Pole structure gives decay information
- Can we predict hadron properties from first principles?



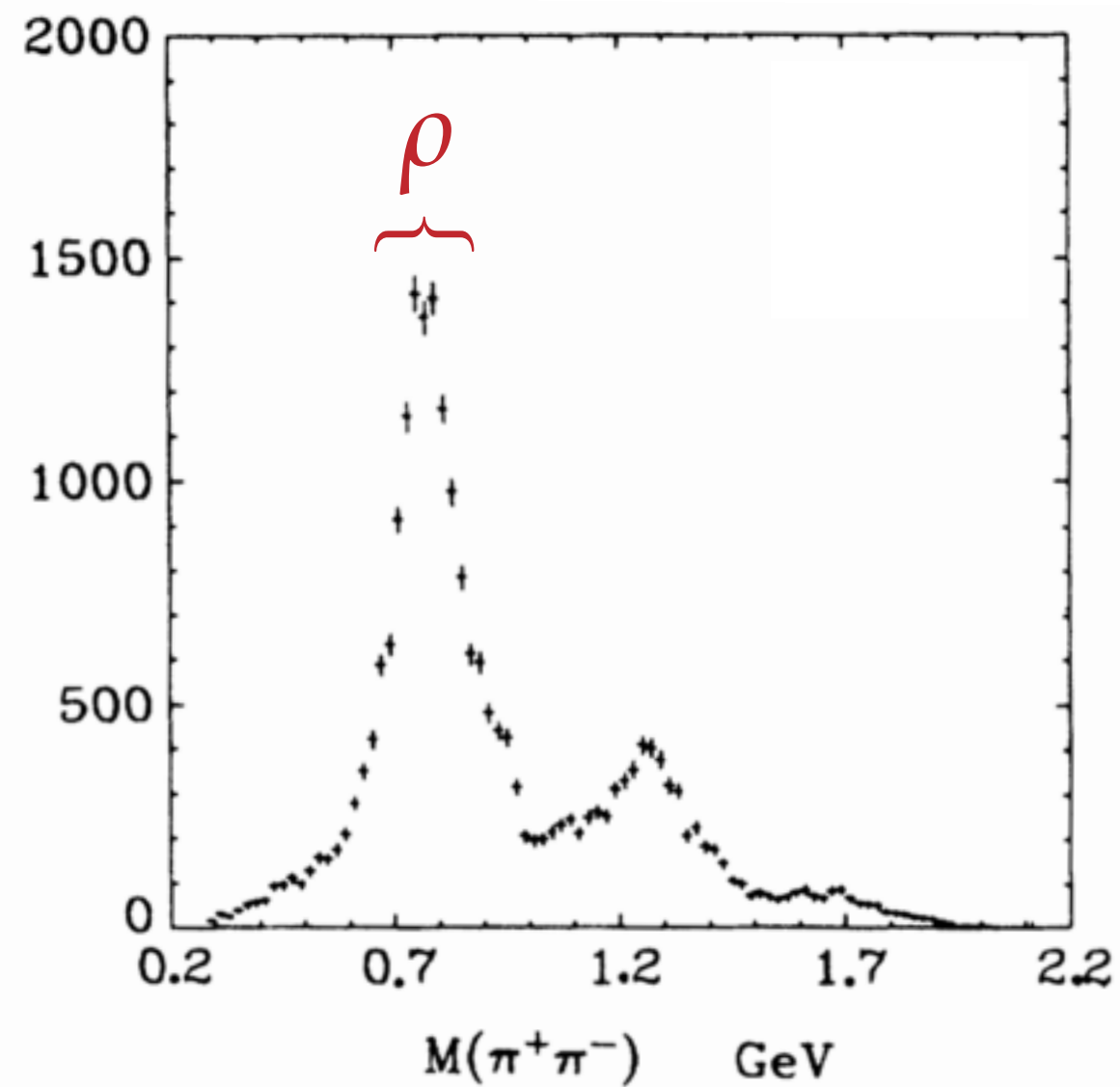
expand angular dependence  
in *partial waves*

## PARTIAL WAVE AMPLITUDE

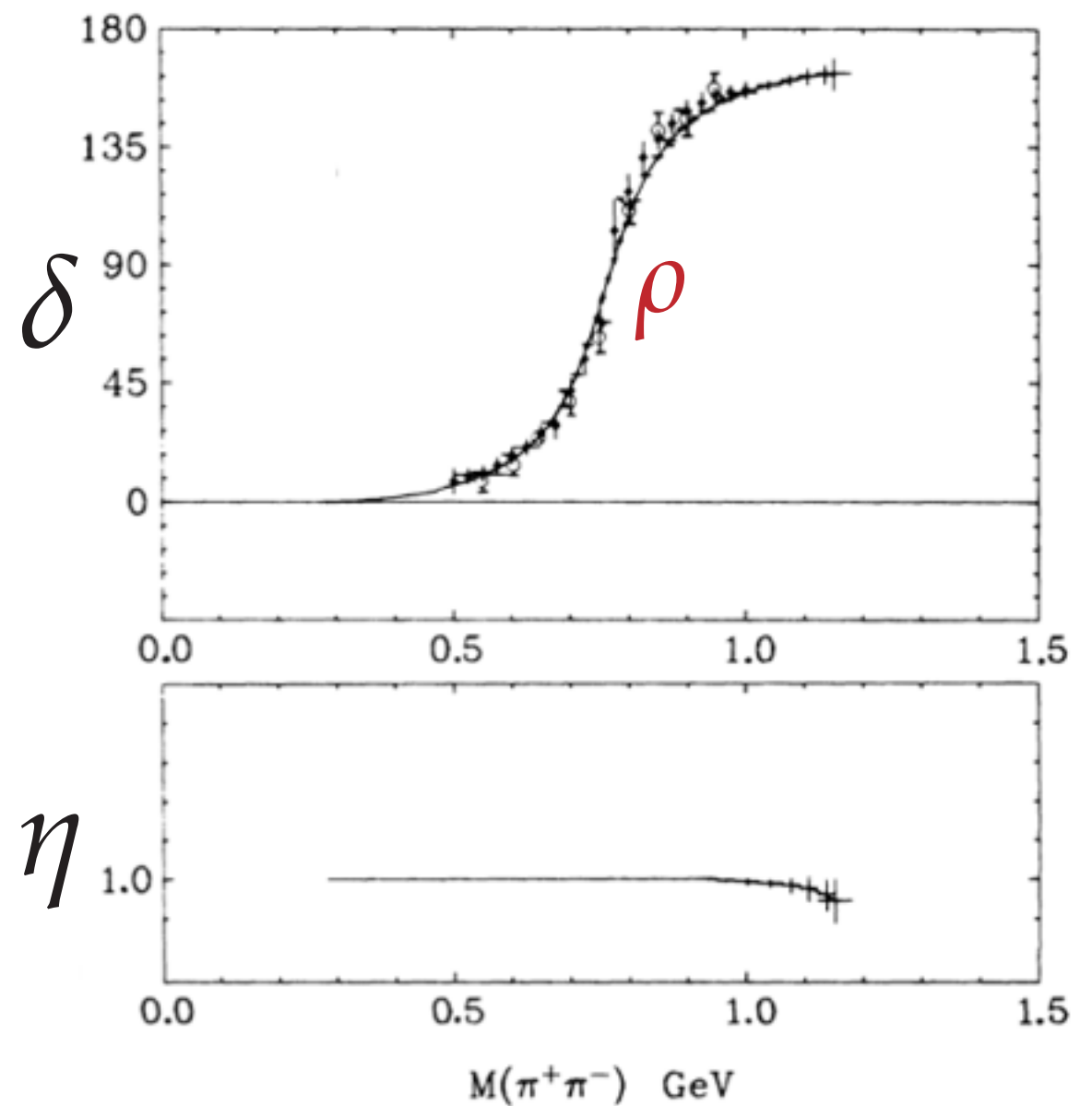
$$f_\ell = \frac{1}{2i} \left( \eta_\ell e^{2i\delta_\ell} - 1 \right)$$

