Fundamental parameters from future lattice calculations

Lattice QCD Executive Committee

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Abstract

We describe the impact that future lattice QCD calculations can have on the determination of the parameters of the standard model (quark masses, coupling constant, and particularly the CKM matrix elements) and the search for new physics beyond. We consider the impact of calculations requiring CPU resources ranging from what is possible by the end of 2007 (≈ 10 TFlop-yr with USQCD resources), through 50 TF-Yr and up to 500 TF-Yr.

We begin with a brief description of progress in the last 5 years and an overview of what should be possible in the next 5 years, focusing on overconstraining the CKM matrix elements. Subsequent sections detail, respectively, the status and future of calculations of the quark masses, the CPU time required for future ensembles of lattice gauge configurations, and the status and future of calculations of electroweak matrix elements which constrain CKM elements. We end with a summary and outlook.[1]

1 Introduction, history and overview

One of the central aims of calculations using lattice QCD is to determine the underlying parameters of the Standard Model (SM) by stripping away the effects of the strong interactions. Lattice calculations aim to provide accurate determinations of the masses of the up, down, strange, charm and bottom quarks[2] the strong coupling constant α_S, and the values of the weak transition couplings between quarks—i.e. the elements of the Cabibbo-Kobayashi-Maskawa (CKM) matrix. These quantities, along with the unknown Higgs mass and coupling, and the well known electroweak coupling and mixing angle, are the parameters of the $SU(3) \times SU(2) \times U(1)$ Lagrangian which defines the SM. Particularly exciting is the possiblity of determining different, inconsistent values of the CKM matrix elements from different decay processes. This would indicate a breakdown in the Standard Model and thus the need for new physics. This approach is complementary to the direct discovery searches to be undertaken at the Large Hadron Collider at CERN (LHC), but to be successful requires reliable and precise lattice QCD calculations.

[1] Companion white papers discuss *"Nuclear Physics from Lattice QCD: The Spectrum, Stucture and Interactions of Hadrons", "Opportunities for Lattice QCD Thermodynamics with Petaflops Resources"*, and *"Challenges for lattice field theory in the LHC era"*.

[2] The top quark decays before it can form hadronic bound states so lattice calculations are not needed for the determination of its mass.

The last five years have seen lattice QCD (LQCD) calculations mature to the point that accurate determinations of some of the fundamental parameters are possible, with all errors controlled (as will be reviewed in sec. 4). Prior to this the methodology had been developed, but calculations had uncontrolled systematic errors, particularly due to the exclusion of the effects of light-quark loops (the so-called "quenched" approximation). A key aspect of the recent progresss has been the creation of an ensemble of gauge configurations generated including the full quantum measure ("unquenched" or "dynamical" configurations including the effects of light quark loops) with a series of values for the lattice spacing (a) and the light quark masses.[3] This ensemble uses "improved" lattice fields (so as to reduce the discretization errors) and staggered fermions, and has been generated by the MILC collaboration under the auspices of the USQCD lattice collaboration. It has been made available to lattice researchers worldwide who have used it to calculate a wide variety of physical quantities. Errors at the few percent level are possible in the best cases, and the ensemble is being extended (to smaller a, smaller m_ℓ, and increased statistics) so as to allow further improvements in accuracy.

In addition to these extensive lattice configurations generated using staggered fermions, there are also an increasing number of lattice configuration generated using domain wall (DW) quarks. This lattice fermion formulation yields accurate chiral symmetry for the quarks and directly provides the correct number of quark flavors. Available configurations have two lattice volumes but a single lattice spacing. This DW fermion approach will be discussed further below, both as the method of choice for computing particular weak matrix elements as well as an important test of the results obtained using staggered fermions.

A crucial aspect of LQCD calculations is validation. There are many sources of error in the calculations and, just as with experimental measurements, cross-checks using different methods and comparison with known results must be used to validate the error estimates. Estimates of statistical errors require a correct understanding of the correlations between configurations. Further errors arise from fitting (e.g. of Euclidean correlation functions to a sum of exponentials) and from the need to make extrapolations (in particular $a \to 0$, $m_\ell \to m_\ell^{\rm phys}$ and box size $L \to \infty$). The methods used to simulate heavy quarks (i.e. b and, to some extent, c quarks) are approximate (e.g non-relativistic QCD) and require theoretical estimates of errors. The calculation of electroweak matrix elements require matching of continuum and lattice operators, which introduces a further error. And, finally, in the case of staggered fermions an additional assumption is made, namely the use of a rooted determinant to cancel the effects of the additional varieties ("tastes") of fermion that are intrinsic to this formulation. There are theoretical arguments that this approach yields the correct continuum ($a \to 0$) limit, but the required fitting is to complicated theoretical forms (derived from "staggered" chiral perturbation theory), and numerical validation is essential.

Validation has been carried out to date by comparing predictions for those quantities which are calculable with the smallest errors. The number of such quantities increases with time, and Fig. 1 gives a recent update [1] on the original figure [2]. The figure shows that the quantities used are sufficiently sensitive to quark loops to provide a stringent test of the methodology. In particular, including quark loops one finds agreement within errors of size 1-3%, while calculations

[3] Calculations to date are in the isospin symmetric limit $m_u = m_d = m_\ell$, but have the strange quark mass, m_s, at (or close to) its physical value. The simulated values of m_ℓ are larger than the physical average light quark mass, but range down to $m_\ell/m_s \approx 1/10$, which approaches the physical ratio of $\approx 1/27$.

using the quenched approximation find deviations of $\sim 10\%$ from experiment. The agreement of unquenched results tests both heavy and light quark methodologies as well as the running of the coupling constant between the heavy and light quark scales.

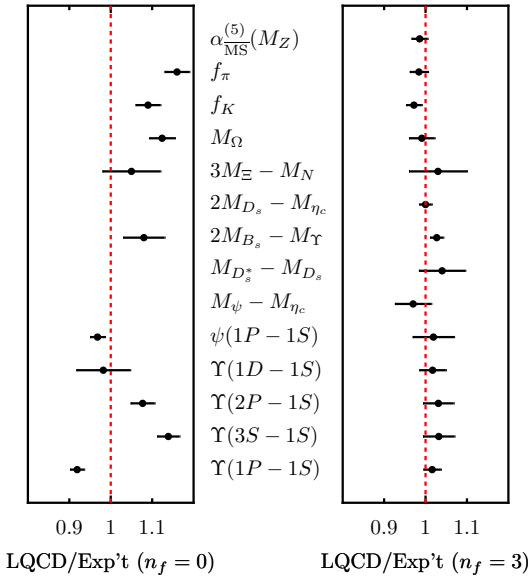

Figure 1: Ratio of lattice results to those from experiment. Left panel uses quenched and right panel unquenched ($2+1$ flavor) lattice calculations.

Another crucial validation is provided by the successful comparison of the strong coupling constant obtained from lattice calculations. $\alpha_S(m_Z) = 0.1170 \pm 0.0012$ [3], with the world average from other methods (in which α_S is determined by matching perturbative QCD predictions to collider results at high energies) $\alpha_S(M_Z) = 0.1185 \pm 0.0015$.[4] This comparison is also shown in Fig. 1.

Perhaps the most convincing test of methods is to make successful predictions in advance of experimental measurements. Three such predictions have been tested to date: the mass of the B_c meson, the decay constant of the D^+, and the shape and normalization of the $D \to K$ semileptonic form factor. All three were successful [5]. These comparisons are not, however, at the level of precision of those discussed above. Both experimental and lattice errors for the D and D_s decay constants and form factors are presently of order 6 to 10%. The experimental errors are typically statistics-limited and may drop by a factor of ~ 2 over the next few years. It it therefore crucial that the lattice errors also be reduced by a similar factor. Computation of these quantities is discussed in more detail in sec. 4.

We note also that lattice calculations have led to controlled predictions of quark masses, with those of the light quarks being the most precise. This will be discussed in sec. 2. Although the connection between quark masses and experimental observables is subtle, pinning down these fundamental parameters is a notable success, and feeds into the details of unification schemes involving new physics.

[4] This number is obtained from Ref. [4] with the lattice result dropped from average.

