Searching for Physics Beyond the Standard Model: Strongly-Coupled Field Theories at the Intensity and Energy Frontiers

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Abstract

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In this proposal we request resources to develop the software and algorithmic infrastructure needed for the numerical study of quantum chromodynamics (QCD), and of theories that have been proposed to describe physics beyond the Standard Model (BSM) of high energy physics, on current and future computers. This infrastructure will enable users (1) to improve the accuracy of QCD calculations to the point where they no longer limit what can be learned from high precision experiments that seek to test the Standard Model, and (2) to determine the predictions of BSM theories in order to understand which of them are consistent with the data that will soon be available from the LHC. Work will include the extension and optimizations of community codes for the next generation of leadership class computers, the IBM Blue Gene/Q and the Cray XE/XK, and for the dedicated hardware funded for our field by the Department of Energy. Members of our collaboration at Brookhaven National Laboratory and Columbia University worked on the design of the Blue Gene/Q, and have begun to develop software for it. Under this grant we will build upon their experience to produce high efficiency production codes for this machine. Cray XE/XK computers with many thousands of GPU accelerators will soon be available, and the dedicated commodity clusters we obtain with DOE funding include growing numbers of GPUs. We will work with our partners in NVIDIA's Emerging Technology group to scale our existing software to thousands of GPUs, and to produce highly efficient production codes for these machines. We will collaborate with our nuclear physics colleagues in USQCD on software for Intel's Many Integrated Core (MIC) architecture. Work under this grant will also include the development of new algorithms for the effective use of heterogeneous computers, and their integration into our codes. It will include im-

provements of Krylov solvers and the development of new multigrid methods in collaboration with members of the FASTMath SciDAC Institute, using their HYPRE framework, as well as work on improved symplectic integrators.

Contents

1 Introduction

The long term goals of high energy physicists are to identify the fundamental building blocks of matter, and to determine the interactions among them that give rise to the physical world we observe. Major progress has been made towards these goals through the development of the Standard Model of high energy physics. It consists of two quantum field theories: the Weinberg-Salam theory of weak and electromagnetic interactions, and quantum chromodynamics (QCD), the theory of the strong interactions. The Standard Model has been enormously successful in explaining a wealth of data produced in accelerator experiments and astrophysical observations over the past thirty years. Despite these successes, it is believed by high energy physicists that to understand physics at the energy scales being probed by the Large Hadron Collider (LHC) will require a more general theory capable of resolving fundamental questions, such as the origin of mass, the nature of dark matter and energy, and the pattern of symmetries found in nature. The Standard Model is expected to be a limiting case of this more general theory, just as classical mechanics is a limiting case of the more general quantum mechanics.

Central objectives of the search for a more general theory of fundamental interactions are to determine the range of validity of the Standard Model, and to search for physical phenomena that will require new theoretical ideas for their understanding. Two complementary approaches are being employed. Accelerators with the highest available beam intensity are being used to make precision tests of the Standard Model in the hope of finding contradictions, while accelerators with the highest available energy are being used to search directly for physical phenomena not predicted by the Standard Model. Numerical simulations within the framework of lattice gauge theory play important roles in both of these approaches. In work at the *intensity frontier,* precise calculations of the effects of the strong interactions on weak and electromagnetic transition amplitudes are often needed for Standard Model tests. For most such cases, the precision of the lattice calculations lag behind those of the experiments. One of the major goals of this project is to improve the precision of these calculations to the point where they no longer limit what can be learned from experiments. The role of lattice gauge theory simulations in work at the *energy frontier* is somewhat different. A number of theories have been proposed to describe beyond the Standard Model (BSM) physics, most of which contain strong coupling regimes that can be studied from first principles only with lattice simulations. Our goal in this area is to determine the predictions of these theories in order to understand which of them are consistent with the data that will soon be available from the LHC. BSM calculations are at least as challenging as QCD ones because of the great range of scales that must be covered for these conformal or nearly conformal theories. Thus, in work at both the intensity and energy frontiers, lattice gauge theory simulations play important roles in efforts to obtain a deeper understanding of the fundamental laws of physics. Detailed descriptions of our scientific goals are presented in Section 2.

In this proposal we request resources to develop the software and algorithmic infrastructure needed to study QCD and BSM theories on current and future computers. The proposed work is a project of the USQCD Collaboration, which consists of nearly all of the high energy and nuclear physicists in the United States working on the numerical study of lattice gauge theories. It will be carried out by high energy physicists within USQCD, in collaboration with nuclear physicists within USQCD, members of the SciDAC FASTMath Institute and colleagues in the NVIDIA Emerging Applications Group.

2 Scientific Goals

2.1 Flavor Physics at the Intensity Frontier

A major component of the experimental program in high energy physics lies at the intensity frontier. It is devoted to making precise tests of the Standard Model (SM), in order to determine its range of validity, and to searching for indications of new physics beyond the SM. Many of these tests require both accurate experiments *and* accurate lattice QCD (LQCD) calculations of the effects of the strong interactions on electroweak processes. In most cases, the precision of the tests are limited by the uncertainties in the lattice calculations, rather than in the experiments. One objective of our calculations is to reduce lattice errors down to, or below, the level of the experimental errors.