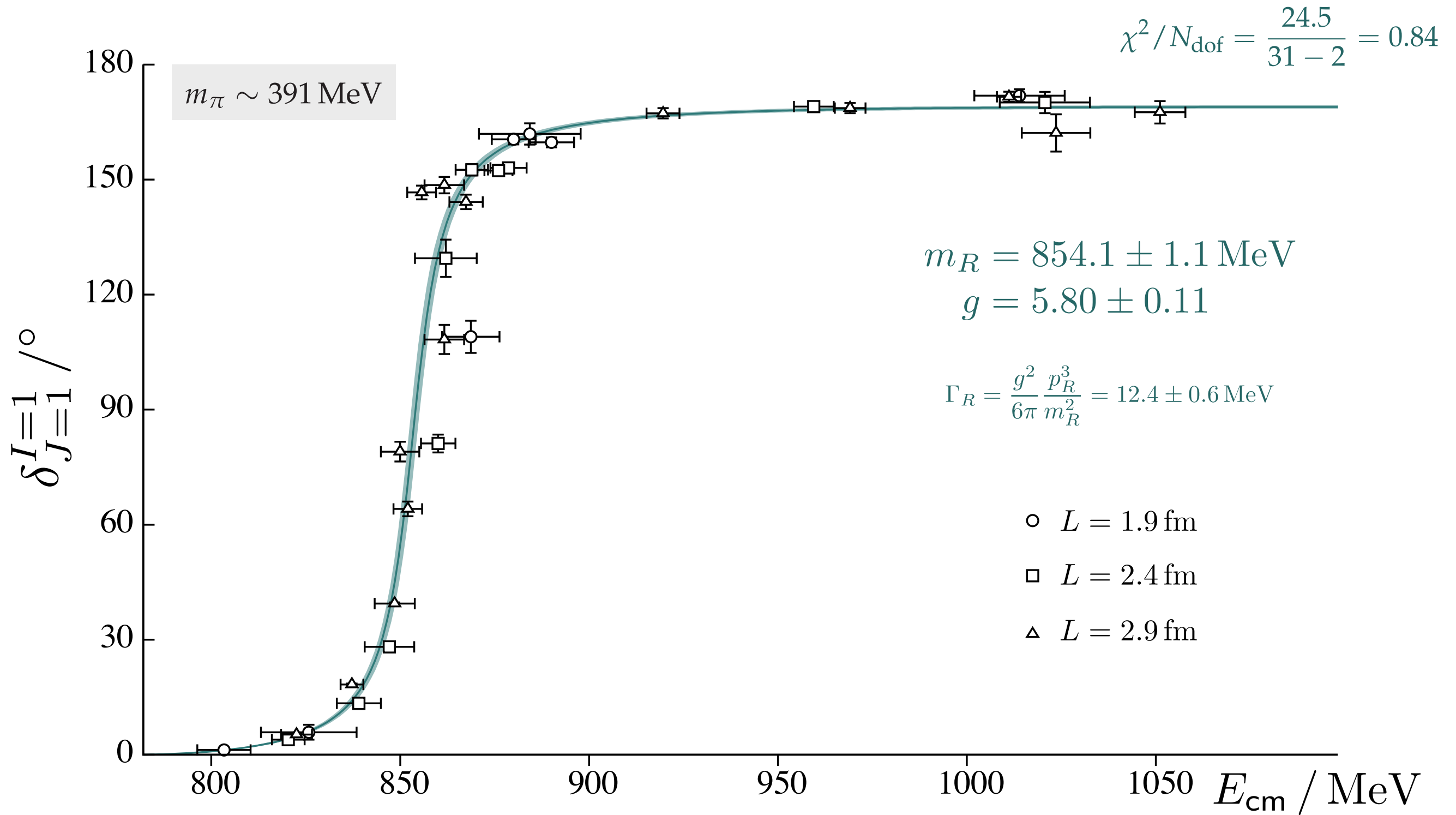
$\eta = 1$  elastic

$\eta \leq 1$  inelastic



## RESONANT PHASE SHIFT

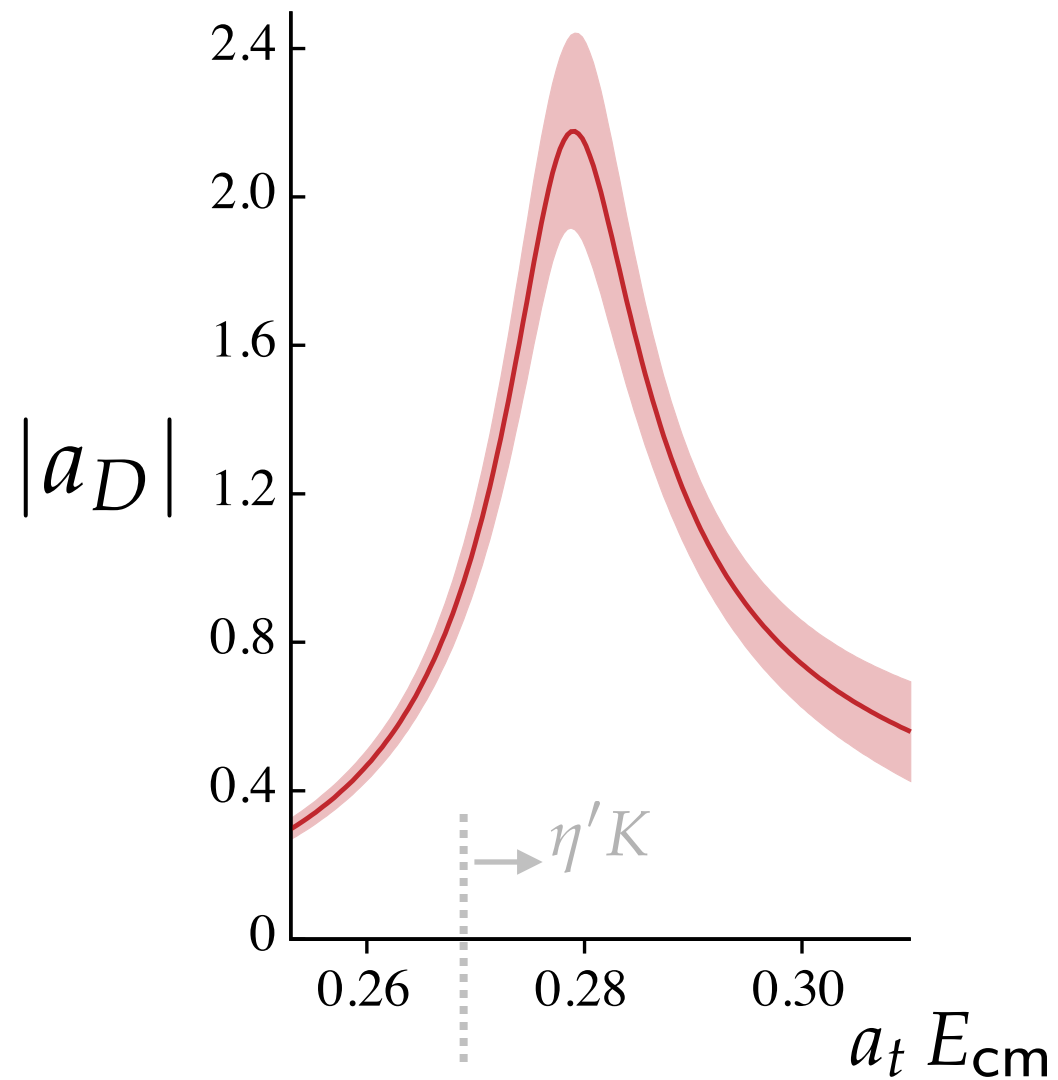




$g_{\text{phys.}} = 5.95(2)$  *PDG*

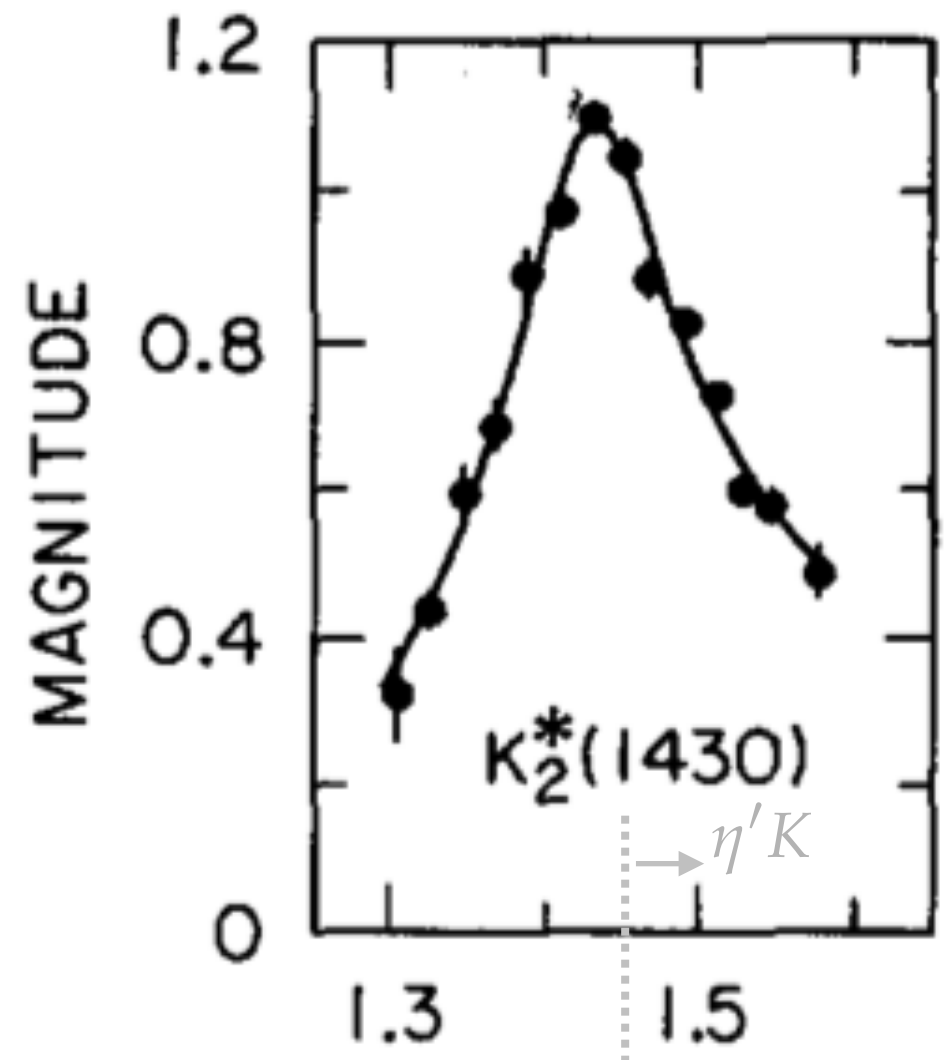


## D-WAVE $\pi K \rightarrow \pi K$ AMPLITUDE



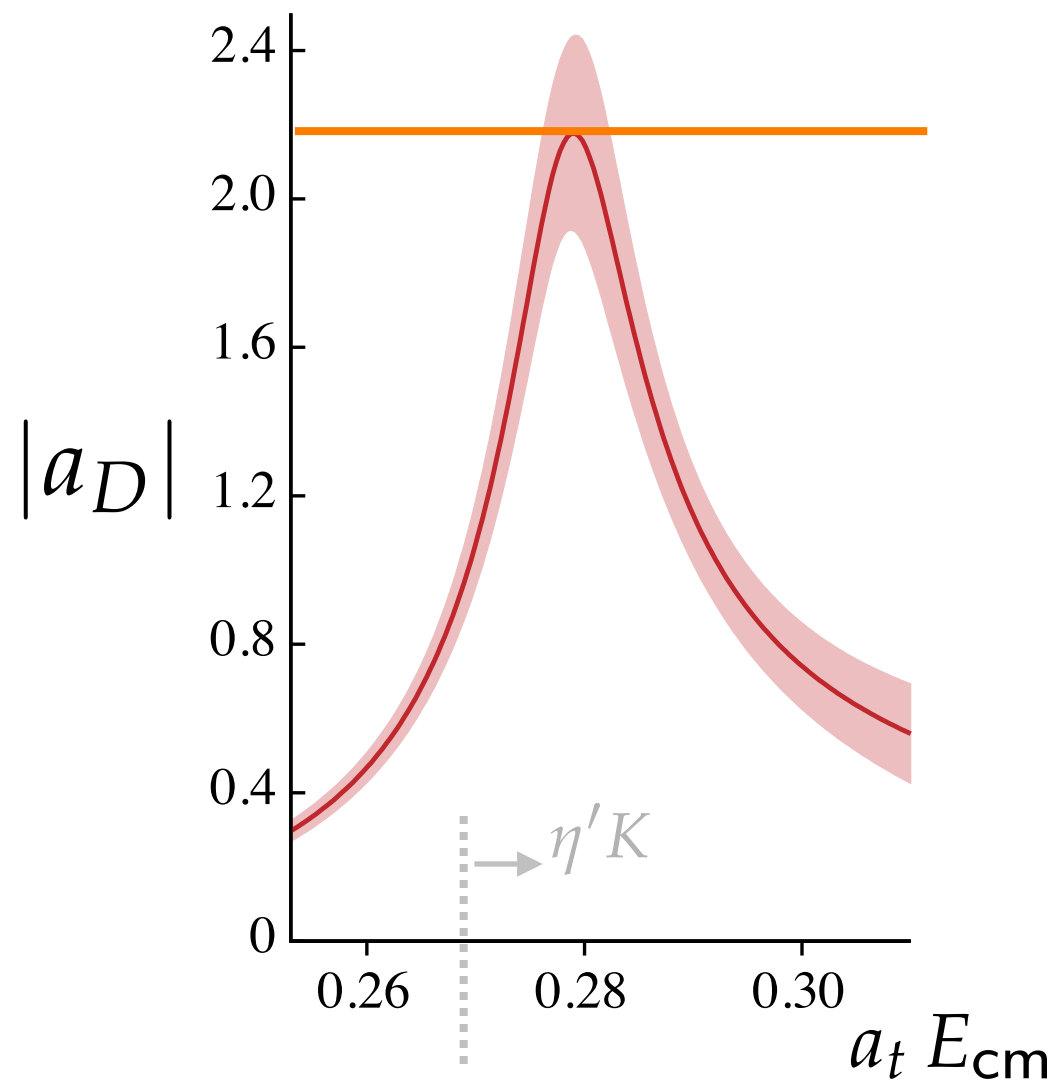
$$m_\pi \sim 391 \text{ MeV}$$

## LASS D-WAVE



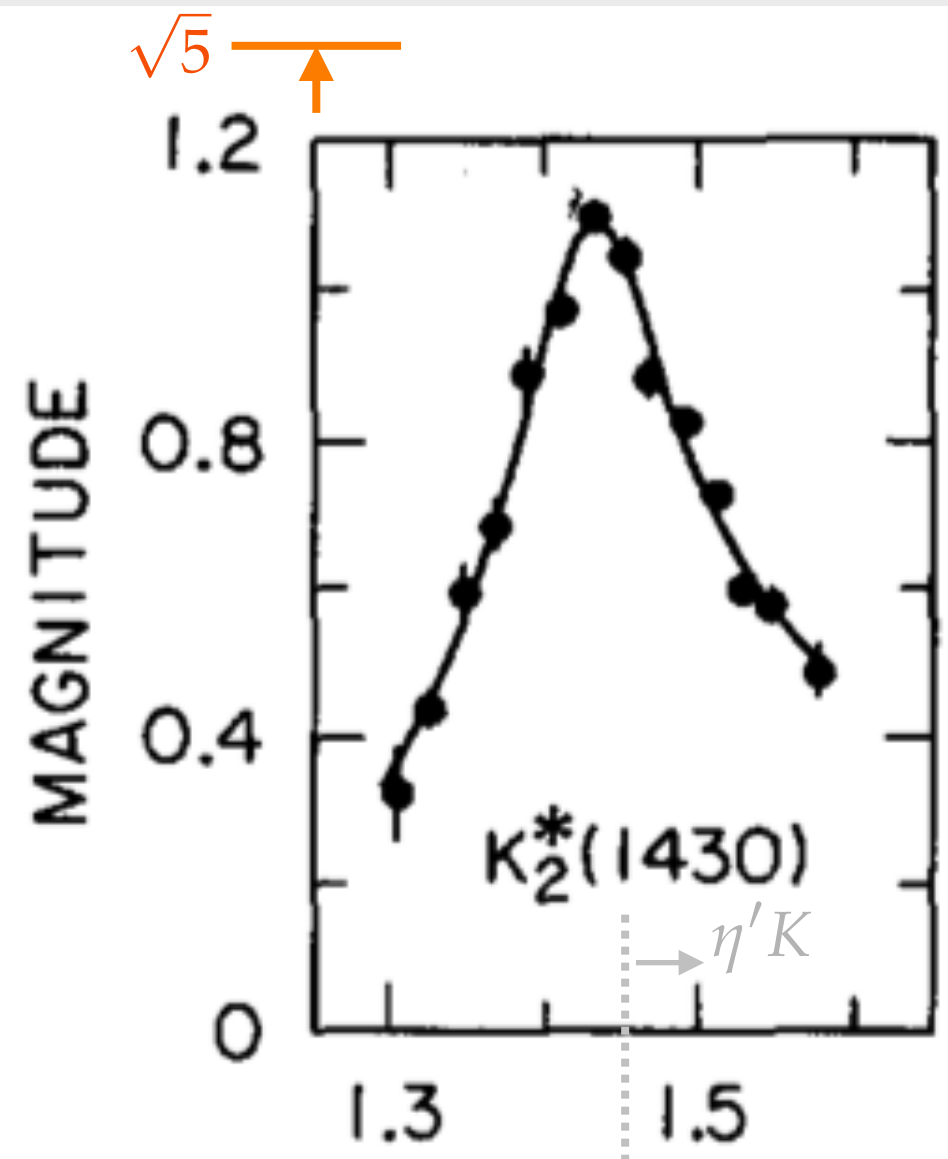
LASS, NPB296 493