A final success, albeit a more qualitative one, concerns the calculations of electroweak matrix elements needed to constrain the CKM matrix. Three constraints that rely solely on lattice methods are (i) the kaon B-parameter B_K, which determines the CP violating part of the $K-\overline{K}$ mixing, ε_K; (ii) the matrix element $f_{B_s}\sqrt{\widehat{B}_{B_s}}$, which controls the rate of $B_s-\overline{B_s}$ mixing; and (iii) the ratio ξ (defined in sec. 4) which determines the relative size of $B_s-\overline{B_s}$ and $B_d-\overline{B_d}$ mixing. Early lattice estimates for these quantities (as collected in Ref. [6] in 2000 and given in the second column of Table 1 below) led (along with other theoretical and experimental input) to predictions for the CKM angle β (see the Appendix for notation for the CKM matrix) and $B_s-\overline{B_s}$ mixing,

$$\tan(2\beta) = 0.698 \pm 0.066\,, \qquad \Delta m_s = (16.3 \pm 3.4)\text{ps}^{-1}\,, \qquad \text{[Lattice Pred. (2000)]} \qquad (1)$$

that are in agreement with the subsequent experimental measurements [7, 8],

$$\sin(2\beta) = 0.674 \pm 0.026\,, \qquad \Delta m_s = (17.77 \pm 0.12)\text{ps}^{-1}\,, \qquad \text{[Experiment (2006)]} \qquad (2)$$

This success has been important in convincing the wider particle physics community of the utility of lattice calculations [9].

Hadronic Matrix Element	Quenched Estimate in 2000	Lattice Result Current	UTA Result Current	Lattice Errors 10. TF-Yr	Lattice Errors 50. TF-Yr
$\widehat{B}_K$	0.87 ± 0.15	0.77 ± 0.08	0.75 ± 0.09	± 0.05	± 0.03
$f_{B_s}\sqrt{\widehat{B}_{B_s}}$	262 ± 40 MeV	282 ± 21 MeV	261 ± 6 MeV	± 16 MeV	± 9 MeV
ξ	1.14 ± 0.07	1.23 ± 0.06	1.24 ± 0.08	± 0.04	± 0.02

Table 1: History, status and future of lattice QCD calculations of three matrix elements which play a key role in the determination of CKM matrix elements. Quenched estimates from 2000 taken from Ref. [6], UTA values from Ref. [9, 10]. Present results are from Refs. [11, 12] (B_K), [13] ($f_{B_s}\sqrt{B_{B_s}}$), and [14] (ξ), and will be discussed further in sec. 4. Note that none of the present lattice results include fully controlled estimates of all errors.

A major focus of the USQCD calculations in the last five years has been the improvement of electroweak matrix calculations. A more extensive status report is given in sec. 4 below, but here we describe the progress of calculations of three matrix elements which play a key role in constraining the SM. Table 1 shows how present lattice results (third column) compare to the estimates from 2000. The main progress has been the use of unquenched, $2+1$ flavor gauge configurations instead of the quenched approximation (except for ξ, for which the present result is based on partially unquenched [2 light flavor] ensembles). While this progress shows up as a reduction in the errors, what cannot be seen from numbers alone is that the *reliability* of the error estimates has improved (since, except for ξ, it is now no longer necessary to estimate the impact of quenching, which can be done, at best, semi-quantitatively). Nevertheless, there are as yet no unquenched calculations of the matrix elements having all errors are controlled. For example, fully unquenched results for $\widehat{B}_K$ and $f_{B_s}\sqrt{\widehat{B}_{B_s}}$ are available only at one lattice spacing and volume, so that discretization and finite volume errors are estimated from previous partially unquenched, or quenched, calculations or using theoretical arguments, and are not yet directly calculated.

There are three main reasons why calculations of matrix elements like these three lag behind those of the quantities discussed above that have been used for validation and prediction. First, they are more complicated to calculate: they involve four-fermion operators rather than bilinears, and, in some cases, non-trivial operator mixing. Second, there is a non-trivial overhead in CPU (and human) time required to progress from the generation of gauge configurations to the calculation of the valence propagators required to extract the matrix elements. And, third, there is a theoretical overhead needed to understand operator mixing and determine the appropriate functional forms to use when doing chiral extrapolations. The net result is that matrix element calculations lag the generation of configurations by one or two years. In fact, in the case of B_K, a light-quark quantity for which chiral symmetry plays an important role, calculations using DW fermions have leap-frogged those using staggered fermions, although (as discussed below) the DW ensemble is only in the early stages of completion.

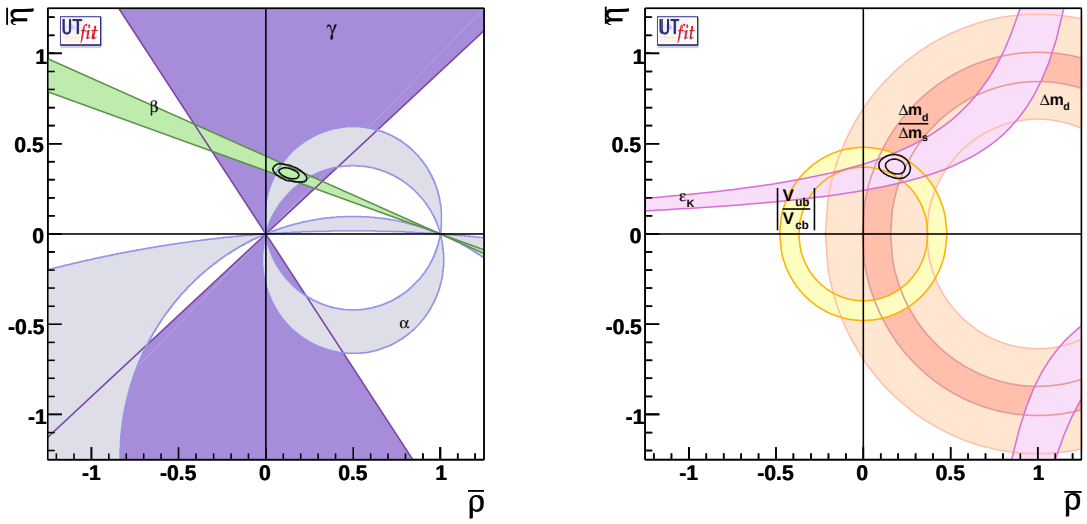

Figure 2: Present constraints on $\bar{\rho}$ and $\bar{\eta}$ from the UTA analysis (left panel) and from matrix elements involving lattice QCD input (right panel). Contours of 68% and 95% probability are shown, together with the 95% probability regions from individual constraints. Consistency of the allowed regions in the two panels provides precision confirmation of the SM. From Ref. [9].

In the last 5 years, there has been tremendous progress in experimental measurements of B-meson properties. This has allowed a determination of the CKM elements using methods that require little or no knowledge of hadronic matrix elements, and, in particular, no input from lattice calculations. Following Ref. [9], we call this the Unitarity Triangle-Angles (UTA) approach. Its status is shown in the left panel of Fig. 2. Combining the results with the measured values for ε_K, Δm_d and Δm_s allows one to *predict* the values of three matrix elements discussed above (assuming that the SM is correct). The results are given in the fourth column of Table 1.[5] The agreement between present UTA and lattice results shows that the SM description of flavor physics, including CP violation, is consistent with experiment. An alternative way of seeing this consistency is to

[5]In this article we use the UTfit collaboration analysis of unitarity triangle constraints as results are given of direct relevance to the needed precision of lattice calculations. See the CKM fitter web site for an alternative approach to imposing the constraints, which leads to similar conclusions [15].

compare the constraints on $\bar{\rho}$ and $\bar{\eta}$ from the two approaches, as is done in Fig. 2. It should be stressed that the agreement is a highly non-trivial test of the SM, involving both electroweak and strong-scale physics.

This comparison shows, furthermore, that the next 5 years presents a tremendous opportunity. If the errors in lattice results can be reduced to a level below those of the UTA predictions (which will themselves be gradually reduced), then there is the potential for stringent tests of the SM. The first stage is to obtain lattice results with all errors controlled, and this should occur in the next year (i.e. by the end of 2007) doing analyses on existing unquenched ensembles. Our estimated errors are given in the Table in the "10 TF-Yr" (10 TeraFlop-Year) column—this is roughly the accumulated computational resource that will have been devoted to the calculation (including configuration generation) by the USQCD collaboration by the end of 2007. The basis for these estimates is described in sec. 4. The needed calculations will likely use both staggered and DW fermions. The expected errors in $\widehat{B}_K$ and ξ are smaller than the present UTA errors, while those in $f_{B_s}\sqrt{\widehat{B}_{B_s}}$ are larger (although comparable). All errors can be further reduced by using a more extensive ensemble (utilizing both staggered and DW fermions). As an example, we give estimates for an accumulated CPU time of 50 TF-Yr. At this stage, the theoretical errors are at the few percent level, allowing precision tests of the SM.