The US LQCD community has developed a very successful program to calculate the needed electroweak matrix elements. Indeed, it is now appropriate to provide world averages for key quantities to the wider particle physics community [1, 2]. Results obtained by members of the USQCD collaboration play the leading role in the determination of the averages for many of these quantities. The software and algorithmic development funded by the SciDAC-2 award has been crucial to obtaining these results.

Under SciDAC-3, we propose to extend this work in two ways. First, to improve the errors on quantities for which results with fully-controlled errors exist, but for which the errors are still larger than or comparable to those from other sources. And, second, to expand our program of calculations to meet the needs of upcoming intensity-frontier experiments, for example the muon $g-2$ experiment at Fermilab, the Project X kaon program at Fermilab, LHC-b, Belle II and Super-B. We describe both these extensions below, focusing on opportunities where we expect LQCD calculations to play a key role in searches for (and possibly discovery of) new physics in the quark-flavor sector.

This effort is complementary to direct searches for new physics at the energy frontier, such as those underway at the LHC. Indeed, if indications for beyond-the-Standard Model (BSM) physics are found at the LHC, then one expects that there will also be significant new contributions to flavor-changing processes such as kaon and B-meson mixing. To determine the size of these contributions predicted by a given model of new physics one also requires LQCD calculations.

The period of the SciDAC-2 award has seen a rapid maturation in LQCD calculations relevant for flavor physics. At the time of our previous proposal, calculations by members of the USQCD collaboration had given the first results in which all sources of errors were controlled. Typical errors were at the few percent level, and the results provided validation of LQCD methods both by finding agreement with experimental quantities and by making predictions that were subsequently confirmed. Examples of successful "postdictions" are the results for f_π, f_K, and splittings in charmonium and bottomonium systems presented in Ref. [3]. Examples of successful predictions are those of D meson semileptonic form factors, D- and D_s-meson leptonic-decay constants, and the mass of the B_c meson, as summarized in Ref. [4].

During the SciDAC-2 award, the number of lattice predictions has substantially increased, the accuracy of the results has significantly improved (with sub-percent accuracy achieved in some cases), and results have been obtained using several different methods for discretizing fermions (providing crucial cross-checks). It is thus now appropriate to provide world averages, as noted above. We mention several important examples in which USQCD has played the major role.

The kaon mixing parameter B_K is the hadronic matrix element needed to determine the constraint on the Cabibbo-Kobayashi-Maskawa (CKM) matrix arising from the observation of CP-violation in kaon mixing. We emphasize that this measurement was made in 1964 [5], and it is only now, after nearly 50 years, that we can use this result to study the properties of quarks. Specifically, combining the measurement with B_K (and some other known inputs) one can determine $\mathrm{Im}V_{td}^2$. (Here the quantity V_{ab} is an element of the 3×3 CKM matrix where the index a identifies one of the three "up"-type quarks, u, c, and t while b labels one of the three "down"-type quarks d, s and b.) At the time of our SciDAC-2 proposal, there was only a single calculation of B_K using the physical complement of light-quark loops (up, down and strange quarks), and several sources of error were not controlled. The total error was estimated to be $12 - 20\%$, It was forecast in the SciDAC-2 proposal that the error would be reduced to $\sim 4\%$ during the award. The present error, in fact, exceeds this goal. Using the average from Ref. [1] ("end of 2010"), which includes only published results using the physical complement of light-quark loops and which control all sources of error, the error is now 2.7%.[1] This is based on three results all obtained using USQCD resources, all using different methods and giving consistent values. The lattice result has improved enough that it is no longer the dominant source of uncertainty in the constraint on CKM matrix elements based on the measured CP-violation in kaon mixing.

The second quantity we highlight is ξ, the ratio of mixing matrix elements for strange and non-strange neutral B-mesons. This ratio, combined with the experimental result for $\Delta M_B/\Delta M_{B_s}$, allows one to determine $|V_{td}/V_{ts}|^2$. Based on a single calculation, the error in this ratio was estimated to be 8% at the time of our SciDAC-2 proposal, and we forecast that the error would be reduced to $\sim 4\%$ during the award. The present error of 2.6% [1] again exceeds this goal. This is based on 3 independent calculations, all using USQCD software and hardware resources, and all with a complete error budget.

The impact of these and other LQCD results can be seen from Fig. 1, which shows a recent global fit to the parameters of the CKM unitarity triangle. New quark flavor-changing interactions or CP-violating phases would manifest themselves as apparent inconsistencies between measurements of the apex $(\overline{\rho}, \overline{\eta})$ that are predicted to be the same within the SM framework. Four of the constraints require LQCD input—for example, those labeled "$\epsilon_K + |V_{cb}|$" and "$\Delta M_s/\Delta M_d$" require B_K and ξ, respectively. The widths of the bands requiring lattice inputs have been significantly reduced over the period of the SciDAC-2 award, primarily due to the reduction in LQCD errors. As can be seen from the figure, this combination of precise experiments and theoretical calculations has established the CKM paradigm of CP-violation at the few-percent level. At the same time, an $\sim 3\sigma$ tension in the fit has been revealed [7, 8, 9, 10]. This may indicate the presence of sources of CP-violation beyond the SM.