## D-WAVE $\pi K \rightarrow \pi K$ AMPLITUDE



$$m_\pi \sim 391 \text{ MeV}$$

## LASS D-WAVE

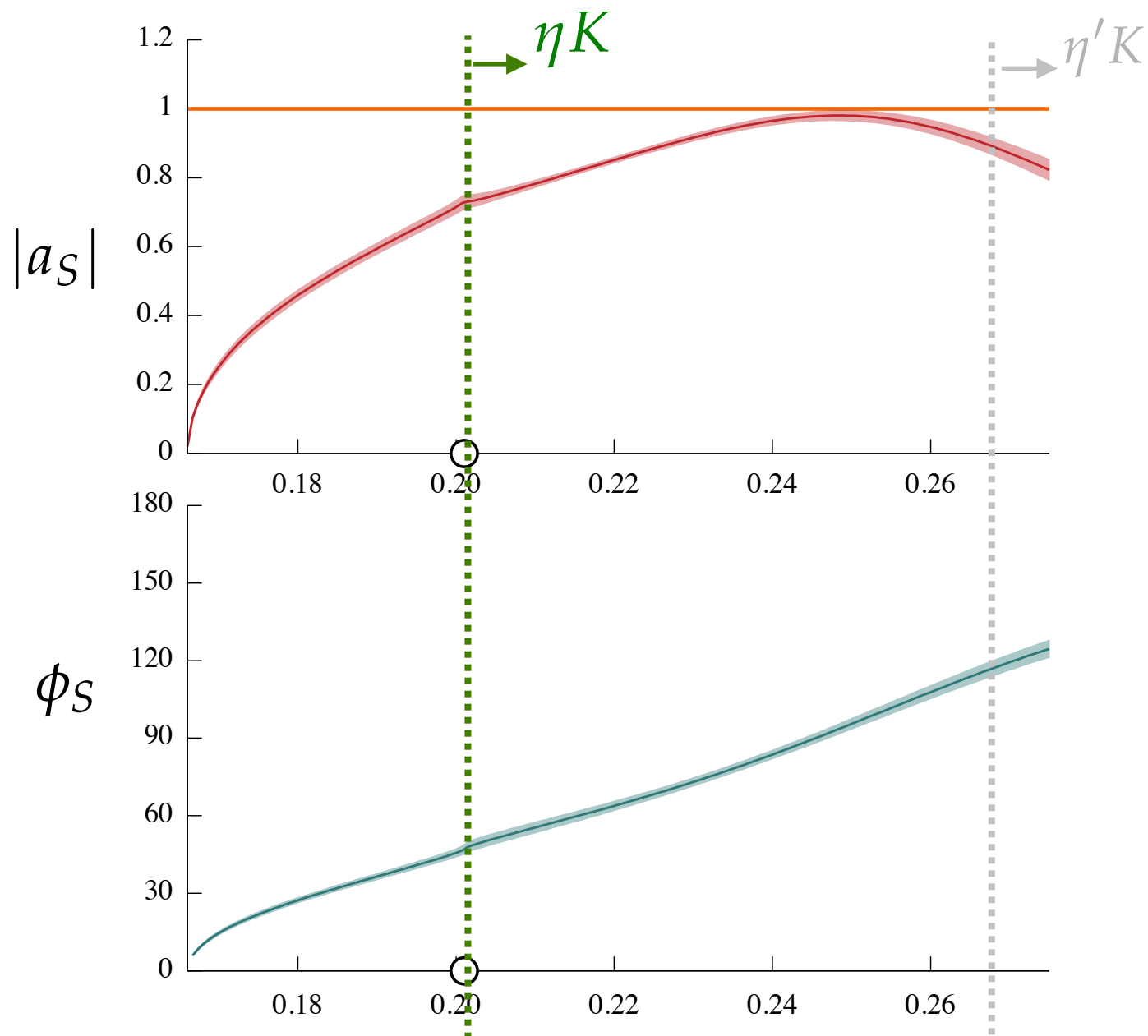


LASS, NPB296 493

# S-wave $\pi K/\eta K$ more complicated

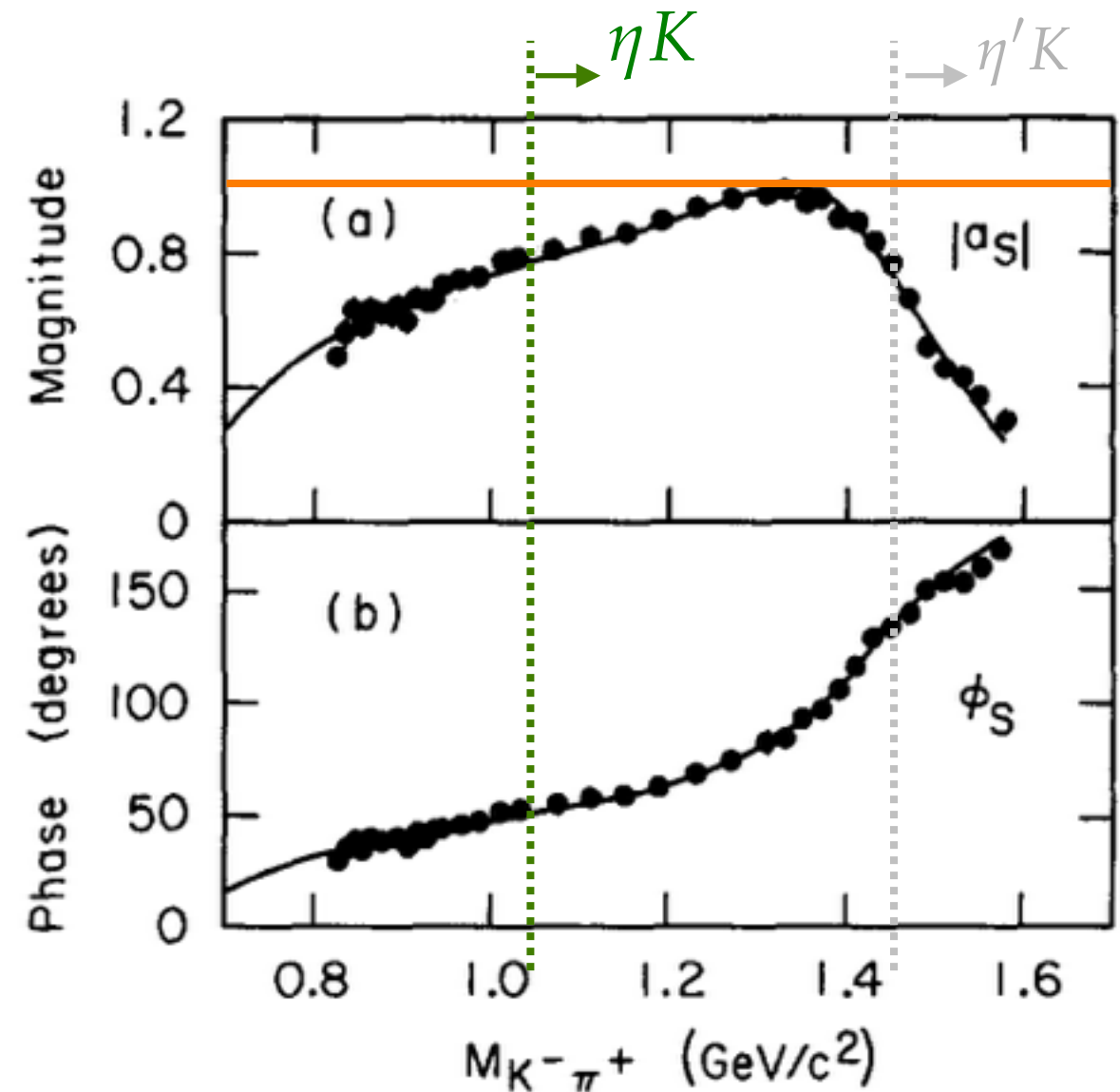
10

## S-WAVE $\pi K \rightarrow \pi K$ AMPLITUDE



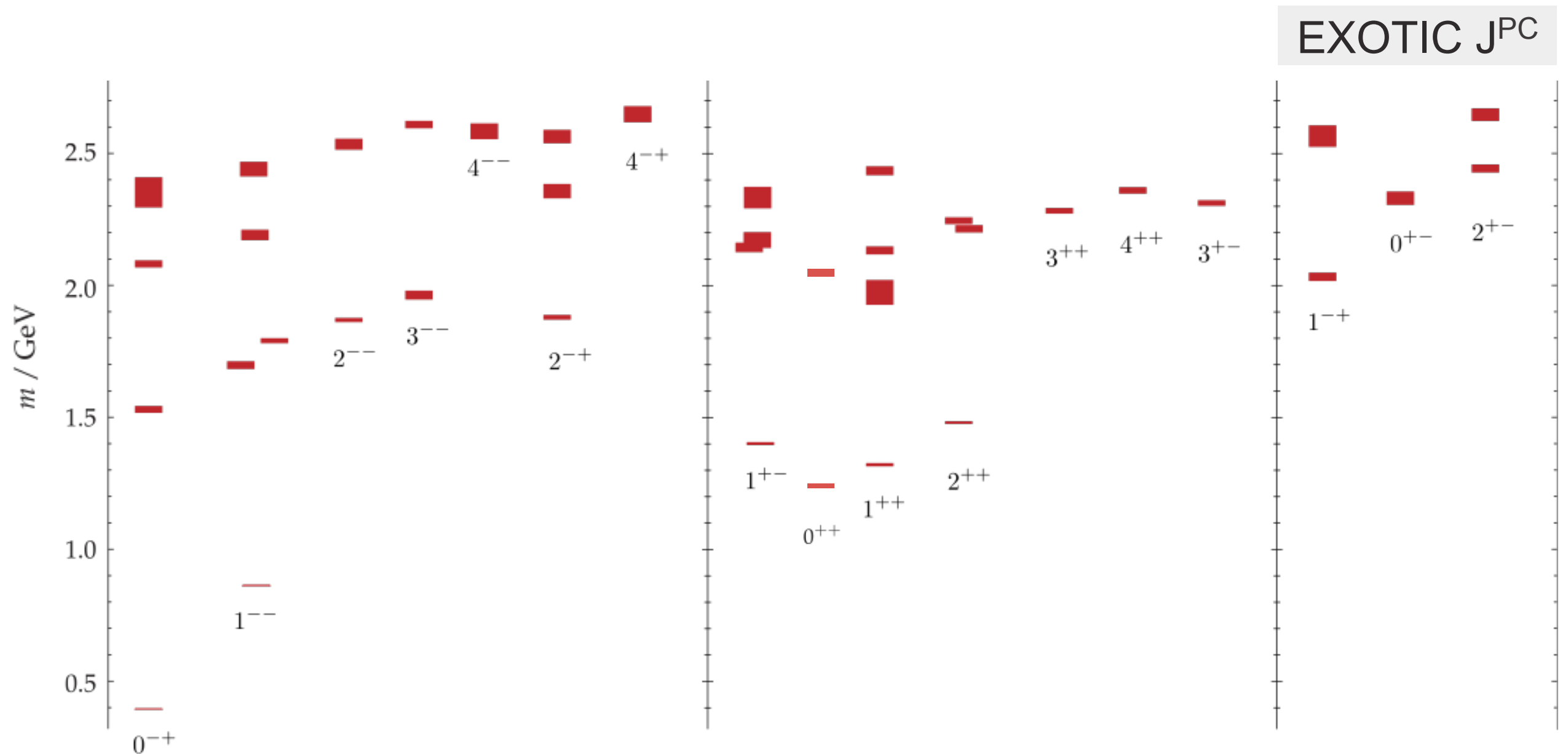
$$m_\pi \sim 391 \text{ MeV}$$

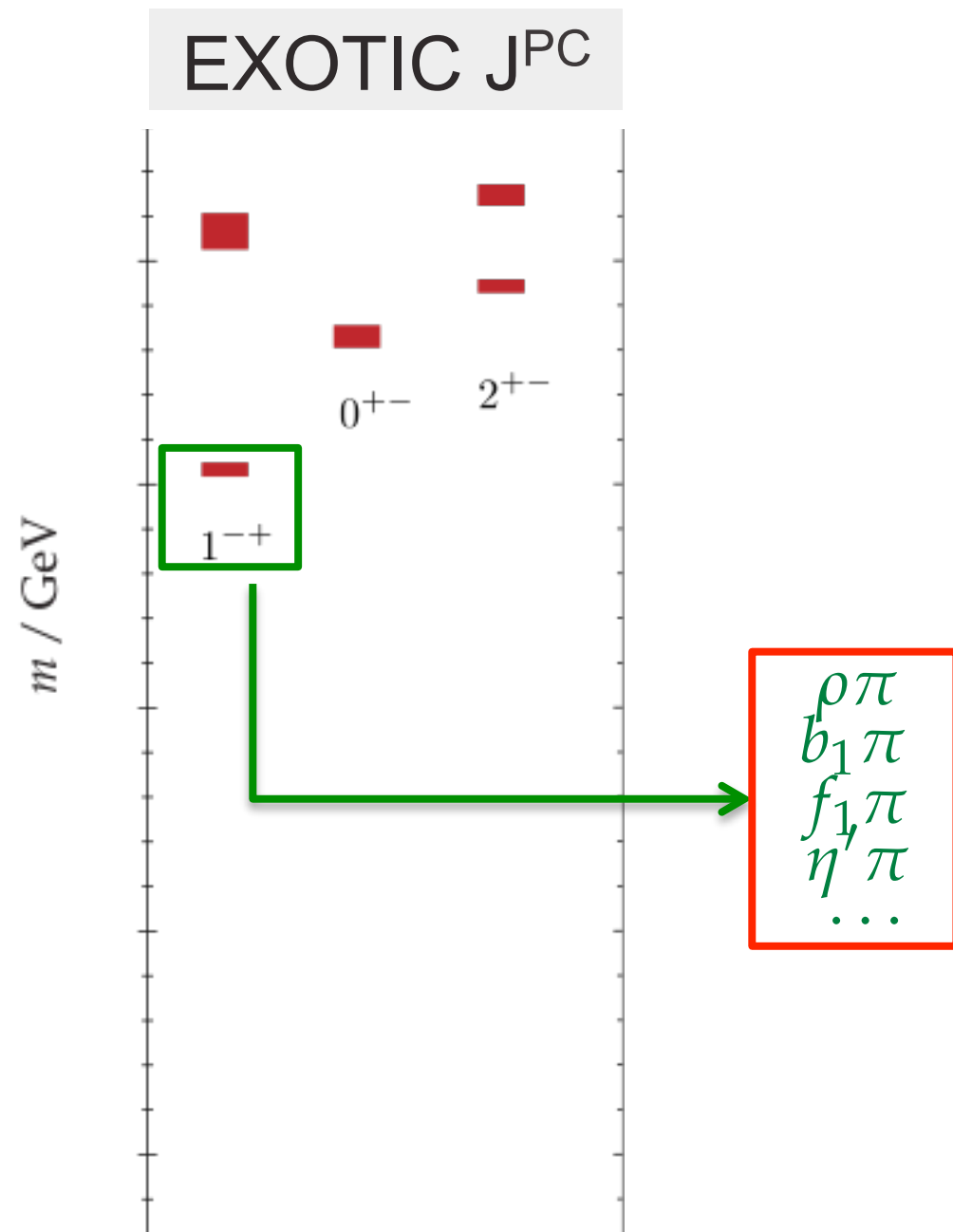
## LASS S-WAVE



LASS, NPB296 493

- Compute decays (branching fractions) of exotic mesons:





- LQCD suggests existence of exotic mesons
- Expt. determination requires measurement in many decay channels
- Present LQCD calculations missing this info

- Objective is to compute them ahead of expt.
  - ➔ Guide expt. analysis

JLab expt. beam has started

- Getting to the physical pion mass **not the most pressing concern here**
- Need to establish feasibility of techniques for resonances



- Getting to the physical pion mass **not the most pressing concern here**
- Need to establish feasibility of techniques for resonances
  - Hard to do with physical kinematics

e.g. Some of the simple low-lying resonances:  $f_2(1270)$      $M > 8m_\pi$

$a_2(1320)$      $M > 9m_\pi$

the number of open channels  
is too large to start here

- Getting to the physical pion mass **not the most pressing concern here**
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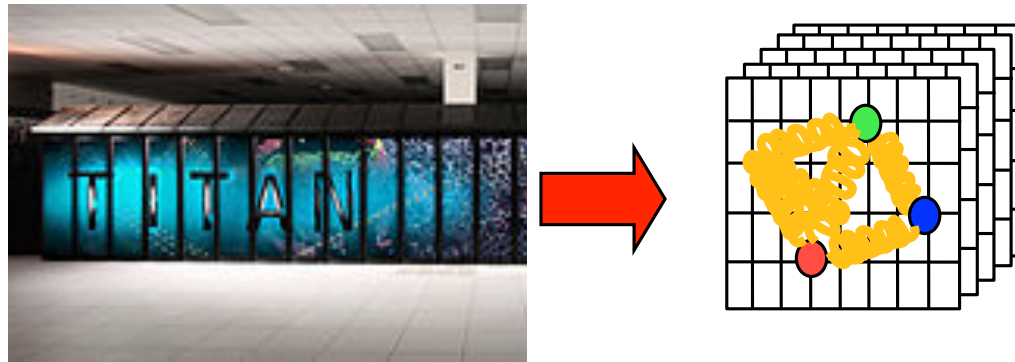
- Development of three-body formalism required

*HANSEN & SHARPE - MUCH PROGRESS*



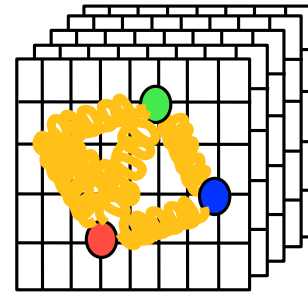
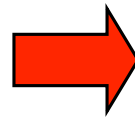
## Generate the configurations

- Leadership level
- 60K cores, 10's TF-yr

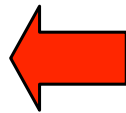
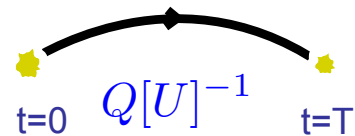


## Generate the configurations

- Leadership level
- 60K cores, 10's TF-yr



## Propagators



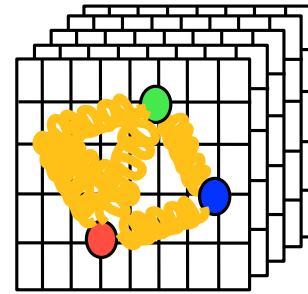
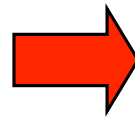
+



Analyze  
100K copies  
4 Kepler GPUs

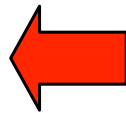
## Generate the configurations

- Leadership level
- 60K cores, 10's TF-yr



## Propagators

$$t=0 \quad \overbrace{Q[U]^{-1}} \quad t=T$$



+



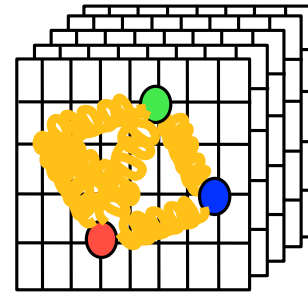
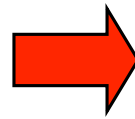
## Analyze

100K copies  
4 Kepler GPUs  
Now also AMG!

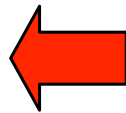
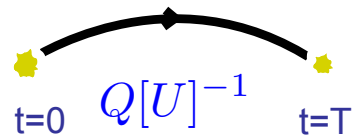


## Generate the configurations

- Leadership level
- 60K cores, 10's TF-yr



## Propagators



+



## Analyze

100K copies  
4 Kepler GPUs  
Now also AMG!



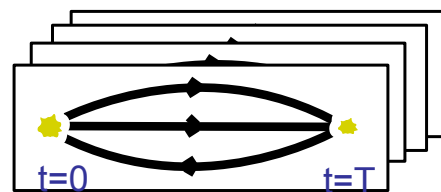
## Contract

- 8 cores, CPUs



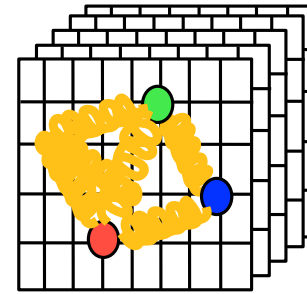
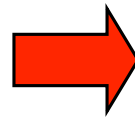
## Correlators

100K – 1M copies

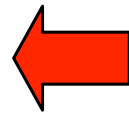
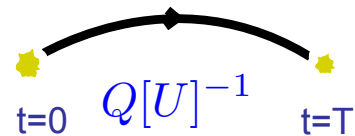


## Generate the configurations

- Leadership level
- 60K cores, 10's TF-yr



## Propagators



+



## Analyze

100K copies  
4 Kepler GPUs  
Now also AMG!

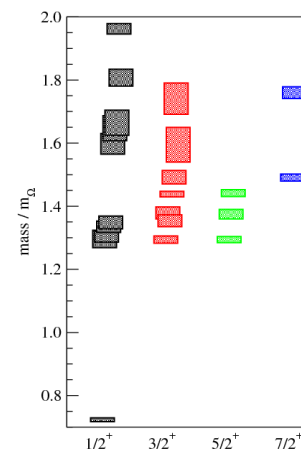
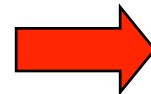
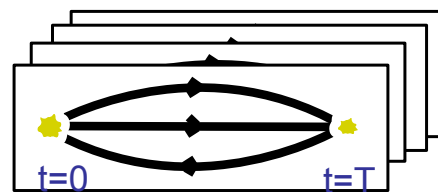
## Contract

- 8 cores, CPUs



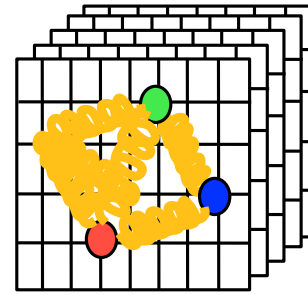
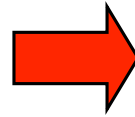
## Correlators

100K – 1M copies



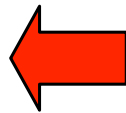
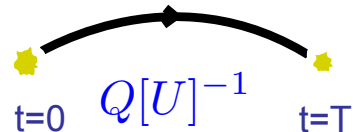
## Generate the configurations

- Leadership level
- 60K cores, 10's TF-yr



Few big jobs  
Few big files

## Propagators



+



## Analyze

100K copies  
4 Kepler GPUs  
Now also AMG!

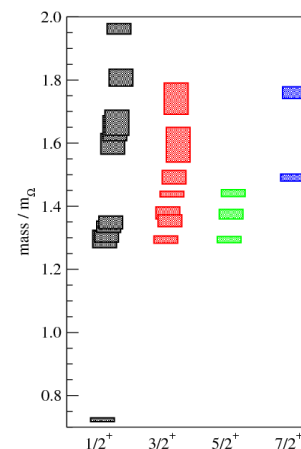
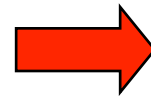
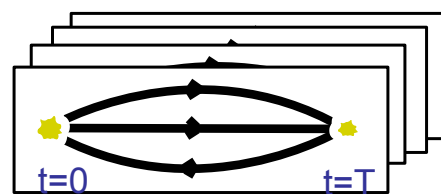
## Contract

- 8 cores, CPUs



## Correlators

100K – 1M copies

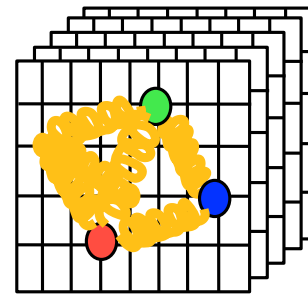
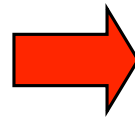


Many small jobs  
Many big files

I/O movement

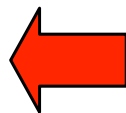
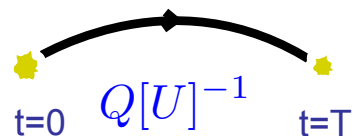
## Generate the configurations

- Leadership level
- 60K cores, 10's TF-yr



~25%  
Leadership level

## Propagators



+



Analyze  
100K copies  
4 Kepler GPUs  
Now also AMG!

~75%  
Throughput mode

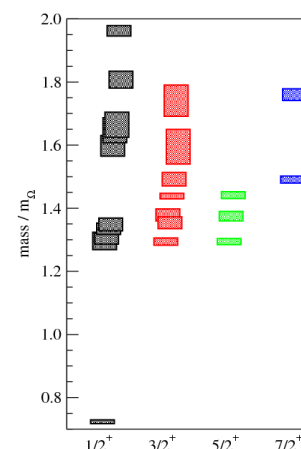
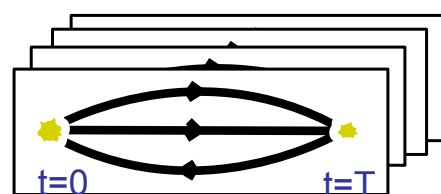
## Contract

- 8 cores, CPUs



## Correlators

100K – 1M copies



> 5%  
New analysis cost

- Expect a discrete spectrum in a finite periodic volume

$$\psi(x + L) = \psi(x)$$

e.g. free particle  $e^{ip(x+L)} = e^{ipx}$

quantized momentum  $p = \frac{2\pi}{L}n$

- For an interacting theory  $\cot \delta_\ell(E) = \mathcal{M}_\ell(E, L)$  *LÜSCHER ...*