These estimates show that, for these three key matrix elements, resources at the 50 TF-Yr level allow one to reach the same level of precision as has already been attained for simple quantities. There are, however, many other matrix elements that can provide constraints on both the SM and on theories proposed for physics beyond.[6] Many of these matrix elements are more complicated than the three discussed in this section, and will require greater computational resources to obtain precision results. We discuss the range of such quantities in sec. 4 below. Preceeding that, we first describe recent progess on the calculation of quark masses, and then discuss the computational requirements for possible future ensembles of gauge configurations.

2 Status and future of lattice results for quark masses

Unquenched calculations of light hadron properties have progressed to the point that the light quark masses can be determined with precision. The present calculation uses light quark masses down to $m_\ell/m_s = 0.1$ at three lattice spacings ($a \approx 0.15, 0.12, 0.09\,\text{fm}$) and also includes first results at $a \approx 0.06\,\text{fm}$ (with $m_\ell/m_s = 0.4$). Preliminary results from this ensemble and using two-loop perturbative matching factors are given in Table 2 under the heading "2006 result". Also shown are earlier results using only parts of the present $a \approx 0.12\,\text{fm}$ and $0.09\,\text{fm}$ ensembles (with m_ℓ/m_s values down to $1/10$ and $1/5$, respectively), and either one-loop (listed as "2004 result") or two-loop perturbation theory (listed as "2005 result"). Errors are from statistics, simulation systematics, the truncation of perturbation theory for matching factors, and incomplete inclusion of electromagnetic effects, respectively.

[6] In this white paper we consider only the QCD calculations that are needed to constrain beyond the standard model (BSM) physics. A separate white paper ("*Challenges for lattice field theory in the LHC era*,") is devoted to direct calculations in possible BSM theories.

Quark mass	2004 result	2005 result	2006 result
m_ℓ/MeV	2.8(0)(1)(3)(0)	3.2(0)(2)(2)(0)	3.3(0)(2)(2)(0)
m_u/MeV	1.7(0)(1)(2)(2)	1.9(0)(1)(1)(2)	2.0(0)(1)(2)(1)
m_d/MeV	3.9(0)(1)(4)(2)	4.4(0)(2)(2)(2)	4.6(0)(2)(2)(1)
m_s/MeV	76(0)(3)(7)(0)	87(0)(4)(4)(0)	90(0)(5)(4)(0)

Table 2: History of recent results for light-quark masses using improved staggered fermions. 2004 results are from Ref. [16, 17], 2005 results from Ref. [18], and 2006 results from Ref. [19]. Here $m_\ell = (m_u + m_d)/2$, and all masses are quoted in the $\overline{\rm MS}$ scheme at a renormalization scale of 2 GeV. Details are discussed in the text.

The main conclusion is that lattice calculations have provided, for the first time, accurate results for quark masses. In particular, the overall scale of the quark masses has turned out to be smaller than pre-lattice estimates suggested (e.g. $m_s \approx 150\,$MeV). The result $m_\ell/m_s = 1/27.1 \pm 0.4$ for the $SU(3)$-breaking ratio is in complete accord with estimates using chiral perturbation theory and other model input. It should be mentioned, however, that partially unquenched calculations with Wilson-like fermions find somewhat higher quark masses [20], so it is important to check the results from staggered fermions with other lattice fermions. This will be done over the next few years, in particular using DW fermions.

Table 2 shows that, after the first unquenched result became available in 2004, subsequent improvements have been largely due to the use of more accurate matching factors. In particular, the increase in central values between 2004 and 2005 is due to inclusion of the two-loop contribution. The use of a larger ensemble, with smaller values of a and m_ℓ, has provided an important consistency check, but the statistical power of the additional lattices is insufficient to reduce the extrapolation errors.

Future work is planned in three directions. First, the use of non-perturbative renormalization to calculate the matching factors, thus replacing an estimated truncation error with a known, and smaller, statistical error. Second, the use of smaller lattice spacings and masses to reduce the systematic errors. And, finally, the use of DW and possibly other fermion actions. In this way, results with few percent accuracy should be possible in the next 5 years. These extended calculations will also allow a much more significant improvement in the determination of the unknown coefficients which appear in the QCD effective chiral Lagrangian (e.g. the Gasser-Leutwyler coefficients).

There has also been considerable progress on calculations of m_c and m_b.[7] Unquenched results using the MILC $a \approx 0.12$ and $0.09\,$fm lattices have been obtained, using both the Fermilab action (for c and b quarks) and NRQCD (for the b quark). The former results are $m_c = 1.22 \pm 0.09\,$GeV, $m_b = 4.7 \pm 0.4\,$GeV [21], while the latter are $m_b = 4.4 \pm 0.3\,$GeV [22], in both cases using one-loop matching factors. These are consistent with the PDG averages excluding lattice input: $m_c = 1.24 \pm 0.09\,$GeV and $m_b = 4.20 \pm 0.07\,$GeV [4]. This validates the lattice heavy-quark methodology at the level of 10% precision.

[7] In the following these quark masses are quoted in the $\overline{\rm MS}$ scheme at the scale of the corresponding mass.

It is important to improve the accuracy the calculations of heavy quark masses, both to provide more precise validation and because these masses are fundamental parameters in the SM. For the approaches used to date, the dominant error arises from the truncation of perturbation theory, so further progress requires theoretical, rather than numerical, work. This is, however, unlikely to lead to precision at the percent level. For this one likely needs to use non-perturbative matching, which has been implemented in pilot quenched studies for both charm [23] and bottom quarks. [24] Another approach is to use highly improved staggered fermions [25].

3 Future ensembles of configurations

Key factors determining the future progress of LQCD calculations are the size and parameters of the ensembles of gauge configurations that can be generated. In particular, what values of a, m_ℓ and L are attainable? In this section we describe what is likely to be possible in the next 5 years. We focus on staggered and DW fermions, which are likely to be the primary choice for flavor-related calculations in this period. Staggered fermions are fast, but require complicated fitting and theoretical analysis to deal with the extra tastes. DWF are (relatively) slow to simulate, but the analysis required for most quantities is continuum-like and straightforward, and their enhanced chiral symmetry is important for many quantities related to flavor physics. It is also possible that other fermion actions will be used for flavor-physics calculations, e.g. improved Wilson fermions. The distribution of resources will be adjusted yearly by the collaboration, based on the results and projects proposed to the collaboration, so as to maximize the production of validated, precise results for important quantities. It should also be kept in mind that the required balancing includes the division of resources between the flavor physics aims discussed in this white paper, and other aims of USQCD, in particular hadronic and nuclear physics, finite temperature/density QCD, and BSM physics.

As noted above, the present staggered fermion ensemble includes lattices at $a = 0.15$, 0.12, 0.09 fm with light masses ranging down to $m_\ell/m_s \approx 0.1$, as well as partial sets at $a = 0.06$ with $m_\ell/m_s = 0.4$ and 0.2. Table 3 shows the cost of extending this ensemble to smaller a and m_ℓ/m_s. The labels (borrowed and extended from our 2004 white paper [26]) indicate how one might progress stepwise as CPU resources increase. MILC1 lattice generation should be completed before the end of 2007, to be followed by MILC2 generation, etc. Each step involves a reduction in a, or in m_ℓ/m_s, but not both, and requires an increase in CPU time ranging from 2-9.

An important and very welcome result shown in the table is that the CPU estimates from 2004 have proven to be too high by factors of 3-10. This is due to two factors. First, the use of the rational hybrid Monte-Carlo (RHMC) algorithm [27] instead of the R-algorithm, which reduces the CPU time by factors $2-6$ for the quark masses in the table (with the reduction increasing with smaller m_ℓ).[8] Second, the earlier estimates used asymptotic formulae which proved overly conservative in practice. The combined improvements bring the possibility of a direct simulation of physical quark masses significantly closer.