Two other highlights of recent lattice calculations are the determination of α_S and the quark masses. LQCD provides the most accurate result for α_S [11], and competitive results for the charm and bottom masses. LQCD provides the only *ab initio* determination of the up, down and strange masses [1, 2]. All these quantities are fundamental parameters of the SM, and are needed as inputs into models of new physics such as possible grand unified theories.

A major goal for the present proposal is to extend these successes to other hadronic matrix elements. The following matrix shows which hadronic processes can be used to obtain each CKM matrix

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[1] Not included in this average is a very recent result using Wilson fermions from the BMW collaboration which further reduces the overall error [6].

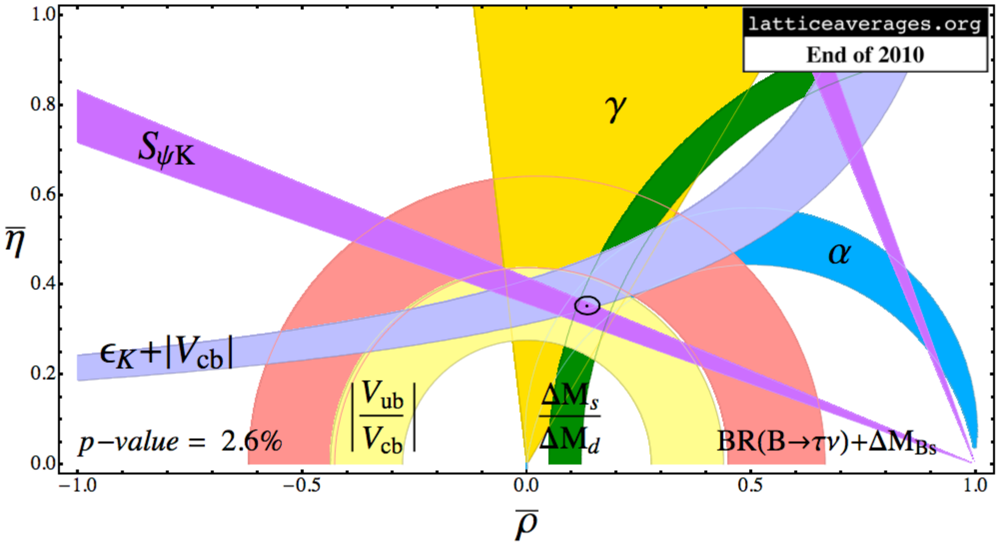

Figure 1: Global fit of the CKM unitarity triangle [1]. The constraints labeled $\epsilon_K + |V_{cb}|$, $|V_{ub}/V_{cb}|$, $\Delta M_s/\Delta M_d$, and $\text{BR}(B \to \tau\nu) + \Delta M_{B_s}$ all require LQCD input, while the others require minimal or non-lattice theoretical input. The solid ellipse encloses the 1σ region.

element, in each case using LQCD calculations:

$$\begin{pmatrix}
\mathbf{V_{ud}} & \mathbf{V_{us}} & \mathbf{V_{ub}} \\
\pi \to \ell\nu & K \to \ell\nu & B \to \ell\nu \\
 & K \to \pi\ell\nu & B \to \pi\ell\nu \\
\mathbf{V_{cd}} & \mathbf{V_{cs}} & \mathbf{V_{cb}} \\
D \to \ell\nu & D_s \to \ell\nu & B \to D\ell\nu \\
D \to \pi\ell\nu & D \to K\ell\nu & B \to D^*\ell\nu \\
\mathbf{V_{td}} & \mathbf{V_{ts}} & \mathbf{V_{tb}} \\
\Delta M_d & \Delta M_s &
\end{pmatrix}$$

In Table 1 we compare, for many of these processes, the present experimental and lattice errors, and give an estimate of what LQCD can achieve in ~ 3 years as well as a forecast for the end of the decade. Note that in the few cases where the effects of low energy QCD can be estimated using non-lattice methods, the resulting uncertainties are at least as large as those in the present lattice determinations.

These forecasts assume computations running at the 10's of petaflop/s by 2014 and approaching the exascale by 2020. Achieving such computational throughput will require extensive software development, for example the extension of our high-performance code for the current, leadership class machines to the much more challenging multi-thread/multi-core architectures of next generation HPC machines such as the Blue Gene/Q, the adaptation of codes to an increasingly heterogeneous, multi-GPU environment such as Titan and Blue Waters, the development of improved multi-scale solvers to avoid critical slowing down as the quark masses are decreased, and the development of improved integrators such as the force-gradient method. This algorithm and software development will be a major focus of our SciDAC-3 program.

A major milestone achieved during the SciDAC-3 program will be the widespread use of simulations at the physical quark masses, first results from which have recently appeared [12, 13, 14]. We plan

Table 1: *Impact of improved LQCD calculations on the determination of CKM matrix elements.*

Quantity	CKM element	Present expt. error	Present lattice error	2014 lattice error	2020 lattice error
f_K/f_π	$\|V_{us}\|$	0.2%	0.6%	0.3%	0.1%
$f_+^{K\pi}(0)$	$\|V_{us}\|$	0.2%	0.5%	0.2%	0.1%
$D \to \pi\ell\nu$	$\|V_{cd}\|$	2.6%	10.5%	4%	1%
$D \to K\ell\nu$	$\|V_{cs}\|$	1.1%	2.5%	2%	$< 1\%$
$B \to D^{(*)}\ell\nu$	$\|V_{cb}\|$	1.8%	1.8%	0.8%	$< 0.5\%$
$B \to \pi\ell\nu$	$\|V_{ub}\|$	4.1%	8.7%	4%	2%
$B \to \tau\nu$	$\|V_{ub}\|$	21%	6.4%	2%	$< 1\%$
ξ	$\|V_{ts}/V_{td}\|$	1.0%	2.5%	1.5%	$< 1\%$
ΔM_s	$\|V_{ts}V_{tb}\|^2$	0.7%	10.5%	5%	3%

to undertake such simulations for both highly improved staggered quarks (HISQ) and domain wall fermions (DWF) and these play a major role in the reduction of errors forecast in Table 1.