elastic scattering  
phase-shift

known  
function

Discrete energies  
in a finite-volume



Discrete values  
of the phase-shift

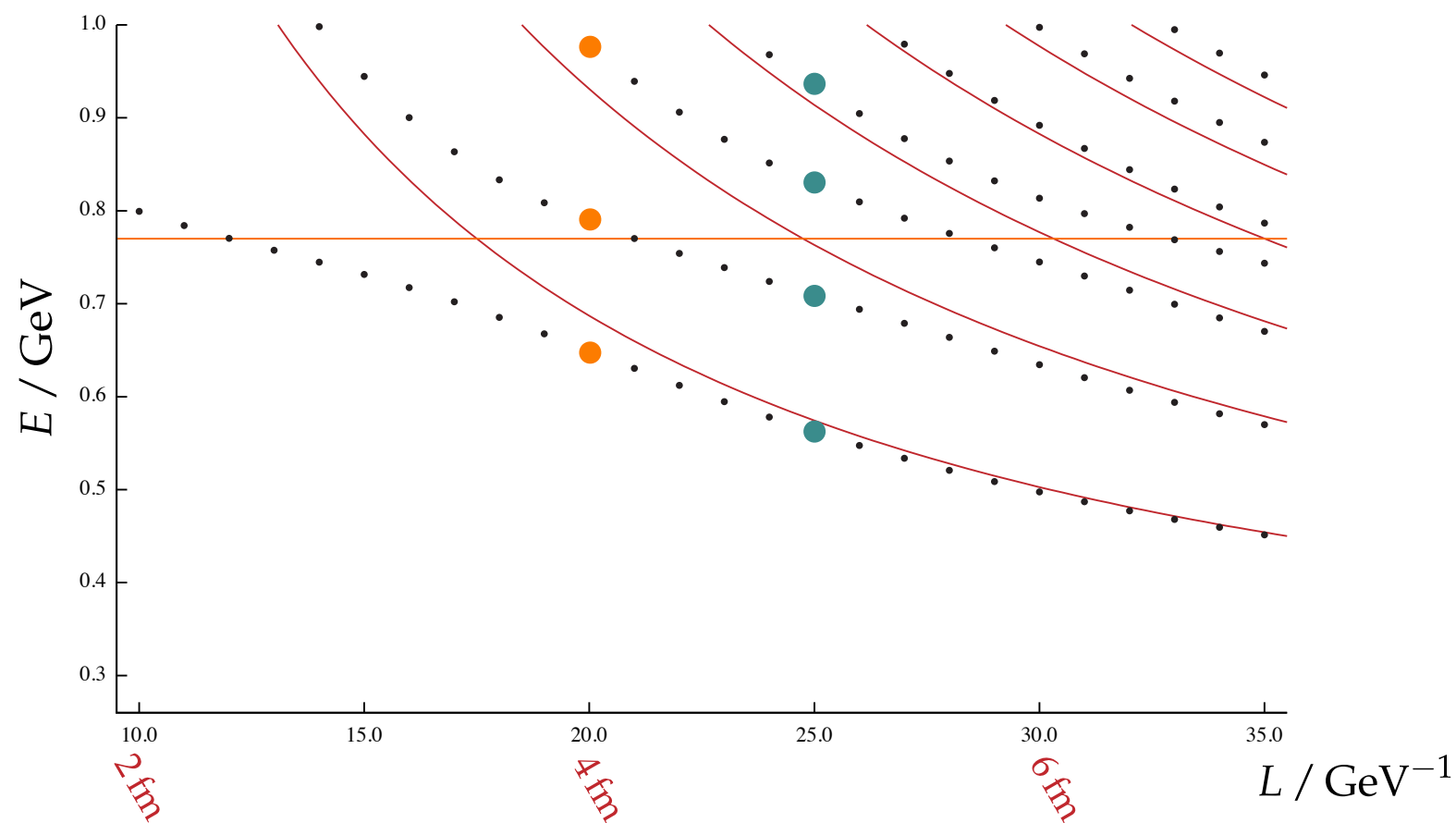


# Scattering in a finite cubic volume

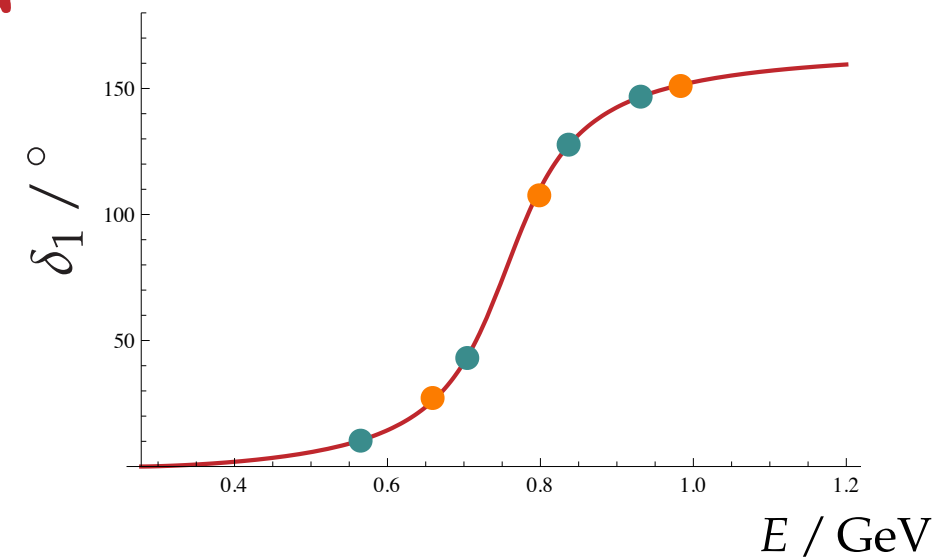
17

- Experimental  $\pi\pi$   $I=1$   $P$ -wave scattering amplitude

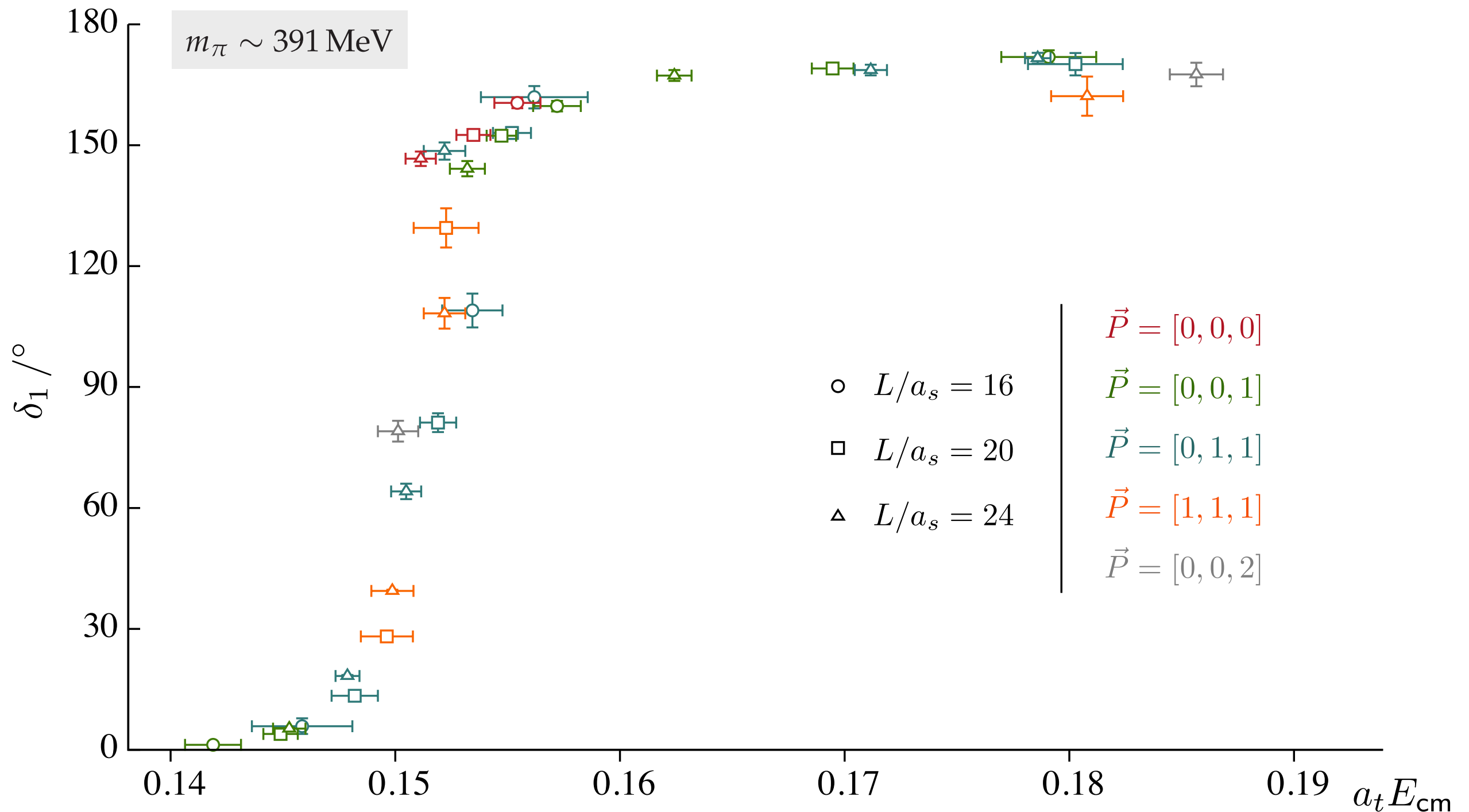
CUBIC BOX SPECTRUM



$P$ -WAVE PHASE SHIFT

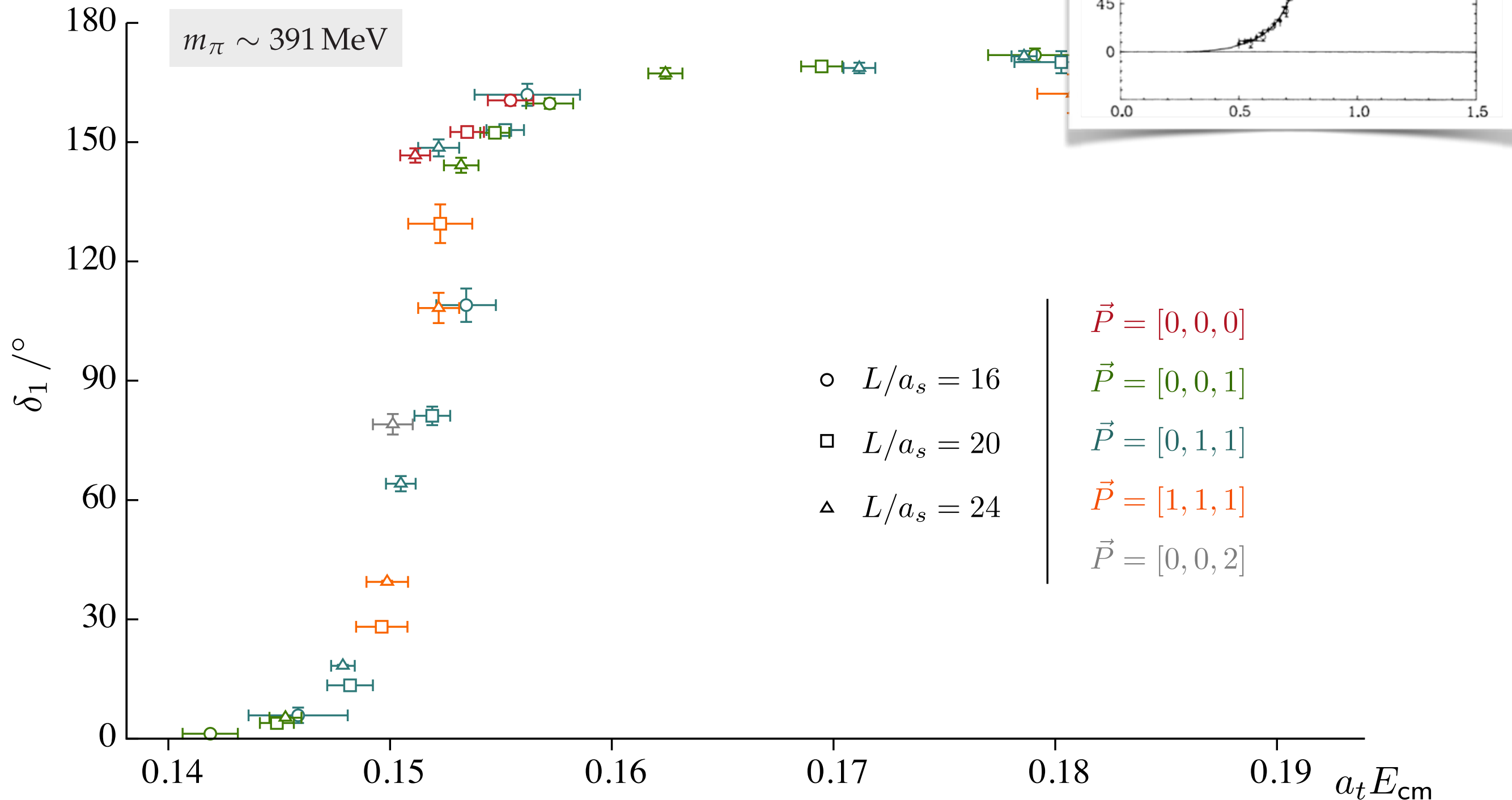




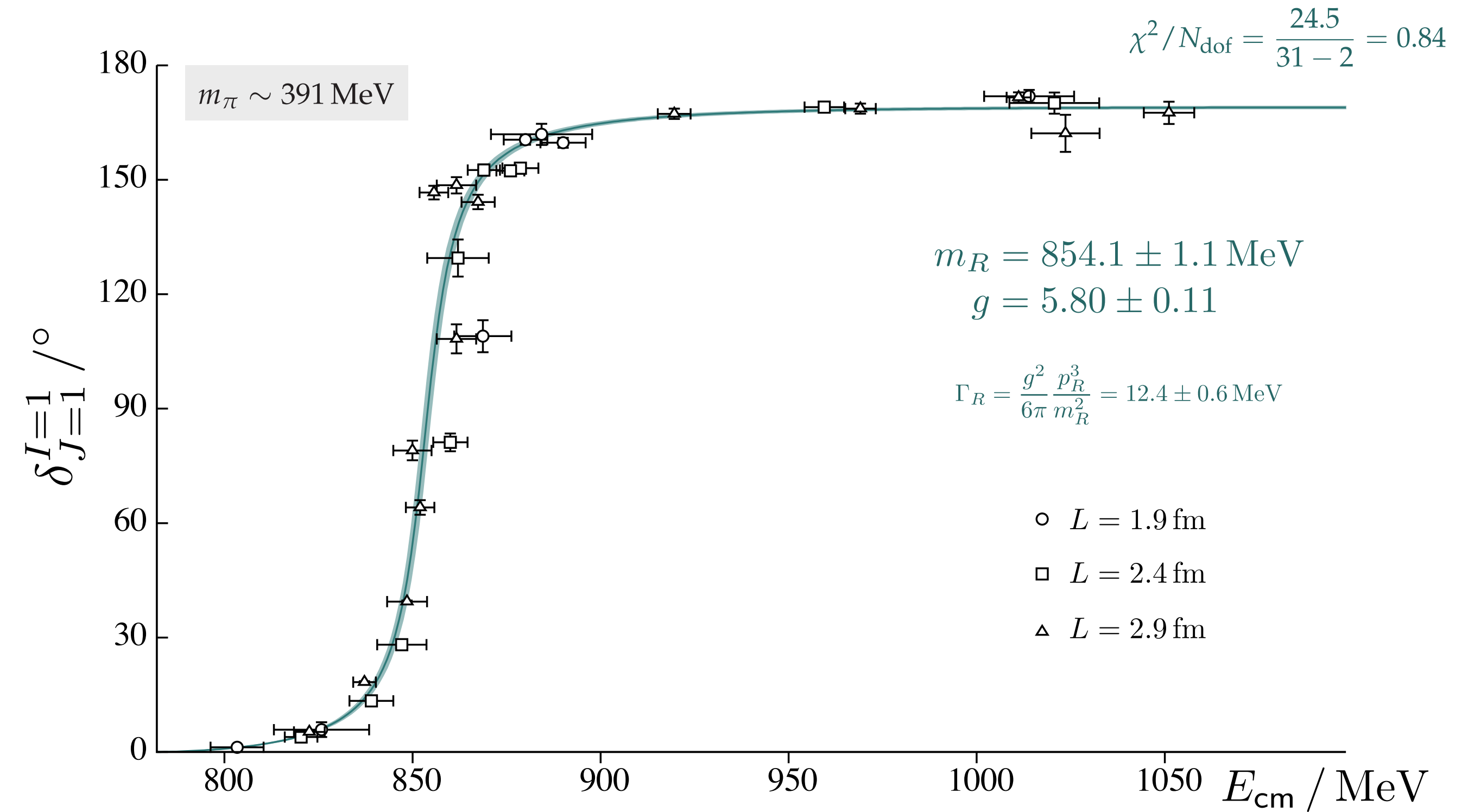


PRD87 034505 (2013)