[8] In practice for the staggered fermion evolutions the final accept-reject step is dropped from the RHMC algorithm, leading to what is called the rational hybrid molecular dynamics (RHMD) algorithm.

a (fm)	m_ℓ/m_s	Size	L (fm)	MC traj.	TF-Yr R-2004	TF-Yr RHMD-2007	Label
0.09	0.10	$40^3 \times 96$	3.6	3000*	1.5	0.14	MILC1
0.09	0.05	$56^3 \times 96$	5.0	4200	23	1.2	MILC2
0.06	0.20	$48^3 \times 144$	2.9	3750†	1.9	0.7	MILC1
0.06	0.10	$64^3 \times 144$	3.8	4500	22	2.4	MILC2
0.06	0.05	$84^3 \times 144$	5.0	6300	280	19	MILC3
0.06	1/27	$100^3 \times 144$	6.0	7454	–	55	MILC4
0.045	0.40	$56^3 \times 192$	2.5	4000	–	1.1	MILC2
0.045	0.20	$56^3 \times 192$	2.5	5000	10	3.0	MILC2
0.045	0.10	$80^3 \times 192$	3.6	6000	135	14	MILC3
0.045	0.05	$112^3 \times 192$	5.0	8400	2100	130	MILC4
0.045	1/27	$124^3 \times 192$	5.6	9940	–	320	MILC5

Table 3: CPU requirements (in TFlop-years) for future generation of unquenched configurations with improved staggered ("asqtad") fermions. Lattice sizes are chosen so that finite volume effects are roughly constant (and small). "MC traj." gives the lengths of the runs (in number of trajectories)—these are chosen so that statistical errors should be sub-dominant for quantities of interest. An asterix indicates that generation is complete, while a dagger that it is underway. The present estimates (labelled 2007) assume the RHMD algorithm. For comparison, we give the estimates (labeled 2004) from our 2004 white paper [26] (appropriately scaled for changes in lattice size and trajectory lengths), which assumed the R algorithm and conservative extrapolations. All estimates are for two degenerate light quarks of mass m_l and a strange quark at its physical mass. The ratio $m_l/m_s = 1/27$ is the physical value. The labels indicate a progression of increasingly demanding calculations, and are used in the text.

To estimate the resources needed to progress to each stage of this program one must include not only the configuration generation but the significant time required to calculate valence propagators. We have previously multiplied by a factor of 2 to account for this, but here use the factor of 4. This increase is appropriate because some of the algorithmic advances do not carry over to propagator calculations, and because of the increasing number of quantities that are being be calculated. With this factor, the costs of the stages are roughly as follows:

$$\text{MILC1: 4 TF-Yr;} \quad \text{MILC2: 30 TF-Yr;} \quad \text{MILC3: 100 TF-Yr; and MILC4: 750 TF-Yr.} \quad (3)$$

In assessing these numbers, it should be kept in mind that these estimates apply for calculations focused on fundamental parameters alone. Such calculations use 1/3-1/2 of USQCD resources, with the remainder focused on spectroscopy, finite temperature calculations, and nucleon structure and interactions.

An important feature of the RHMC algorithm is that there is essentially no additional cost if one simulates with non-degenerate u and d quarks. Thus the estimates for $m_\ell/m_s \approx 1/27$ apply as well to simulations with all three light quarks at their physical values.[9] Direct simulations with

[9]The pion masses are essentially the same for degenerate or non-degenerate light quarks, so one does not need to

physical light quarks are thus attainable once resources reach the PetaFlop level. These would represent a major milestone, avoiding the need for chiral extrapolations, but requiring the inclusion of electromagnetic effects.

Estimates of required CPU time for configuration generation with DW fermions are given in Table 4. In this case we do not have earlier estimates to compare to, but it should be noted that in the last two years a speed-up in DWF codes by about a factor of 6 has been achieved by algorithmic and coding work [27]. The timing estimates are made by extrapolations from present simulations, which are currently being run and analyzed on lattices of spacing $a = 0.122$ and 0.093 fm. The tables show that the CPU cost is 10-20 times more for DW than for staggered fermions.

a (fm)	m_ℓ/m_s	Size	L_5	L (fm)	MC traj.	TF-Yr	Label
0.12	0.3	$24^3 \times 64$	16	3.0	9000†	0.7	DWF1
0.12	0.19	$24^3 \times 64$	16	3.0	9000†	0.8	DWF1
0.09	0.20	$32^3 \times 64$	16	3.0	4500	1.3	DWF1
0.09	0.136	$32^3 \times 64$	16	3.0	4500	1.4	DWF2
0.09	0.136	$48^3 \times 64$	16	4.4	5000	7.0	DWF2
0.09	0.065	$48^3 \times 64$	16	4.4	5000	8.6	DWF3
0.09	1/27	$64^3 \times 128$	24	5.9	10000	230	DWF5
0.06	0.144	$48^3 \times 64$	16	3.0	10000	18	DWF3
0.06	0.084	$64^3 \times 128$	16	4.0	10000	130	DWF4
0.06	1/27	$96^3 \times 128$	16	5.9	10000	680	DWF6

Table 4: CPU requirements for future lattice generation of unquenched configurations with DWF using the RHMC algorithm [27]. Notation as in tab. 3, except that earlier estimates are not available, and that L_5 is the number of sites in the fifth dimension.

The labels indicate an analogous progression to that for staggered fermions, with the resources needed for DWF1 roughly corresponding to those for MILC1, etc. The present ensemble (DWF0) consists of lattices at $a = 0.122$ fm, with $m_\ell/m_s \geq 1/7$, and spatial sizes of $L = 2$ and 3 fm. The DWF2 level is particularly important because there will then be two lattice spacings and small quark masses so all extrapolations (and corresponding errors) should be controlled. This is thus the threshold for precision predictions from DWF. Note that, although the progression for DWF in terms of a and m_ℓ lags that for staggered fermions due to the extra CPU cost, this is counterbalanced by the simplified fitting, improved chiral symmetry, and possibly by smaller discretization errors. At the DWF2 stage, which is attainable when total resources applied to flavor physics are of order 50 TF-Yr, we will have precision results with both staggered and DW fermions. This is a very important milestone both because it allows a crucial check of the methods and thus the predictions, and because it will allow a considered decision on which fermion method to pursue for subsequent calculations.

increase the physical volume or expect longer decorrelation times. Note that although the rooting method breaks down for a massless quark [28], the physical masses are likely positive enough for this not to be a problem [29, 30].

4 Future calculations of electroweak matrix elements

In this section we list the most important matrix elements of electroweak operators that can be calculated using LQCD, describing in some detail the signficance, status and future prospects for each. We proceed roughly from the best to the least well known.

In describing some of the following results it is useful to introduce some notation for different subsets of the present MILC ensemble. We call the $a = 0.12\,\mathrm{fm}$ and $a = 0.09\,\mathrm{fm}$ lattices the "coarse" and "fine" MILC lattices, respectively. Some calculations have been done only on the coarse lattices, others on the coarse and a subset of the fine (with m_ℓ/m_s down to 0.2). We call the latter collection the "MILC0" ensemble. The most up-do-date calculations use also the fine lattices with $m_\ell/m_s = 0.1$, and the $a = 0.06\,\mathrm{fm}$, m_ℓ/m_s lattices—we refer to these together with the MILC0 lattices as the "present MILC ensemble".

We stress that the best fermion discretization to use depends on the quantity, and that, as the following descriptions show, a mix of staggered and DW fermion calculations is likely to be optimal for the next few years.

The notation for CKM matrix elements is summarized in the Appendix.

4.1 Bilinear matrix elements

- f_π and f_K. These quantities have been used for validation of lattice methods in the light meson sector by comparing to the experimental leptonic decay rates. They can be calculated to good accuracy, with errors of size 2.6% using staggered fermions on the present MILC ensemble [19]. Some errors cancel in the ratio f_K/f_π, in which the present error is about 1%.

  Although these are among the most accurate results from LQCD, it is important to further reduce the errors. Improving the calculation of f_π will allow more precise validation. (The CKM element which enters, V_{ud}, is known to very high accuracy from nuclear decays.) This is particularly important for staggered fermions, in order to test whether the complications due to taking roots of the determinant are understood. Another important test, which can be done at unphysical quark masses to avoid chiral extrapolations, is to compare precision results obtained with staggered and DW fermions.