Table 1 shows that LQCD calculations can by 2014 match the present experimental errors in many quantities and do substantially better than the alternative methods. Such improvements promise to push the SM beyond its limits—possibly raising the present $\sim 3\sigma$ tension to the 5σ discovery level. We give three examples of the essential role that LQCD calculations play in this program.

The first concerns the lattice results for B-$\bar{B}$ mixing. The lattice quantity ξ^2 determines the width of the $\Delta M_s/\Delta M_d$ band in Fig. 1. By 2014 we expect that this width should be roughly halved, which will significantly reduce the horizontal width of the allowed ellipse. It will also be possible to include a separate constraint from ΔM_s alone. This requires a LQCD calculation of the matrix element $f_{B_s}^2 B_{B_s}$. Only one calculation with the full complement of light sea quarks is available and, as noted in the bottom row of Table 1, it has an error of 10.5%. We expect several independent calculations to be completed by 2014, with errors roughly halved. Although the lattice error will still dominate over the sub-percent experimental error, the input will add an important further constraint on the unitarity triangle.

Our second example concerns the importance of improving the determination of $|V_{cb}|$ using lattice calculations of $B \to D^{(*)}\ell\nu$ form factors. $|V_{cb}|$ plays a key role in tightening unitarity constraints, since, now that B_K is well determined, the width of the ϵ_K band in Fig. 1 arises principally from the uncertainty in $|V_{cb}|^4$. We expect a reduction in this width by about a factor of two by 2014. Reducing the uncertainty in $|V_{cb}|$ also has a major impact on searches for new physics in the rare decays $K^+ \to \pi^+\nu\bar{\nu}$ and $K_L \to \pi^0\nu\bar{\nu}$. The leading SM contribution to these decays is from penguin diagrams, and is thus very small, so that new physics can lead to substantial deviations from SM predictions. It is thus important to nail down the SM predictions; and these are also proportional to $|V_{cb}|^4$.

Our final example concerns the determination of $|V_{ub}|$, using lattice results for the $B \to \pi\ell\nu$ form factor and of f_B. At present there is a $\sim 2\sigma$ discrepancy between the value of $|V_{ub}|$ determined from lattice inputs and that determined from comparing inclusive B decays with perturbative QCD predictions based on heavy-quark effective theory. Reducing the lattice errors will allow us to determine whether this long-standing discrepancy is real—indicating new physics such as right-handed currents—or not. We forecast very significant improvements in the lattice results. This is in part because one will be able to work on very fine lattices, allowing the use of automatically normalized vector and axial currents.

The calculations discussed so far involve standard methods, and their improvement with increasing resources can be estimated with reasonable accuracy. We now turn to the second extension of our SciDAC-2 program, namely to calculations of new quantities. For most of these, the methodology is at an earlier stage of development, and it is hard to predict future errors. We expect these calculations to take up an increasing part of our computational effort over the next few years. We describe three main examples. (Other promising topics which are not discussed include $B \to K\ell^+\ell^-$, nucleon matrix elements required for dark matter searches, and hadronic contributions to the determination of the fine structure constant α.)

The first is the most straightforward. New physics is expected to enhance mixing in neutral kaon, D and B systems. This would show up, for example, as a failure of the unitarity triangle fit. If such a failure occurs, we need to be able to determine whether any given model of new physics, e.g. supersymmetry or extra dimensions, is consistent with the observed mixing. This involves both determining the SM contribution—which for K and B systems has been discussed above—and the contribution from new physics. The latter enters through four-fermion operators which have different Dirac structures than the left-handed operators of the SM. Thus, we must determine the generalizations of B_K, B_D and B_B for these new operators. The calculations are, in fact, being done in parallel with those for B_K etc., and we expect that results with comparable errors will be available during the SciDAC-3 award.

Our second example concerns kaon properties. It has been a long-standing aim of lattice calculations to calculate the $K \to \pi\pi$ decay amplitudes. Calculating the CP-conserving parts will allow us to test whether QCD explains the $\Delta I = 1/2$ rule (i.e. the dominance of the $I = 0$ final state pions over $I = 2$). Calculating the CP-violating parts will allow us to use the experimental result for direct CP-violation in kaon decays (ϵ'_K/ϵ_K) in order to further constrain the parameters of the SM. We have recently made very significant progress on the lattice calculations of these decay amplitudes using domain wall fermions. A complete methodology is now in place, and the path to precise calculations is clear. Indeed, results with $\sim 20\%$ precision for the $\Delta I = 3/2$ amplitudes have already been obtained [15], and a pilot calculation of all parts of the more challenging $\Delta I = 1/2$ amplitudes has been completed [16]. We expect that by 2014 the error in the latter amplitudes will have dropped to $\sim 30\%$.