# $\pi\pi$ $P$ -wave phase-shift



PRD87 034505 (2013)



$$g_{\text{phys.}} = 5.95(2) \text{ PDG}$$

HE, JHEP 0507 011  
HANSEN, PRD86 016007  
BRICENO, PRD88 094507  
GUO, PRD88 014051

- Finite-volume formalism recently derived (multiple methods)

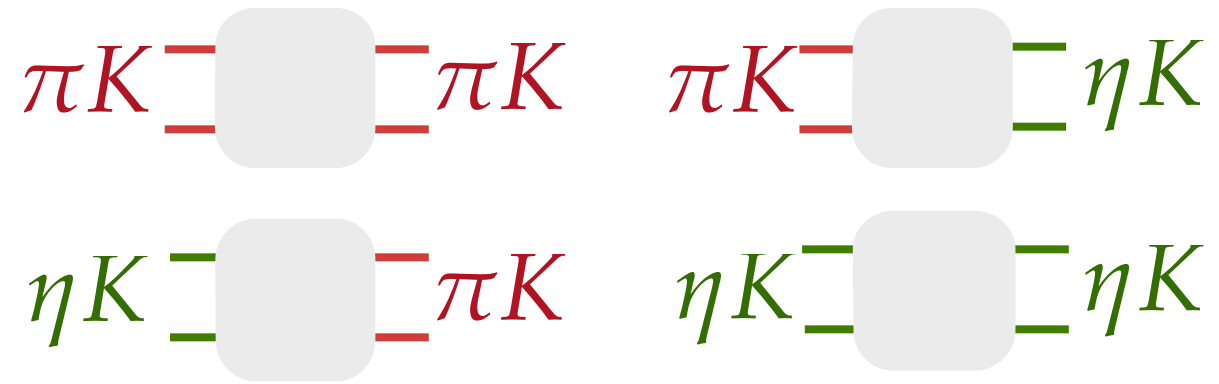
$$\det \left[ \underbrace{([t^{(\ell)}(E)]_{ij})^{-1}}_{\text{scattering matrix}} + \underbrace{i\rho_i(E) \delta_{ij}}_{\text{phase space}} - \underbrace{\delta_{ij} \mathcal{M}_\ell(p_i(E)L)}_{\text{known functions}} \right] = 0$$

matrices in  
partial-wave space ...

- However, this is **one equation for multiple unknowns** (per energy level)  $\frac{1}{2}N(N+1)$  for  $N$  channels
  - parameterize the energy dependence of  $t$
  - try to describe a spectrum globally

“Energy-dependent” analysis

- Example of coupled-channel scattering



- Compute finite-volume spectrum

$$\bar{u}\Gamma s$$

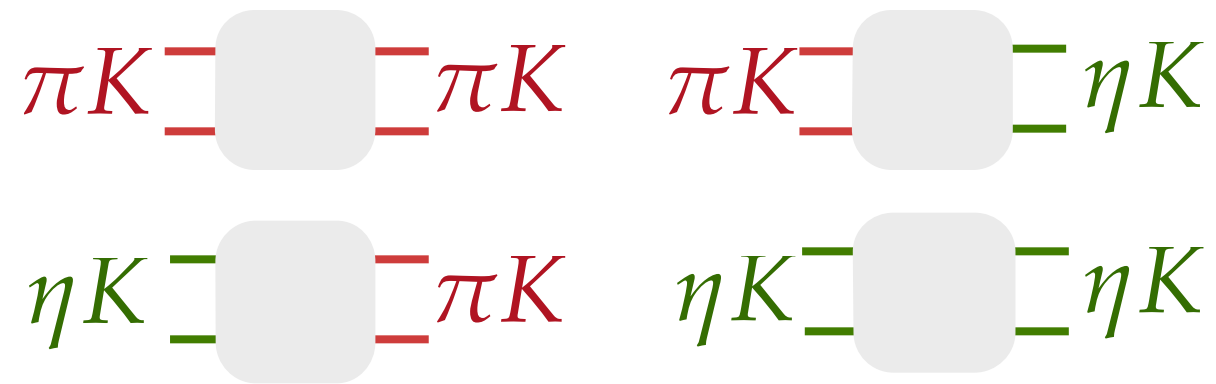
$$\sum_{\hat{k}_1, \hat{k}_2} C(\Lambda, \vec{P}; \vec{k}_1, \vec{k}_2) \pi^+(\vec{k}_1) K^+(\vec{k}_2)$$

$$\sum_{\hat{k}_1, \hat{k}_2} C(\Lambda, \vec{P}; \vec{k}_1, \vec{k}_2) \eta^+(\vec{k}_1) K^+(\vec{k}_2)$$

PRL 113 182001

PRD 91 054008

- Example of coupled-channel scattering



- Compute finite-volume spectrum

 $\bar{u}\Gamma s$ 

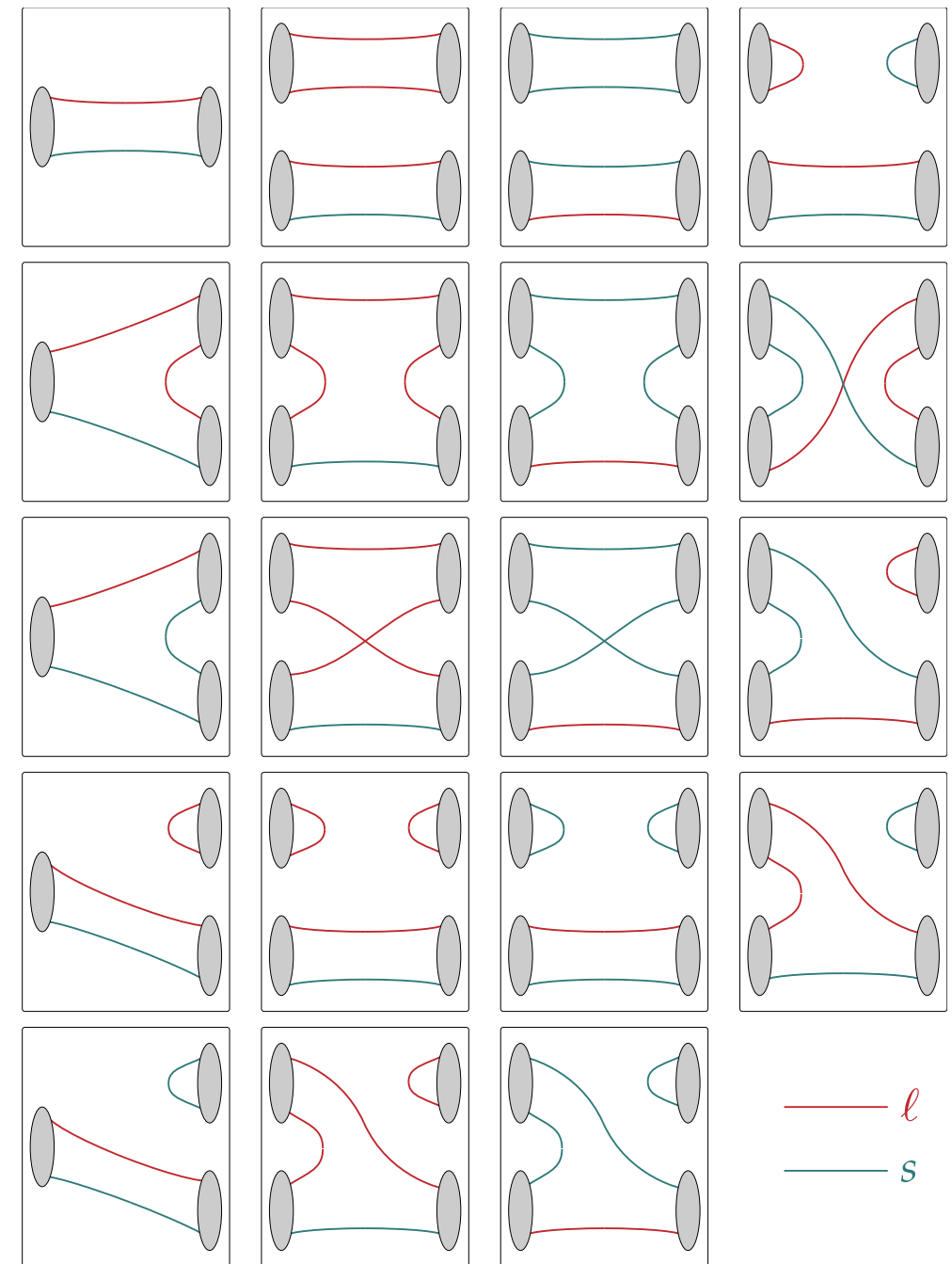
$$\sum_{\hat{k}_1, \hat{k}_2} C(\Lambda, \vec{P}; \vec{k}_1, \vec{k}_2) \pi^+(\vec{k}_1) K^+(\vec{k}_2)$$

$$\sum_{\hat{k}_1, \hat{k}_2} C(\Lambda, \vec{P}; \vec{k}_1, \vec{k}_2) \eta^+(\vec{k}_1) K^+(\vec{k}_2)$$

PRL 113 182001

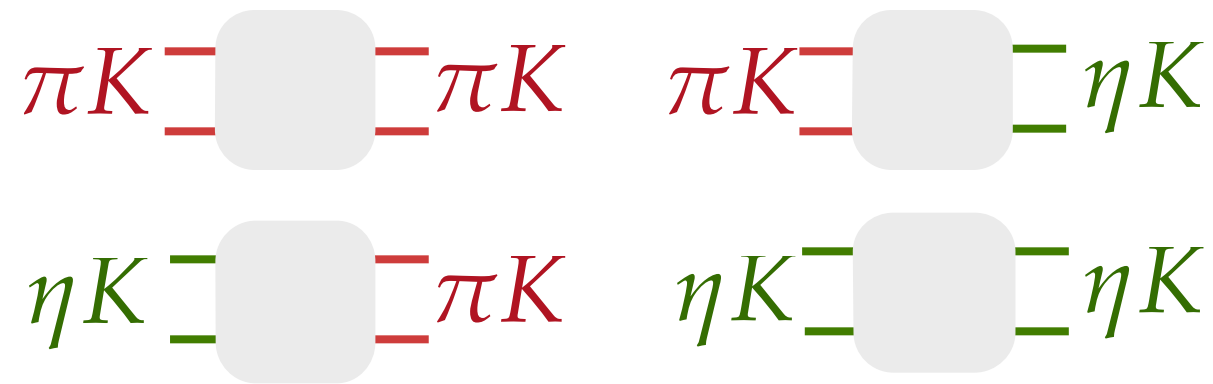
PRD 91 054008

## WICK CONTRACTIONS





- Example of coupled-channel scattering



- Compute finite-volume spectrum

 $\bar{u}\Gamma s$ 

$$\sum_{\hat{k}_1, \hat{k}_2} C(\Lambda, \vec{P}; \vec{k}_1, \vec{k}_2) \pi^+(\vec{k}_1) K^+(\vec{k}_2)$$