  The ratio f_K/f_π can be used to determine V_{us}, the present value leading to

$$|V_{us}| = 0.2223 \begin{pmatrix} +26 \\ -14 \end{pmatrix} \qquad (\text{Lattice } f_K/f_\pi)\,. \tag{4}$$

  This has comparable errors to that from the standard method using semileptonic $K \to \pi$ decays (discussed below): $0.2257(21)$ [4]. Thus further improvements in LQCD calculations of f_K/f_π will lead to an improvement in the present determination of V_{us}. This is of considerable interest since it would allow a more stringent test of the unitarity of the first row of

the CKM matrix. Present results are

$$|V_{ud}|^2+|V_{us}|^2+|V_{ub}|^2 = 0.9992(5)(9)(0)\ \text{[PDG06]}\ \text{or}\ 0.9977(5)(12)(0)\ [\text{Lattice}\, V_{us}]\,, \quad (5)$$

with the errors coming, respectively, from those in V_{ud}, V_{us} and V_{ub}. Both results are consistent with unitarity, but the larger deviation of the central value from unity when using the lattice V_{us} provides particular motivation to reduce the lattice error.

We can roughly estimate how errors will be reduced in calculations using staggered fermions. This will serve as the standard for subsequent estimates so we provide some details. The dominant errors in f_π come from setting the scale and from the combined chiral-continuum extrapolation. In the detailed 2004 MILC study [17] (which used a somewhat smaller ensemble than the present best results, and had slightly larger errors), these errors were 1.8% and 1.9% respectively. Both errors are reduced by extending the ensemble so as to improve chiral and continuum extrapolations, though the scale error will likely be reduced less as it has a weaker dependence on the light quark mass (being based on calculations of the Υ spectrum). We estimate a reduction of the scale error by 0.8 and 0.6 for the MILC1 and MILC2 stages, respectively, and corresponding reductions by 0.7 and 0.5 in the extrapolation error. This leads to a combined systematic error of 2.0% and 1.4% in f_π (and f_K) at the MILC1 and MILC2 stages, respectively. We expect the statistical error to remain at about the present level of 0.3%, and thus to be subdominant.

For the ratio f_K/f_π, there are essentially no scale errors, and we expect the extrapolation error to reduce as for f_π. The 1% extrapolation error in Ref. [17] is thus estimated to fall to 0.7% and 0.5% in the MILC1 and MILC2 ensembles, respectively. This will likely remain dominant over the statistical error (0.2% at present). These reductions would lead to considerably more stringent test of unitarity. For example, if the central value of f_K/f_π remained unchanged, the unitarity sum would become $0.9977(5)(6)(0) = 0.9977(8)$ at the MILC2 stage, a 3-σ effect.

DWF calculations will also give precision results for f_π and f_K. At this stage, it is difficult to estimate the likely errors, since first results from unquenched simulations have just been presented, and are only at a single lattice spacing [31]. One complication is that the renormalization constant Z_A, which is unity with staggered fermions, must be determined non-perturbatively from the simulations. It is therefore very encouraging that the results for Z_A using non-perturbative renormalization have extremely small errors (0.03% or less).

- $K \to \pi\ell\nu$ form factor. This is needed to convert the experimental measurement of the semileptonic decay rate to a determination of V_{us}. The PDG uses the Leutwyler-Roos value from model calculations, $f_+(0) = 0.961(8)$, but notes that there is a possible 2% theoretical uncertainty in this number, which is not included in the PDG error for V_{us} [4]. An accurate lattice result would greatly improve upon this situation. Lattice results with sub-percent level precision are possible here through the use of appropriate ratios [32]. Indeed, first unquenched results using DWF at $a \approx 0.125\,\text{fm}$, and $m_\ell/m_s = 0.25 - 0.75$, have statistical and estimated chiral extrapolation errors each of ≈ 0.001 in f_+, with continuum extrapolation errors estimated to be smaller [33]. It is crucial to check these estimates by calculations at smaller m_ℓ/m_s and a, and this will be possible using the DWF1-2 ensembles. If confirmed, the error in V_{us} will become dominated by that from the experimental measurement, which contributes $\approx \pm 0.0009$ to V_{us} at present (i.e. somewhat less than half of the present total error

of ± 0.0021). This would lead to a unitarity test of comparable precision to that envisioned above from f_K/f_π.

Calculations with staggered fermions are also possible, although a preliminary unquenched result has considerably larger errors than obtained with DWF [34].

- $D \to (K,\pi)\ell\nu$ form factors. These can be used together with the measured semi-leptonic decay rates to determine V_{cs} and V_{cd}, respectively. Lattice results allow prediction of the differential cross-section, thus allowing a detailed comparison with experiment. The success of this comparison for $D \to K\ell\nu$ is one of three successful lattice predictions noted in the introduction.

  Further work on these calculations is of considerable interest. Accurate determinations of V_{cs} and V_{cd} would allow a precision unitarity test from the second row of the CKM matrix (since V_{cb} is known to be small). Lattice calculations can contribute most usefully to the determination of V_{cs}, which is only known with an error of $\approx 30\%$ from neutrino scattering. An unquenched calculation using Fermilab *c* quarks and staggered light quarks on the coarse MILC lattices yields $V_{cs} = 0.957(17)(93)$, where the first error is from experiment and the second from the lattice calculation [35, 4]. Discretization errors dominate the lattice uncertainty, and should be substantially reduced, as well as firmed up, by extending the calculation to the full present MILC ensemble.

  V_{cd} is known most accurately from neutrino scattering: $|V_{cd}| = 0.230(11)$. The error is less than half that obtained using the lattice form factor from coarse MILC lattices, $|V_{cd}| = 0.213(8)(21)$, where the first error is from experiment and the second from the lattice [35, 4]. If the lattice error could be reduced below that of experiment, then this would become competitive with the neutrino scattering method. This should be possible using the envisioned MILC ensembles. Alternatively, one can view this as a moderate-precision test of lattice methods for charmed quarks.

  Calculations using DWF both for the charm and light quarks are also envisaged in the future.

- f_{D_s} and f_D. These quantities provide important validation of lattice methods for heavy quarks, by comparing predicted and measured semileptonic decay rates of the D_s and D^+ (using V_{cu} from neutrino scattering and V_{cs} from unitarity). In the case of the D^+, the unquenched lattice prediction preceeded the first measurement. Experimental results from the CLEO collaboration are statistics limited and are at present [36]

$$f_{D^+} = 223(17)(3)\,\mathrm{MeV}, \qquad f_{D_s} = 282(16)(7)\,\mathrm{MeV}, \qquad f_{D_s}/f_{D^+} = 1.27(12)(3), \quad (6)$$

  with errors being statistical and systematic, respectively. The unquenched lattice calculations using Fermilab *c* quarks and staggered light quarks on the MILC lattices give

$$f_{D^+} = 201(3)(17)\,\mathrm{MeV}, \qquad f_{D_s} = 249(3)(16)\,\mathrm{MeV}, \qquad f_{D_s}/f_{D^+} = 1.21(1)(4), \quad (7)$$

  where the first two results use the MILC0 ensemble [37] while the ratio is a recent update using more of the $a = 0.09\,\mathrm{fm}$ lattices [38]. The method relies on a partially non-perturbative normalization of currents [39].

  It is important to improve the lattice calculation, since the experimental errors are likely to fall with time. The dominant lattice uncertainties come from charm quark discretization errors and the chiral extrapolation, both of which can be substantially reduced by reducing *a*

and m_ℓ. We expect roughly a 50% reduction in the lattice error by the MILC2 stage, giving a total lattice error of $\approx 5\%$ in the decay constants and $\approx 2\%$ in their ratio.

- f_{B_s} and f_B. These quantities have long served as benchmarks for the progress of lattice methods for heavy-light systems. A recent development is the direct measurement of the $B \to \tau\nu$ branching ratio, allowing a determination of f_B given a value for V_{ub}. The errors in this measurement, presently about 35%, are unlikely to become small enough for a precision test of methods, although the measurement does provide a consistency check.