Another quantity that can be used to constrain new physics is the CP-conserving part of the kaon mixing amplitude, ΔM_K. In the SM, this quantity receives a large contribution from the long-distance two pion intermediate states. This makes it much harder to calculate using lattice methods than the short-distance dominated CP-violating part (which is proportional to B_K). Nevertheless, a method for the calculation has recently been presented [17], and a pilot study undertaken [18]. We expect an error of $\sim 30\%$ by 2014.

An important area of overlap between the standard quantities where errors on the 1% level are targeted and new directions where the methods of lattice QCD can be applied but are not fully developed is the inclusion of electromagnetic effects and the isospin breaking difference between the masses of the up and down quarks. Here pioneering calculations have demonstrated that the methods of LQCD can indeed be extended to include the electromagnetic as well as the color gauge fields [19, 20, 21]. These methods will be further developed during SciDAC-3 and will be needed for some of the precision goals targeted in 2014.

Finally, we discuss how LQCD calculations will contribute to the calculation of the muon anomalous magnetic moment in the SM. At present, there is a $\sim 3.5\sigma$ discrepancy between the theoretical prediction and experiment (see, e.g., Ref. [22]). If this discrepancy holds up, it is a clear signal of new physics. The new FNAL experiment E989 plans to reduce the experimental error over the next decade from the present 0.5 parts per million (ppm) to 0.14 ppm. To make full use of this experimental effort, it is crucial to reduce the theoretical error to a similar level. This is a major challenge, because there are two hadronic contributions, involving strong interaction physics, that have uncertainties that are presently significantly larger than the planned final accuracy. These are the hadronic vacuum polarization contribution, which is presently obtained using data from $e^+e^- \to$ hadrons and has an error of about 0.4 ppm, and the light-by-light scattering contribution, which is estimated using various theoretical methods and has a comparable error. LQCD calculations can, in principle, provide improved results for both quantities, and methods for these calculations are under development [23, 24, 25, 26, 27]. It is very hard to forecast the future of such pioneering calculations, but we expect significant progress during the period of the SciDAC-3 award.

2.2 Physics Beyond the Standard Model at the Energy Frontier

The era of the Large Hadron Collider (LHC) is likely to expose new Beyond the Standard Model (BSM) physics with important non-perturbative aspects to explain the origin of electroweak symmetry breaking and the origin of mass in the universe. A complete understanding of the new physics will require non-perturbative lattice studies that can be effectively implemented with the USQCD hardware/software infrastructure. The BSM effort at the energy frontier of the LHC is complementary to indirect searches for new physics at the intensity frontier. Precise lattice calculations are capable of making important connections between the two frontiers since BSM models lead to direct predictions that can be observed at the intensity frontier. The BSM paradigm also offers potential new insight into the origin of the electroweak phase transition in the early universe relevant to the cosmic frontier.

In the Standard Model (SM) electroweak symmetry breaking is accomplished by coupling the theory to the elementary scalar Higgs field - and the related Higgs particle. From a theoretical viewpoint the problematic and unnatural fine tuning of the scalar Higgs field can be replaced by two known solutions: 1) replace the elementary Higgs boson by some new composite Higgs mechanism, or 2) assume that the Standard Model is embedded in an appropriate supersymmetric theory. In both cases new strong dynamics is required, either to break supersymmetry, thus ensuring that any new super particles are sufficiently heavy to have evaded detection thus far, or to provide the new strong force (with roots in past technicolor studies) necessary to break chiral symmetry and trigger a composite Higgs mechanism when the electroweak gauge interactions are turned on [28, 29, 30]. In the following subsections we will describe a lattice program designed to explore important nearly conformal BSM models and SUSY scenarios.

2.2.1 The composite Higgs mechanism close to the conformal window

For theories of dynamical weak symmetry breaking, such as technicolor, to be viable, they must have nearly conformal, or ”walking”, behavior over a large range of energy scales, as explained below. Walking technicolor theories generically have many techniquarks, and therefore many light techni-hadrons. The most copiously produced are likely to be singly-produced techni-rhos and techni-omegas, techni-hadron analogs of the spin-one meson states of QCD. These generically decay into pairs of Ws and Zs, or into a W or a Z and a pair of jets via a techni-pion. Singly-produced narrow resonances do not occur in SUSY theories with R parity, so discovery of such a signal would give high priority to non-perturbative lattice search for nearly conformal field theories representing a very interesting and promising class of BSM theories with the realization of the composite Higgs mechanism [31, 32, 33, 34, 35].

These theories are built on new fundamental particles, historically known as "techniquarks", that are different from the QCD quarks. They are massless and occur in an unknown number of N_c colors and N_f flavors. Candidate models also differ in the choice of fermion representation of the technicolor gauge group in the BSM Lagrangian. The theories exhibit a fundamental chiral symmetry on the Lagrangian level, which is dynamically broken in the vacuum by the new strong force, forming a condensate of fermions carrying the quantum numbers of the SM Higgs and providing the replacement for the old Higgs mechanism. The new strong force, similar to QCD, but operating at the TeV scale, plays a central role, replacing the SM Higgs particle as the source of electroweak symmetry breaking. The composite Higgs particle, if it can be observed in the new theory, will exhibit modified couplings to the electroweak gauge fields, like the dilaton of broken scale invariance close to conformality. The "techniquarks" are bound by the confining force of the new theory into heavy and colorless composite particles on the TeV scale providing interesting LHC signatures.