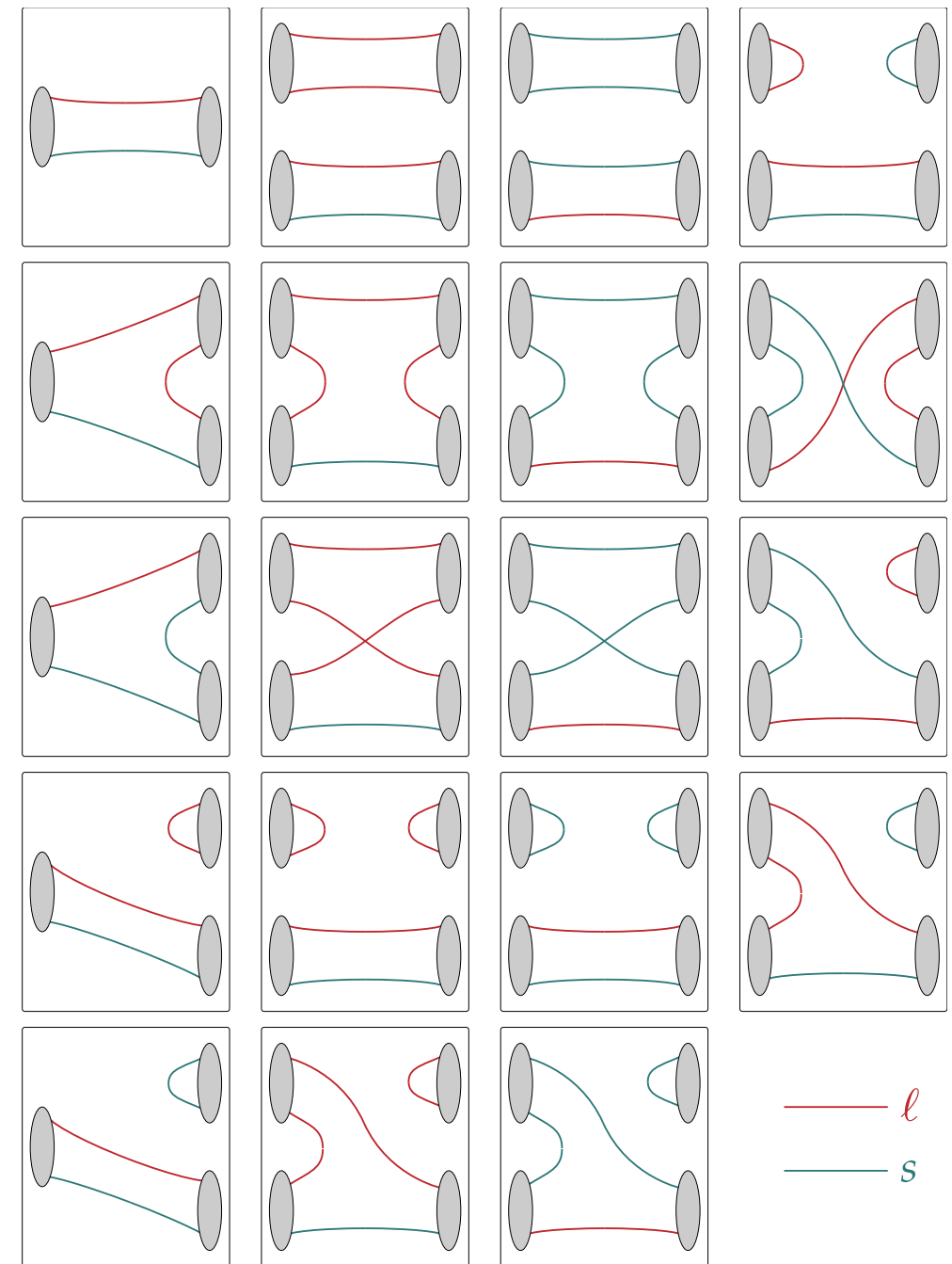
$$\sum_{\hat{k}_1, \hat{k}_2} C(\Lambda, \vec{P}; \vec{k}_1, \vec{k}_2) \eta^+(\vec{k}_1) K^+(\vec{k}_2)$$

PRL 113 182001

PRD 91 054008

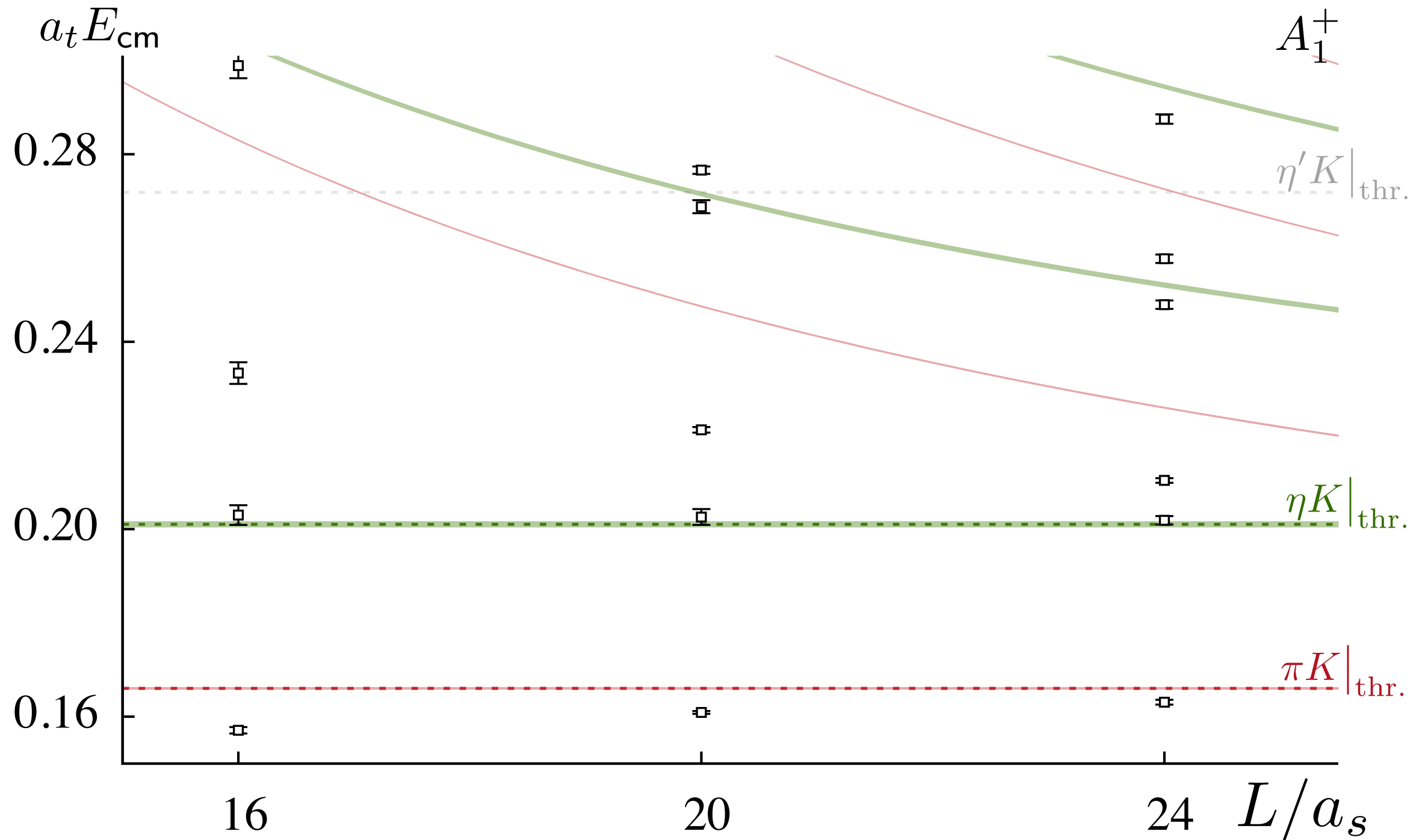
Large combinatoric factors - contractions expensive

## WICK CONTRACTIONS

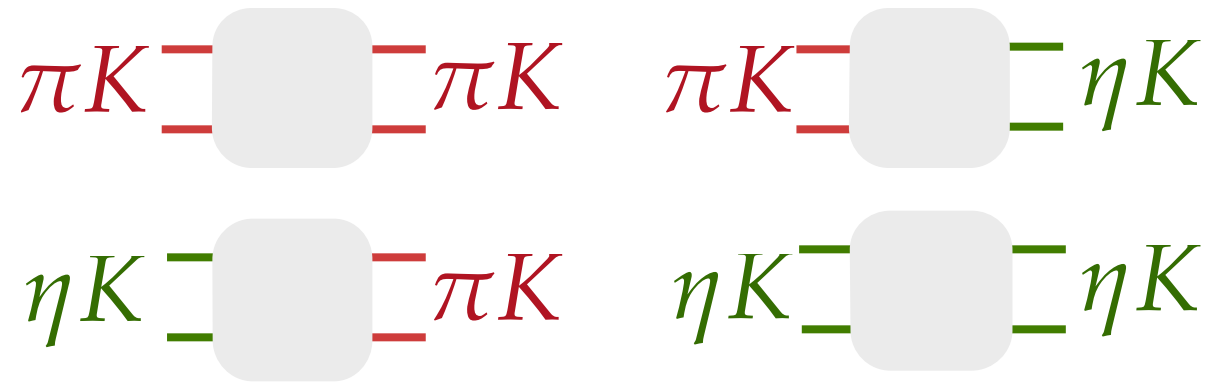


- Rest frame “S-wave” spectrum

$$m_\pi \sim 391 \text{ MeV}$$



- Parameterize the  $t$ -matrix in a unitarity conserving way



$$t_{ij}^{-1}(E) = K_{ij}^{-1}(E) + \delta_{ij} I_i(E)$$

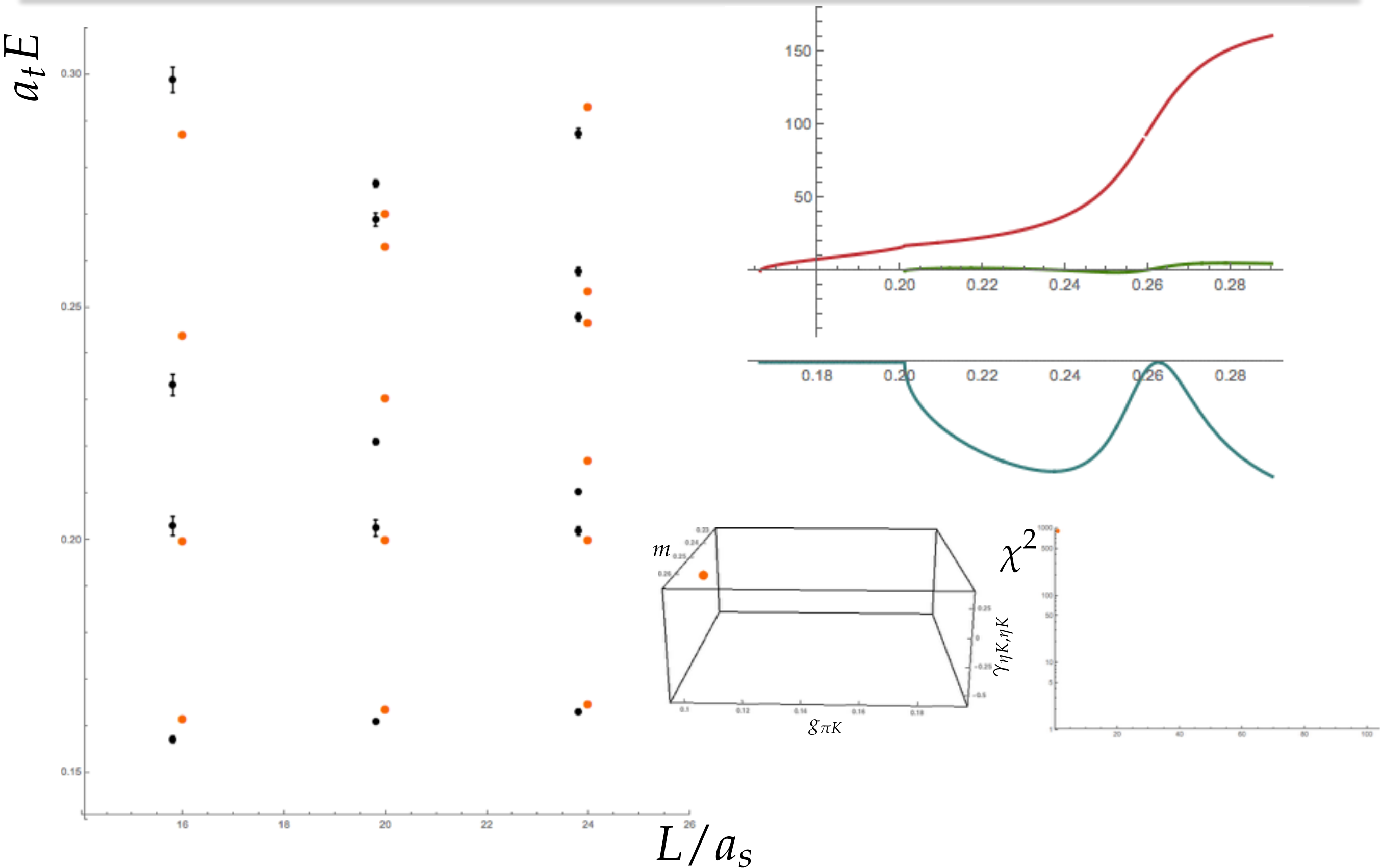
$$K_{ij}(E) = \frac{g_i g_j}{m^2 - E^2} + \gamma_{ij}$$