  Unquenched results are available using staggered light quarks and both NRQCD and the Fermilab action to simulate the b quark [40, 41, 42]. The dominant error is from perturbative matching—using one-loop results the error is estimated to be $\approx 9\%$ out of a total error of $\approx 10\%$. The matching error cancels in the ratio f_{B_s}/f_B, which is known to $\approx 3\%$. These calculations use the MILC0 ensemble. Significant improvement will require higher order matching. A longer-range alternative method offering higher precision uses HQET and the $1/m_b$ expansion for the b quark [43], an approach in which non-perturbative matching is possible. This approach is at an earlier stage of development, however, with no unquenched results available. A related approach is presented in Ref. [44].

- $B \to (D, D^\star)\ell\nu$ form factors. Combined with experimental measurements, these allow a determination of V_{cb}. Using the double-ratio method to cancel matching factors [45], and with confidence in the heavy-light methods provided by validation from decay constant calculations, percent-level accuracy is possible. This has been achieved for the $B \to D$ case [46], where unquenched results have errors smaller than those of experiments. The best data, however, are for the $B \to D^\star$ decays, where no unquenched result is yet available. The necessary staggered heavy-light chiral perturbation theory has been completed [47] and results with all errors controlled should be available in 2007 [48]. This will provide a very important check on the result for V_{cb} from inclusive $b \to c$ decays analyzed using HQET and short-distance methods. The latter is projected to ultimately reach 1% accuracy, and to match this the MILC2 ensemble will likely be needed.

- $B \to \pi\ell\nu$ form factors. These can be calculated for large q^2 (so that $\vec{p}_\pi$ is small in the B rest frame) and used to extract V_{ub}. This provides one of the key constraints on the $\bar{\rho} - \bar{\eta}$ plane. Results using NRQCD b quarks and staggered light quarks with all errors controlled are available [49]. The total error is $\approx 12\%$, dominated by matching ($\approx 9\%$) and statistics/chiral extrapolation ($\approx 8\%$). The experimental error (using $q^2 > 16\,\mathrm{GeV}^2$) is smaller, $\approx 6\%$. One can also use data from smaller q^2 either by using QCD sum-rule results for the corresponding form factors or including the constraints of unitarity. The present results for $10^3 \times V_{ub}$ from two independent analyses are $3.85 + 0.67 - 0.49$ [4] and 3.50 ± 0.40 [10].

  It is particularly interesting to improve this calculation, because there is tension with the result from inclusive $b \to u$ decays (to which lattice calculations do not contribute): two independent analyses find $10^3 \times V_{ub}$ to be $4.4 \pm 0.2 \pm 0.27$ [4] or 4.49 ± 0.33 [7, 10]. An important goal is thus to reduce the lattice error in the form factor by a factor of 2 (down to the level of the experimental error). At this stage exclusive and inclusive methods would have comparable errors and the significance of the present tension would be much clearer. To achieve this two-loop matching and the use of at least the MILC1 ensemble is needed.

4.2 Four-fermion matrix elements I: well studied examples

As noted above, these matrix elements involve operator mixing and more complicated contractions, and tend to lag calculations of bilinear matrix elements by 1-2 years.

- B_K. This is defined through

$$\frac{8}{3} m_K^2 f_K^2 B_K = \langle \bar{K} | \bar{s} \gamma_\mu (1-\gamma_5) d \bar{s} \gamma_\mu (1-\gamma_5) d | K \rangle \,. \tag{8}$$

  and determines the CP violating part of the $K_0 \leftrightarrow \bar{K_0}$ mixing amplitude. The latter is parameterized by the measure quantity ε_K, which is predicted in the SM to be (see e.g. Ref. [6])

$$|\varepsilon_K| = C_\varepsilon A^2 \lambda^6 \bar{\eta} \left[\eta_2 S(x_t) A^2 \lambda^4 (1-\bar{\rho}) + \text{charm}-\text{contribs} \right] \hat{B}_K \,. \tag{9}$$

  Here $\hat{B}_K$ is the renormalization-group invariant B-parameter,

$$C_\varepsilon = G_F^2 f_K^2 m_K m_W^2 / (6\sqrt{2}\pi^2 \Delta m_K) \,, \tag{10}$$

  is a well measured combination of quantities, and η_2 is a QCD Wilson coefficient, known to next-to-leading-order (NLO). Only the dominant top quark loop contribution is shown. Knowledge of B_K thus constrains the allowed region in the $\bar{\rho} - \bar{\eta}$ plane, or, conversely, knowledge of $\bar{\rho}$ and $\bar{\eta}$ constrain B_K.

  In particular, using direct measurements of the UT angles (from exclusive hadronic B decays), and eq. (9), one finds the constraint given in Table 1: $\widehat{B}_K = 0.78 \pm 0.09$ [10]. A lattice result with an error at or below this level will serve to constrain the SM.

  As noted in the introduction, there is as yet no complete calculation of B_K with all errors controlled. The state-of-the-art uses DWF at $a \approx 0.125\,\text{fm}$ and m_ℓ/m_s down to 0.25, and NPR for matching. This is a precision calculation, with statistical errors and matching uncertainties each $\approx 2\%$ [11]. What is lacking is a complete estimate of the errors due to chiral and continuum extrapolations. Taking the conservative estimate of these from Ref. [12] gives $\widehat{B}_K = 0.77 \pm 0.02 \pm 0.08$. This second error can be replaced by a controlled (and likely much smaller) error by the use of smaller m_ℓ and a second value for a. This should be provided by the DWF1 ensemble, and leads to the estimated error of ± 0.05 in the "10 TF-Yr" column of Table 1. Further reduction in systematics will occur upon reaching the DWF2-3 stages, and we estimate an error of ± 0.03 when resources reach 50 TF-Yr.

  It is necessary to have cross-checks on calculations of such important quantities, and fortunately this will be provided by at least two other methods. One uses valence DWF on MILC lattices, determining the form of mixed-action errors using chiral perturbation theory [50]. Results on the MILC0 data set should be available in 2007, and should have errors similar to those with the DWF1 ensemble.

  A second approach uses valence staggered fermions on the MILC lattices [51, 52]. It suffers from the extra systematics of taste-breaking, but gains from smaller CPU requirements. The main drawback is the need, at this stage, to use one-loop perturbative matching, which limits the accuracy to $\approx 10\%$ on the MILC0 ensemble. Two-loop, or non-perturbative, matching will be needed to make this method competitive.

- $f_{B_s}\sqrt{B_{B_s}}$.[10] B_{B_s} is defined in an analogous way to B_K, eq. (8). The combination $f_{B_s}\sqrt{B_{B_s}}$ appears, squared, in the $B_s - \overline{B}_s$ mass difference

$$\Delta M_s = \frac{G_F^2 M_W^2 M_{B_s}}{6\pi^2}\eta_c S(x_t)|V_{ts}V_{tb}|^2 f_{B_s}^2 B_{B_s}\,, \tag{11}$$

  where η_c is a perturbative coefficient known at NLO. Since ΔM_s is now well measured (at the 2% level) this provides an indirect determination of $|V_{ts}|$, which is predicted to high accuracy by the unitarity of the CKM matrix, $V_{ts} = -A\lambda^2 + \ldots$. That it provides a potentially stringent test of the SM is shown by the accuracy of the UTA prediction $f_{B_s}\sqrt{B_{B_s}} = 261 \pm 6\,\text{MeV}$ [10]. Thus it is very important to reduce the present error of $\pm 21\,\text{MeV}$ (7.5%).

  As indicated in Table 1, substantial progress is possible with future ensembles. The present calculation uses only (a subset of) coarse MILC lattices, and NRQCD for the b quark [13]. The next step is to use the MILC0 ensemble, including the fine lattices, which we estimate will reduce the error to $\approx 6\%$, or $\pm 16\,\text{MeV}$.[11] This should occur during 2007. The most important subsequent improvement would be the use of two-loop matching. Combining this with the reductions in other errors that should be possible by working on the MILC2 ensemble we estimate a total error of $\pm 9\,\text{MeV}$.[12] This brings the lattice error almost down to that of the present UTA result.

  To reduce errors below this level would likely require other methods, and, in particular, non-perturbative matching as part of the HQET expansion.