Early technicolor efforts employed models that were scaled up versions of QCD, but these models have since been ruled problematic by precision tests of the Standard Model. In contrast, nearly conformal Yang-Mills theories require good non-perturbative understanding of how precise properties of the theory depend on N_f, N_c and the fermion representation. At fixed N_c in a given fermion representation and low flavor number N_f, the models exhibit confinement and chiral symmetry breaking, which can in turn be used to describe electroweak symmetry breaking (EWSB). On the other hand, a perturbative study of the beta function indicates that these properties are lost for large N_f, giving way to conformal behavior in the infrared that can no longer be used for EWSB. The transition between these two phases occurs at some critical value $N_f^c(N_c)$ for a given fermion representation. Walking gauge coupling is expected just before $N_f^c(N_c)$ and the conformal phase is reached. Several such theories have been studied in the last few years and USQCD has played a major and successful role in these efforts [36, 37, 38, 39, 40, 41, 42, 43, 44]. The left side of Figure 2 is a visual summary of ongoing and planned USQCD BSM investigations of the theory space while the right side displays the first results on the variation of the S-parameter moving toward the conformal window.

Recent first-principles lattice calculations at $N_c = 3$ indicate that in the fundamental fermion representation N_f^c is close to $N_f^c = 12$ [37, 40, 41, 42, 46, 47]. A theory with N_f just below the critical value N_f^c may show approximately conformal infrared behavior, making it a good candidate model for walking technicolor. A very important added dimension to this exploration is the choice of the fermion representation. For example, N_f^c in the color sextet fermion representation has been found to be close to $N_f^c = 2$ [43, 44, 40]. There is reason to believe that phenomenologically viable models would prefer low N_f^c values, hence the significance of our proposed search in the theory space of higher fermion representations. Recent simulations with $SU(2)$ color gauge group have also made considerable progress exploring the BSM theory around the lower end of the conformal window [48, 49, 50, 51, 52, 53].

New BSM physics can affect low-energy precision measurements. Two such quantum effects are the contribution of the new interactions to the S parameter of electroweak precision tests and to flavor changing neutral currents (FCNCs). There are theoretical arguments that nearly conformal gauge theories might lead to an acceptable S parameter and naturally suppress contributions to FCNC's through a large anomalous dimension of the fermion condensate. All these constraints are deeply non-perturbative issues demanding large scale computations and the design of new BSM software infrastructure.

Composite Higgs model plans for the next three years include studies with two, three, and four colors. Studies with the $SU(2)$ color group will focus on the fundamental and adjoint fermion representations close to the conformal window. Studies with the $SU(3)$ color group will focus