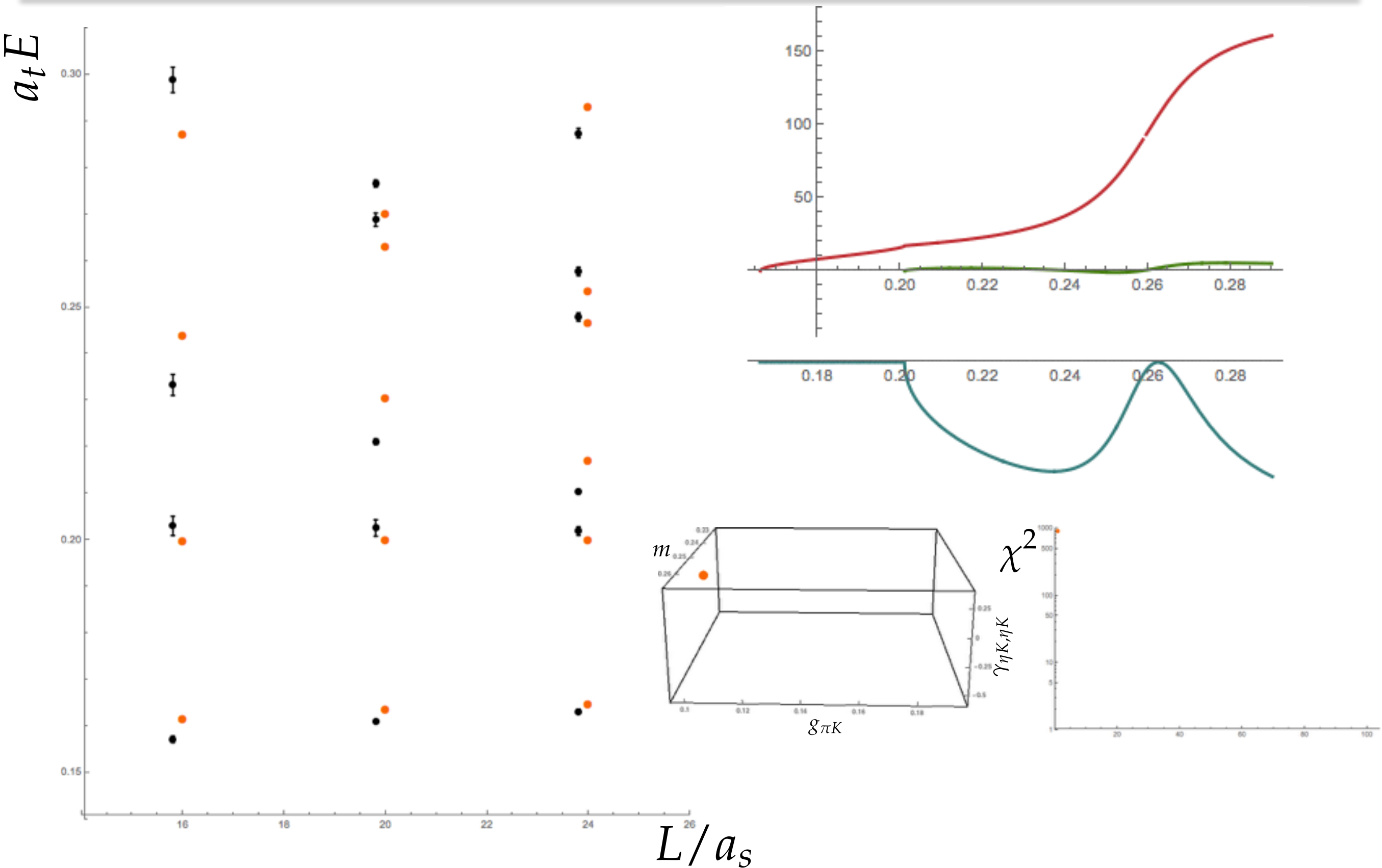
- vary the parameters, solving

$$\det \left[ ([t^{(\ell)}(E)]_{ij}^{-1} + i\rho_i(E) \delta_{ij}) - \delta_{ij} \mathcal{M}_\ell(E, L) \right] = 0$$

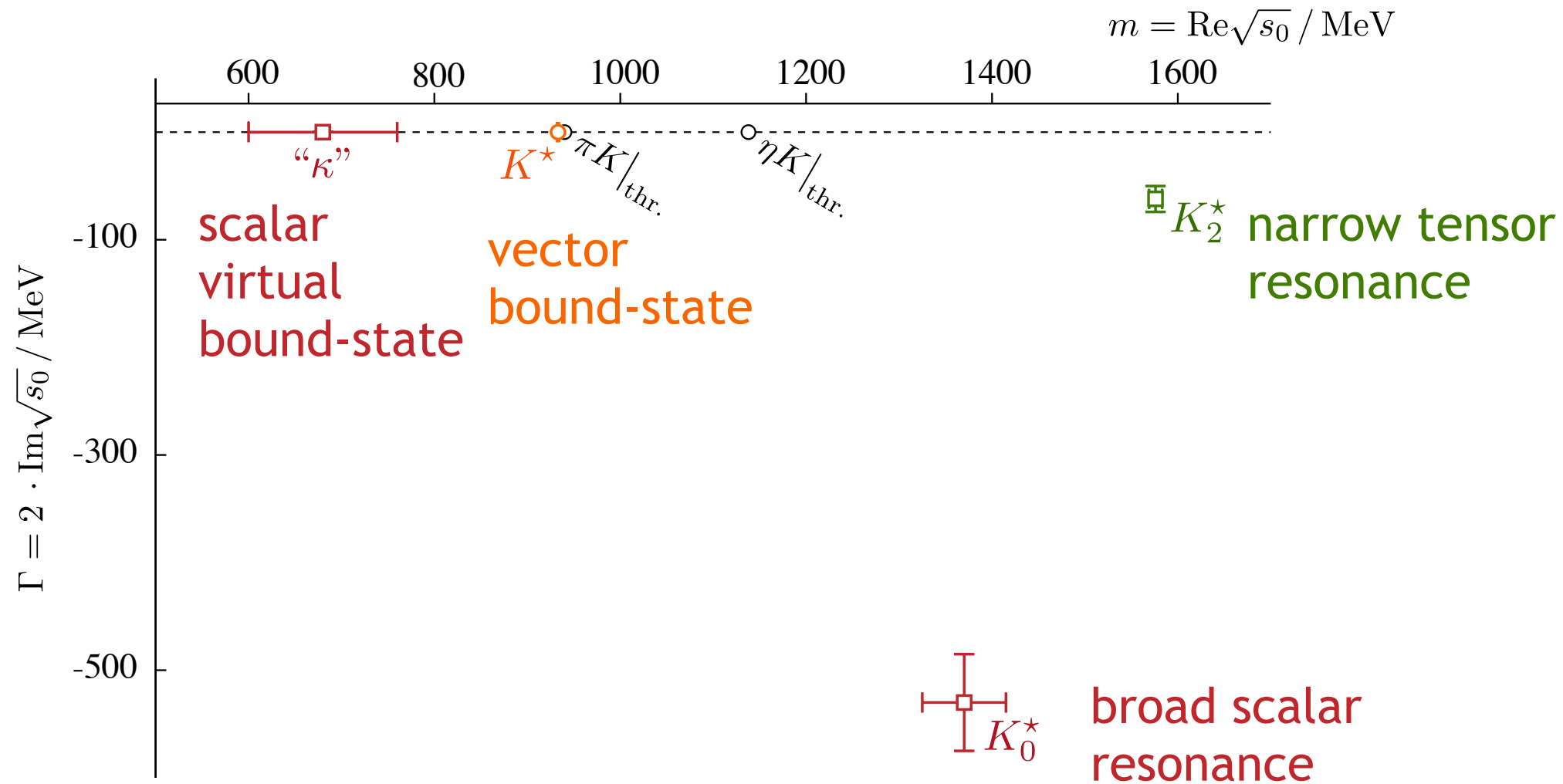
for the spectrum in each irreducible representation & momentum

Want pole mass and couplings of  $t$ -matrix





- $t$ -matrix poles as least model-dependent characterization of resonances



$$m_\pi \sim 391 \text{ MeV}$$

PRL 113 182001  
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## Physics Opportunities with the 12 GeV Upgrade at Jefferson Lab

Jozef Dudek, Rolf Ent, Rouven Essig, Krishna Kumar, Curtis Meyer, Robert McKeown, Zein Eddine Meziani, Gerald A. Miller, Michael Pennington, David Richards, Larry Weinstein, Glenn Young

Approved expt: second phase of GlueX program

PR12-13-003

### An initial study of mesons and baryons containing strange quarks with GlueX (A proposal to the 40<sup>th</sup> Jefferson Lab Program Advisory Committee)

A. AlekSejevs,<sup>1</sup> S. Barkanova,<sup>1</sup> M. Dugger,<sup>2</sup> B. Ritchie,<sup>2</sup> I. Senderovich,<sup>2</sup> E. Anassontzis,<sup>3</sup> P. Ioannou,<sup>3</sup>  
C. Kourkouveli,<sup>3</sup> G. Voulgaris,<sup>3</sup> N. Jarvis,<sup>4</sup> W. Levine,<sup>4</sup> P. Mattione,<sup>4</sup> W. McGinley,<sup>4</sup> C. A. Meyer,<sup>4,\*</sup>

•  
•  
•

The primary motivation of the GLUEX experiment is to search for and ultimately study the pattern of gluonic excitations in the meson spectrum produced in  $\gamma p$  collisions. Recent lattice QCD calculations predict a rich spectrum of hybrid mesons that have both exotic and non-exotic  $J^{PC}$ , corresponding to  $q\bar{q}$  states ( $q = u, d, \text{ or } s$ ) coupled with a gluonic field. A thorough study of the



Science case for JLab CLAS12 expt

**Studies of Nucleon Resonance Structure in Exclusive Meson Electroproduction**

I. G. Aznauryan,<sup>1,2</sup> A. Bashir,<sup>3</sup> V. M. Braun,<sup>4</sup> S. J. Brodsky,<sup>5,6</sup> V. D. Burkert,<sup>2</sup> L. Chang,<sup>7,8</sup>  
Ch. Chen,<sup>7,9,10</sup> B. El-Bennich,<sup>11,12</sup> I. C. Cloët,<sup>7,13</sup> P. L. Cole,<sup>14</sup> R. G. Edwards,<sup>2</sup>  
G. V. Fedotov,<sup>15,16</sup> M. M. Giannini,<sup>17,18</sup> R. W. Gothe,<sup>15</sup> F. Gross,<sup>2,19</sup> Huey-Wen Lin,<sup>20</sup>  
P. Kroll,<sup>21,4</sup> T.-S. H. Lee,<sup>7</sup> W. Melnitchouk,<sup>2</sup> V. I. Mokeev,<sup>2,16</sup> M. T. Peña,<sup>22,23</sup> G. Ramalho,<sup>22</sup>  
C. D. Roberts,<sup>7,10</sup> E. Santopinto,<sup>18</sup> G. F. de Teramond,<sup>24</sup> K. Tsushima,<sup>13,25</sup> and D. J. Wilson<sup>7,26</sup>

NSAC report prominently featuring exotic meson spectroscopy project

**Report to the  
Nuclear Science Advisory Committee  
Implementing the 2007 Long Range Plan  
January 31, 2013**

New NSAC report in writing now...

## JEFFERSON LAB

Jozef Dudek  
Robert Edwards  
Balint Joo  
David Richards  
Frank Winter

## TRINITY COLLEGE, DUBLIN

Mike Peardon  
Sinead Ryan

## TATA, MUMBAI

Nilmani Mathur

## CAMBRIDGE UNIVERSITY

Christopher Thomas

## U. OF MARYLAND

Steve Wallace

& postdocs, students

## MESON SPECTRUM

*PRL103 262001 (2009)*  $I = 1$   
*PRD82 034508 (2010)*  $I = 1, K^*$   
*PRD83 111502 (2011)*  $I = 0$   
*JHEP07 126 (2011)*  $c\bar{c}$   
*PRD88 094505 (2013)*  $I = 0$   
*JHEP05 021 (2013)*  $D, D_s$

## BARYON SPECTRUM

*PRD84 074508 (2011)*  $(N, \Delta)^*$   
*PRD85 054016 (2012)*  $(N, \Delta)_{\text{hyb}}$   
*PRD87 054506 (2013)*  $(N \dots \Xi)^*$   
*PRD90 074504 (2014)*  $\Omega_{ccc}^*$   
*arXiv:1502.01845*  $\Xi_{cc}^*$

## HADRON SCATTERING

*PRD83 071504 (2011)*  $\pi\pi I = 2$   
*PRD86 034031 (2012)*  $\pi\pi I = 2$   
*PRD87 034505 (2013)*  $\pi\pi I = 1, \rho$   
*PRL113 182001 (2014)*  $\pi K, \eta K$   
*PRD91 054008 (2015)*  $\pi K, \eta K$

## “TECHNOLOGY”

*PRD79 034502 (2009)* lattices  
*PRD80 054506 (2009)* distillation  
*PRD85 014507 (2012)*  $\vec{p} > 0$

## MATRIX ELEMENTS

*arXiv:1501.07457*  $M' \rightarrow \gamma M$   
*PRD90 014511 (2014)*  $f_{\pi^*}$



- Spectroscopy program maturing

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  - Unphysical pion masses
  - Using only “single-hadron” operators gives a sketch of spectrum
  - Suggests rich spectrum of baryons
  - See evidence of exotic and non-exotic mesons - suggests *hybrids*
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  - ➔ Knowledge of even size of branching fractions useful for expt. analysis

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- Switch to isotropic lattices at physical limit