- $\xi = f_{B_s}\sqrt{B_{B_s}}/(f_B\sqrt{B_B})$. This appears, squared, in the ratio of mixing amplitudes for B and B_s mesons,

$$\frac{\Delta M_B}{\Delta M_{B_s}} = \frac{M_B}{M_{B_s}}\lambda^2\left[(1-\bar{\rho})^2 + \bar{\eta}^2\right]\frac{1}{\xi^2}\,. \tag{12}$$

  This ratio is now measured accurately. We have written the result in terms of the Wolfenstein parameters to emphasize that it directly constrains $\bar{\rho}$ and $\bar{\eta}$ (unlike ΔM_{B_s} itself). Present experimental precision is indicated by the UTA prediction of $\xi = 1.24 \pm 0.08$.

  The fact that ξ is a ratio leads to a cancellation of some systematic errors (e.g. scale errors), but this is counterbalanced by the need to do a chiral extrapolation in the light valence quark mass.

  The current lattice result, 1.23 ± 0.06, is in complete accord with the UTA result, but is based on quenched and partially unquenched results. First unquenched results, with all errors controlled, are expected in 2007 by extending the existing calculations of $f_{B_s}\sqrt{B_{B_s}}$ described above. The expectation is for precision of $\approx 3\%$ (± 0.04). This will be a very important step forward, allowing another fully controlled, precision test of the SM. The precision will improve with time on both the experimental and lattice sides—the lattice error will perhaps be reduced by a factor of 2 with the MILC2 ensemble.

[10] Note that, compared to older analyses, it is preferable to use $f_{B_s}\sqrt{B_{B_s}}$ rather than $f_B\sqrt{B_B}$, since the former has smaller chiral extrapolation uncertainties.

[11] This is obtained as follows. Present errors in the squared quantity $f_{B_s}^2 B_{B_s}$ are 9% (matching), 9% (statistics/fitting), 4% (discretization), 3% (relativistic corrections) and 5% (setting scale). Working at a finer lattice should reduce these, respectively, to about 8%, 6%, 2%, 3% and 4%, or 11.4% in total if added in quadrature. The smallest reduction is in the matching error which is due to the use of one-loop perturbation theory and is $\approx \alpha(1/a)^2$.

[12] The respective errors are estimated to be 3%, 4.5%, 1%, 3% and 2.5% in the squared quantity, or 6.7% in total.

- $(\Delta\Gamma/\Gamma)_{B_s}$. The relative width difference in the B_s-$\overline{B}_s$ system provides an another method with which to test the SM. Present experimental measurements give a first indication of a signal, $(\Delta\Gamma/\Gamma)_{B_s} = 0.35 + 0.12 - 0.16$ [7], and precision is expected to improve.

  Using the operator product and heavy quark expansions, this ratio can be written, at leading order, in terms of the matrix elements of local four-fermion $\Delta B = 2$ operators. The operators needed include that contributing to $B_s - \bar{B}_s$ mixing in the SM, and also others with different Dirac structure [53]. These can all be calculated using the same methods employed in the calculation of $f_{B_s}\sqrt{B_{B_s}}$. First unquenched results are now available on the coarse MILC lattices [13]. The precision attained for all matrix elements is comparable to that in $f_{B_s}\sqrt{B_{B_s}}$. The next step will be to repeat the calculation on the fine MILC lattices so as to control all errors.

  The corresponding ratio for B_d mesons is much smaller in the SM and is thus likely to be hard to measure. The present limit is $(\Delta\Gamma/\Gamma)_{B_d} = 0.009 \pm 0.037$ [7]. A lattice calculation of the relevant matrix elements should nevertheless be undertaken, and will proceed in parallel with that for the B_s mesons.

4.3 Four-fermion and other operators II: future directions.

Lattice calculations of the following quantities are at an earlier stage than those discussed above, although in some cases considerable work in the quenched approximation has been done. These quantities can both provide further tests of the SM and are needed to study the impact of BSM physics. For these matrix elements it is difficult to predict the resources needed for a given precision, and we give only general comments on the difficulty of the calculations. We stress that this list is not complete, but gives an indication of the breadth of current and future work.

- BSM $B - \overline{B}$ mixing. Flavor-changing neutral transitions are suppressed in the standard model, and thus provide a window into BSM physics. In particular, new physics could have a very small effect on the B decays used to determined the UT angles, while significantly changing the B mixing amplitudes. In this case the UTA prediction for, say, $f_B\sqrt{B_B}$ would be in conflict with the SM prediction using lattice matrix elements. In order to constrain the nature of the new physics in such a scenario, one will need the matrix elements of four-fermion operators with all possible Dirac and color structures (not just the LL structure of the SM). Calculations of the full set of matrix elements requires a straightforward extension of present work, and indeed three of the matrix elements have been already calculated in Ref. [13]. It is likely that the required accuracy can be obtained with the MILC1 ensemble.

- $D - \bar{D}$ mixing matrix elements. Neutral D-meson mixing is suppressed in the SM, and experiments have barely reached the sensitivity required to test the SM prediction. As above, this makes the mixing a good place to look for BSM physics. In general, the prediction of the mixing rate in BSM models requires the knowledge of a general set of four-fermion operators, as in the B system. Such a calculation is a straightfoward extension of present work, and should be able to attain comparable precision to that envisaged for the B system.

- BSM $K-\bar{K}$ mixing matrix elements. Similar comments apply to neutral K mixing as for B mixing: BSM physics, involving the matrix elements of different four-fermion operator, may lead to a discrepancy between the experimental and SM value for CP-violation in mixing. Thus, as for the B and D systems, a complete set of $\Delta S = 2$ matrix elements should calculated. This is a straightforward extension of the calculation of B_K; in fact, the other matrix elements are less constrained by chiral symmetry and should be easier to calculate precisely. Thus the DWF1 level should be sufficient.

- CP-conserving part of $\mathcal{A}(K \to \pi\pi)$. Calculation of the $K^{\pm}$ and K^0 decay amplitudes from first principles has turned out to be very challenging. They remain quantities of central interest because understanding the $\Delta I = 1/2$ rule observed in these decays would be a milestone for lattice calculations.

  The difficulties include power-divergent mixing of the four-fermion operators with lower-dimension operators, the fact that there are two particles in the final state, and the presence of quark-disconnected diagrams. In addition, the amplitudes are constrained by chiral symmetry. In the next few years it is likely that a calculation using $K \to 0$, $K \to \pi$ and $K \to \bar{K}$ amplitudes and leading order, or perhaps next-to-leading order, chiral perturbation theory will be possible. This method has been successful in quenched studies [54], and should extend straightforwardly to unquenched lattices. DWF appear essential because of their chiral symmetry. The drawback is that, particularly in the $\Delta I = 1/2$ case, the convergence of the chiral expansion may be poor.

  A direct calculation with the two-pion final state using the method of Lellouh and Lüscher [55] should also be possible for the $\Delta I = 3/2$ channel, based on the success of preliminary calculations [56]. It is not clear, however, whether it will be possible in the next 5 years to complete a full direct calculation in the $\Delta I = 1/2$ channel, to which quark-disconnected diagrams contribute.

- CP-violating part of $\mathcal{A}(K \to \pi\pi)$. These amplitudes are needed to test whether the SM can explain the measured value of ε'/ε. While similar to the calculation of the CP-conserving parts of the amplitudes, these calculations are more challenging because it is not possible to ameliorate power divergences by keeping a dynamical charm quark, and because more operators contribute, with possible cancellations. Nevertheless, we expect the calculations to progress only slightly behind those of the CP-conserving part.

- $\tau(\Lambda_b)/\tau(B_d)$ and other lifetime ratios of hadrons containing b-quarks. These differences are well measured and so can provide an important validation of our understanding of heavy-light systems. The difference of the experimental ratios from unity can be systematically studied using the heavy quark and operator product expansions. (A recent review is Ref. [57].) The matrix elements that are required are those of $\Delta B = 0$ operators between b-hadrons, and one needs to include both bilinears and four-fermion operators. The latter are hard to calculate as they involve mixing with lower-dimensional operators, similar to that occuring in $K \to \pi\pi$ amplitudes. Only partial calculations have been done so far, but we expect these calculation to progress at a similar rate to those for $K \to \pi\pi$ amplitudes using chiral perturbation theory (since the problem of quark-disconnected contractions does not arise in either case).