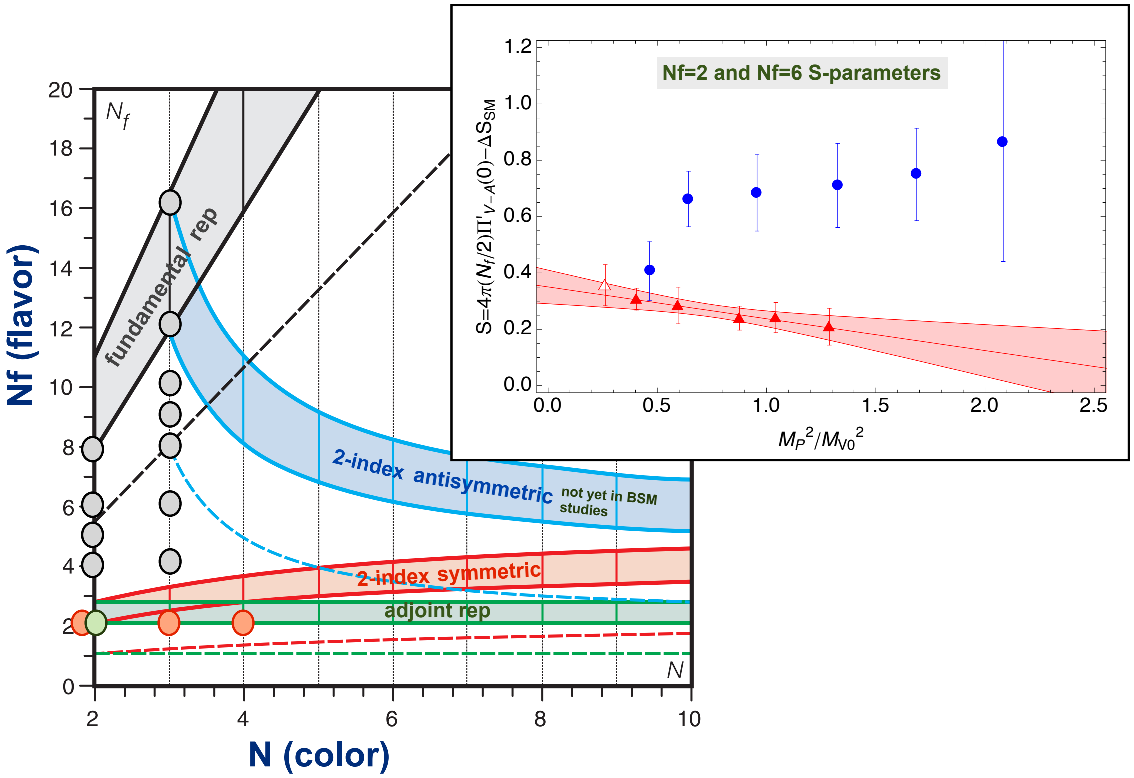

Figure 2: On the left side circles mark USQCD BSM activities in the large space of N_f and N_c with different $R(N_c)$ fermion representations color coded. Solid lines mark the upper and lower boundary of the conformal window from approximate analytic estimates [35], while the dashed line marks the lower boundary from the original Banks-Zaks prediction [31]. The right side plot of the S parameter from the LSD collaboration [45] is the first indication that the theory getting closer to the conformal window is not responding to the variation of N_f as it was expected from a scaled up version of QCD.

on the fundamental and two-index symmetric fermion representations. In addition, We plan to study the two-index symmetric fermion representation with four colors. Finally, we will extend our investigations of strongly coupled gauge theories in several directions, including four-fermion interactions, like gauged NJL type theories, or models related to top quark condensates.

2.2.2 Supersymmetry and supersymmetry breaking

Supersymmetry (SUSY) was proposed as a possible symmetry of nature forty years ago. While it is an extension of the usual symmetries of space-time, such as translations and rotations, its consequences are dramatic; it predicts that every particle is accompanied by a *superpartner* of equal mass and charge but with spin differing by one-half. It thus unifies fermions with bosons. From a phenomenological point of view SUSY can explain why the Higgs particle responsible for breaking electroweak symmetry is light, while from a theoretical perspective it plays a crucial role in constructing consistent string theories, and may play an important role in understanding quantum gravity.

In supersymmetric models, SUSY is usually taken to be broken dynamically, with SUSY breaking communicated to the standard model sector by some mediation mechanism. Superpartners are expected to provide evidence of the mediation mechanism in their decays. For example, copious photon production in superpartner decay would be an evidence of gauge mediation. Light sleptons associated with superpartner decay would be a sign of anomaly mediation, and gravity mediation is associated with missing energy. If superpartners and evidence for a mediation mechanism is found, the investigation of the dynamical breaking of supersymmetry with lattice gauge theory will come to center stage.

Dynamical SUSY breaking and the soft parameters in the MSSM: It is straightforward to "supersymmetrize" the usual theories of particle physics; the Minimal Supersymmetric Standard Model (MSSM) is perhaps the most studied extension of the Standard Model. In this model the Higgs is naturally light since it is accompanied by a fermionic partner whose mass is protected by chiral symmetries. A great deal of effort has been expended on predicting signals for SUSY/MSSM at the LHC.

Of course the low energy world we inhabit is manifestly not supersymmetric; so a key component of any realistic theory of Beyond Standard Model physics must provide a mechanism for spontaneous supersymmetry breaking. In general a variety of no go theorems ensure that any such symmetry breaking must be non-perturbative in nature. In the MSSM the effects of this non-perturbative SUSY breaking is parametrized in terms of a series of "soft breaking terms" which are put in by hand and lead to a large parameter space and a lack of predictivity for the theory.

However, in general we might expect that these parameters are determined from dynamical breaking of SUSY at high energies in a "hidden sector". A supersymmetrized version of QCD - super QCD - with N_c colors and N_f massive flavors is a natural candidate for this hidden sector. For $N_c + 1 \leq N_f < \frac{3}{2}N_c$ it is thought that super QCD has long lived metastable SUSY breaking vacua [54]. The lifetimes of these metastable vacua can exceed the age of the Universe and ensure that the physical vacuum breaks supersymmetry. Within such a vacuum state non-perturbative phenomena, such as confinement and chiral symmetry breaking, precipitate a breaking of supersymmetry. Furthermore, if the quark masses are small compared to the confinement scale, these vacua have extremely long lifetimes. If the Standard Model fields are coupled to the hidden sector fields in an appropriate fashion, then such non-perturbative dynamics arising in the broken phase of this theory can feed down to yield soft supersymmetry breaking terms in the low energy effective theory - the MSSM.

Thus, a detailed understanding of the vacuum structure and strong coupling dynamics of super QCD can strongly constrain possible supersymmetric models of BSM physics, in some cases leading to detailed predictions of the soft parameters of the MSSM in terms of a handful of non-perturbative quantities obtained in the hidden sector super QCD theory. Lattice simulations of supersymmetric lattice QCD thus have the potential to play an important role in constraining the parameter space of the MSSM and in building realistic supersymmetric theories of BSM physics.

The technical problem that must be immediately faced in studying supersymmetric lattice theories is that supersymmetry is broken by discretization and it is non-trivial to regain SUSY as the lattice spacing is sent to zero with e.g. Wilson or staggered fermions. Luckily in the case of $\mathcal{N} = 1$ super Yang-Mills theory, which contains both gluons and their fermionic superpartners gluinos, this generic problem can be avoided by the use of domain wall fermions (DWF). In this case the exact lattice chiral symmetry of the fermion action ensures that SUSY is *automatically* recovered in the chiral limit.

Preliminary work by USQCD has already revealed a non-zero gluino condensate in the SU(2) theory in agreement with theoretical expectations [55, 56, 57]. However, to build a realistic theory capable of yielding the soft parameters of the MSSM, we will need to add N_f quarks and their scalar superpartners - squarks, and additionally extend the gauge group to a larger number of colors N_c.

To restore SUSY in the continuum limit now requires *both* use of DWF *and* tuning of parameters in the squark sector [58]. In principle this can be done by performing a series of runs over a grid in squark parameter space, and using (offline) reweighting techniques in the scalar sector to tune to the supersymmetric point.

We anticipate this program would proceed in a number of steps:

(1) Conduct studies of $\mathcal{N} = 1$ super Yang-Mills, which is arguably the simplest supersymmetric theory and the core of the MSSM. Studies of this theory for $SU(2)$ gauge group have already been started and the relevant range of parameter space has been determined. A computation of physical observables, such as the gaugino condensate and low lying spectrum, are achievable within a couple of years. Code development to support an arbitrary number of colors of adjoint DWF will be necessary.

(2) Extend the previous calculations to super QCD. To tune to the supersymmetric limit a series of runs over a grid in the scalar (squark) parameter space will be necessary. DWF code with support for arbitrary numbers of flavors of fermions in the fundamental representation and additional Yukawa interactions will be needed. At each point in this parameter space a computational effort will be needed which is comparable to the $\mathcal{N} = 1$ super Yang-Mills case.

Lattice supersymmetry and string theory: String theory evolved out of attempts to understand the strong interactions. However, these early string theories were quickly discarded with the development of QCD. In the last decade there has been a huge resurgence of interest in the connections between string theory and gauge theories such as QCD. This interest dates from a seminal paper by Juan Maldacena [59], which conjectured that a particular supersymmetric gauge theory – $\mathcal{N} = 4$ super Yang-Mills – was dual or equivalent to type IIb string theory propagating on a five dimensional anti-de Sitter space.

The number of these so-called AdS/CFT dualities connecting QCD like theories with gravitational theories is now vast and examples exist in many dimensions, for many different space-times and for many types of conformal and non-conformal gauge theories. However, in most cases these dualities are conjectural in nature, and based on calculations in which the number of colors is taken large and the string theory is computed at low energy. Typically the dual Yang-Mills theories are strongly coupled and existing analytical techniques fail. This motivates the use of the lattice, which now offers a new non-perturbative tool to study theories of quantum gravity. This is the general focus of this second thread of work. Specifically we would like to develop codes to simulate $\mathcal{N} = 4$ super Yang-Mills using two strategies.

(1) $\mathcal{N} = 4$ super Yang-Mills is a special case of the super QCD theories discussed above, since it can be thought of as a special case of an $\mathcal{N} = 1$ gauge multiplet coupled to three hyper-multiplets. We can thus hope to study it using domain wall fermions, fine tuning in this case exactly four quartic scalar operators. The computational cost is high, but the approach is somewhat conservative in the sense that the numerical algorithms are well understood and the calculations can piggyback on the LHC physics program described above.

(2) Over the last five years a number of exciting theoretical developments have taken place which have culminated in a lattice action for $\mathcal{N} = 4$ super Yang-Mills in which the supersymmetry is exact even for non-zero lattice spacing [60]. This dramatically reduces the amount of fine tuning need to ensure the continuum limit is supersymmetric. To use this approach new codes would need to be developed. However, single core codes already exist and exploratory calculations have begun with some promising results [61].

These two approaches should allow us to make contact with the wealth of physics applications flowing from the AdS/CFT correspondence - for example the computation of black hole thermodynamics from gauge theory [62].

2.2.3 Physics driven BSM needs of SciDAC-3 software support

Our new SciDAC-3 software infrastructure will transform the early BSM lattice field theory program of USQCD into flexible rapid-response software solutions to answer challenges driven by new theoretical ideas and LHC discoveries. BSM lattice studies are extremely demanding computationally because N_c in the color gauge group $SU(N_c)$, the number of fermion “flavors” N_f, and the dimension of the group representation $R(N_c)$ for fermions vary in the theory space of the BSM paradigm. Some of the special needs of the new SUSY program were already outlined, and they also require the extensions of our application codes to general values of N_c, N_f and $R(N_c)$. There are some very specific physics driven goals of the new BSM software suite:

- With particular importance on the chiral properties of the BSM models, the domain wall, overlap, and staggered fermion solvers have to enable the use of several color groups in the fundamental, adjoint, two-index symmetric and two-index antisymmetric fermion representations.
- Using Random Matrix Theory, the determination of the low-lying eigenvalue spectrum and the related wave-functions of the fermion Dirac operators will help significantly in finite volume studies of BSM models. The eigenvalue spectrum is particularly important for the identification and detailed study of chiral symmetry breaking close to the conformal window. Thus, very large scale and robust eigen-solvers have to be designed and developed, addressing the issues of large grids, different lattice fermion choices, and different fermion representations.
- Studies of the running coupling will require the implementation of Schrödinger functional boundary conditions, twisted gauge and twisted fermion boundary conditions and renormalization group blocking methods in all the important fermion representations.

A Appendix

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