- $B \to (K^*, \rho, \omega)\gamma$ amplitudes. These decays are induced in the SM by penguin diagrams containing the top quark. They are potentially sensitive to BSM physics, which could contribute

at a level approaching that of the SM. There are by now accurate experimental results for $B \to K^*\gamma$, and first observations of $B \to (\rho, \omega)\gamma$. Thus it is important to determine the relevant hadronic matrix elements so as to test the SM.

The required form factor is of the tensor bilinear, e.g. $\bar{s}\sigma_{\mu\nu}d$ for $B \to K^*\gamma$. Thus the calculation is similar to that of the $B \to \pi\ell\nu$ form factor discussed above, but differs in the important respect that one must extrapolate to $q^2 = 0$. Because of this, and also from the conclusions of a recent detailed quenched study [58], we expect this calculation to lag behind that of the $B \to \pi\ell\nu$ form factor by a year or more.

- $B \to K\ell^+\ell^-$ form factors. The motivation here is similar to that for $B \to K^*\gamma$—this decay is sensitive to BSM physics, but the SM "background" must be known accurately. First measurements of the branching ratio are available, and the q^2 dependence of the form-factor should follow. The lattice calculation is easier than that for $B \to K^*\gamma$ because all values of q^2 are of interest. On the other hand, both the vector and tensor operator contribute. First, preliminary results have been recently presented [59], and we expect major progress in the next few years.

- Moving NRQCD. A drawback with present calculations of $B \to \pi$ and related semileptonic form-factors is that the lattice pion cannot have very large momentum due to $a\vec{p}$ errors and poor signal/noise. This restricts the range in q^2 accessible to lattice calculations. One idea to ameliorate this problem is to make the b-quark, and thus $B-$meson, move. This approach is in its infancy, but has considerable promise, as indicated by the first results [60].

- Neutron electric Dipole Moment (d_N). Experiments continue to lower the limit on this quantity, and the limit (or value, if it is subsequently observed) can be used to constrain the value of $\theta_{\rm QCD}$ in the SM, and also the nature of BSM physics.

  Methods to calculate the $\theta_{\rm QCD}$-induced d_N have been developed and tested in the quenched approximation [61]. Calculations using chirally symmetric fermions are preferred, and we expect that a continued effort using DWF will yeild a signal in the next few years.

  Supersymmetric extensions of the standard model lead to a potentially larger contribution to d_N resulting from operators of the form $\bar{q}\sigma_{\mu\nu}F^{\mu\nu}q$ and $\bar{q}\sigma_{\mu\nu}G^{\mu\nu}q$, where F and G are respectively the electromagnetic and gluon field strengths [62]. It would thus be interesting to calculate the nucleon matrix elements of these operators. To date, little work has been done in this direction, but we expect that calculations with moderate precision should be possible in the next few years.

- Proton decay matrix elements. Grand-unified models predict nucleon decay, with modes such as $p \to \pi^0 e^+$. Although to date no such decay has been observed, next generation detectors are under construction. To convert a measurement, or limit, in any given mode into a constraint on the parameters of the underlying model requires (as for the SM) hadronic matrix elements. The required matrix elements do not, in general, have quark-disconnected contributions, so that, in principle, these are straightforward extensions of mature calculations such as that of nucleon form-factors and B_K. The extra challenge compared to a form-factor calculation is the multi-fermion nature of the operator, while, compared to B_K, the final-state meson has significant momentum, and the inital state is a nucleon, both of which increase the noise. On the other hand, less precision is needed to make a significant impact, since model calculations vary in their predictions by a factor of ~ 10.

Quenched calculations have demonstrated that the needed methodology exists [63], and first unquenched calculations using DWF are beginning. We expect results with moderate precision by the DWF1 stage, and good precision at the DWF2 level.

- The nucleon matrix elements $\langle N|\bar{u}u + \bar{d}d|N\rangle$ and $\langle N|\bar{s}s|N\rangle$. These are important for BSM physics because they control the sensitivity of detectors to certain types of dark matter. (They are also of considerable interest to the hadronic physics community, being the simplest indicators of the strange content of the nucleons.)

  These matrix elements are difficult to calculate because they involve quark-disconnected contractions. Several attempts are underway at present using new methods and, in part, USQCD computational resources. Results with fairly poor precision ($\approx 30\%$) would be useful, since present phenomenological estimates vary by a factor of 3. We expect that 30% precision is attainable in the next few years, although not enough is known to give a more detailed prediction of precision.

- Hadronic contributions to $g-2$. There is a continuing uncertainty in the size of the hadronic "bubble" contribution to the muon $g-2$: data from e^+e^- production leads to results differing by about 3-σ from that using τ decays. It is possible that the lattice can help resolve this discrepancy, by directly calculating the appropriate integral over the Euclidean vector-vector correlator [64].

  A lattice calculation of the light-by-light contribution has also been considered [65], and could be developed further with increased resources.

5 Summary

In the last five years lattice QCD calculations have begun to fulfill their promise, with precision calculations of the simplest quantities in which all errors are controlled. This has been due to a combination of increased computer resources and signficant improvements in algorithms and actions.

The next 5 years will allow precision calculations of many more complicated quantities, providing several stringent constraints on the SM, and validation using at least two types of lattice fermions. In addition, if the SM fails, lattice calculations of matrix elements will help map out the parameters of BSM physics. We stress that the improved precision relies not only on increased computer resources but also improved theoretical calculations (matching, chiral perturbation theory) and on the further development of numerical methods (particularly for precision calculations of heavy quark matrix elements).

Acknowledgements: We thank Claude Bernard, and especially Stephen Sharpe for their contributions to this paper.

A CKM matrix and some notation

The elements of the CKM matrix are as follows,

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}. \tag{13}$$

In the SM there are three generations and V_{CKM} is unitary. A standard way to implement this, and at the same time to remove unphysical phases, is the Wolfenstein parameterization in terms of four real quantities λ, A, ρ and η [66] (shown here in its higher-order form [67]):

$$V_{CKM} = \begin{pmatrix} 1-\frac{\lambda^2}{2}-\frac{\lambda^4}{8} & \lambda & A\lambda^3(\rho-i\eta) \\ -\lambda+\frac{A^2\lambda^5}{2}[1-2(\rho+i\eta)] & 1-\frac{\lambda^2}{2}+\frac{\lambda^4}{8}(1+4A^2) & A\lambda^2 \\ A\lambda^3(1-\bar{\rho}-i\bar{\eta}) & -A\lambda^2+\frac{A\lambda^4}{2}(1-2\rho)-i\eta A\lambda^4 & 1-\frac{A^2\lambda^2}{2} \end{pmatrix} + O(\lambda^6), \tag{14}$$

where

$$\bar{\rho} = \rho(1-\lambda^2/2) \qquad \text{and} \qquad \bar{\eta} = \eta(1-\lambda^2/2). \tag{15}$$

CP violation is proportional to the imaginary part of V_{CKM}, i.e. to η. The least well known parameters are ρ and η, and it is common to show plots of the constraints in the $\bar{\rho}-\bar{\eta}$ plane. The position of $\bar{\rho}$ and $\bar{\eta}$ define the apex of a triangle, as shown in Fig. 3. The angles of this triangle have been given conventional names, as shown. Note that another naming convention is $\phi_1 = \beta$, $\phi_2 = \alpha$ and $\phi_3 = \gamma$.

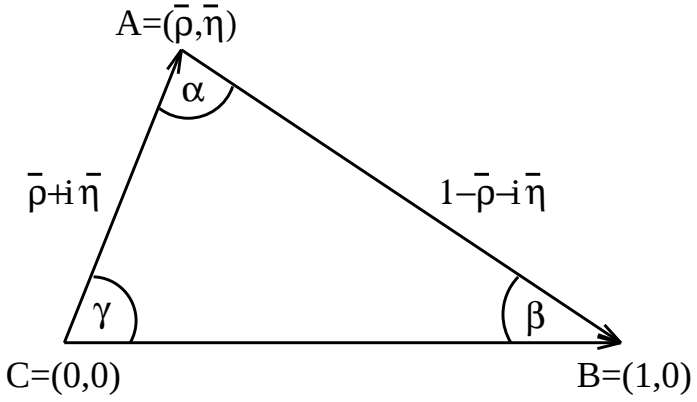

Figure 3: Unitarity triangle, giving definitions of the angles α, β and γ.

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